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A *metric graph* is a graph in which each edge has a length $L \in (0, \inf)$. A *quantum graph* is a metric graph together with a Hamiltonian operator and appropriate vertex condition. The most studied Hamiltonian operators on quantum graphs are the negative second derivative $f(x) \mapsto -\frac{d^2f}{dx^2}$ and the Schrodinger operator $f(x) \mapsto -\frac{d^2f}{dx^2} + V(x)f(x)$. These graphs can be used to model the behavior of particles restricted to a network, and have been used in recent years to study the electrical properties of systems such as carbon nanostructures.

One major question that arises is what boundary conditions can be applied to the graph, how these boundary conditions are prescribed by the physics of the system, and what effect they have on the development of the physical system. A complete characterization of the permissible boundary conditions (i.e. those that make the Hamiltonian operator self-adjoint) has been proved. I am studying the relation between this result and the physical constraints of the system, in an effort to understand what physical properties rule out certain boundary conditions. Other questions I am looking into are how these conditions affect the behavior of the system and what boundary conditions can be practically enforced in an actual physical system.

There are several possible directions this research may lead. One idea is to examine the the Cayley graph of a finite group as a quantum graph and looks for connections between the quantum graph structure and the group structure. Another is to examine random walk on quantum graphs and their possible relations to scattering theory.

References:

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