ARTU Spring 2014 Report *

Philip Bonneville

April 16, 2014

In preparation for research in some area of algebraic topology (likely the theory of covering spaces) under the guidance of Dr. Shareshian, I have been primarily reading through [4]. In addition, I have consulted [1, 2, 3, 5, 6, 7]. At present, I am studying the problem of how the types of groups that are realized as monodromy groups of coverings of Riemann surfaces depend upon the restrictions placed on the genus, with the hope that either extending this question to positive characteristic or applying existing results to special cases of the Inverse Galois Problem may serve as a situable research project for the summer.

Given topological spaces E and B and a surjective map $p\colon E \to B$, we say that p is a cover of the base space B by E if, for each point $b \in B$, there exists an open neighborhood O of B whose inverse image under p consists entirely of components that are each mapped homeomorphically onto O by p [4]. This definition yields many nice properties for coverings, especially in relation to uniqueness of liftings of paths, that can be abstracted to the case of covers of groupoids as done in [4]. One important fact about any covering of groupoids (and thus any covering of spaces) is that over all b in the base groupoid B, the fiber $F_b = p^{-1}(b)$ is fixed in size.

When the spaces in question are real manifolds, we are able to speak of the universal cover. This leads, for instance, to a bijective correspondence between "isomorphism classes of connected coverings" and "conjugacy classes of subgroups" [5]. In addition, we may define the monodromy representation of a covering map of finite degree by taking each element $[f]$ of the fundamental group $\pi_1(B, b)$ at a fixed point in the base space to the permutation it defines on F_b through the liftings of the conjugacy class $[f]$ to E; that is, the element of the symmetric group on F_b associated with $[f]$ takes $e \in F_b$ to the unique element $e' \in F_b$ at which every path in E starting at e and taken into the class $[f]$ by p, ends [5].

However, in the case of a Riemann surface and a holomorphic map, we do not necessarily have a covering of spaces due to ramification [5]. In particular, the size of the fiber of a point in the image might not always be the same for a given holomorphic map. Monodromy representations can, however, still be defined, though in a slightly different sense. In fact, the associated monodromy group has deep implications for the structure of the Riemann surfaces in question. My next step is to study how the abstract approach of [4] may be generalized to this situation.

References

[1] Askold Khovanskii. *Galois Thoery, Coverings, and Riemann Surfaces*. Springer, 2013.

*This material is based upon work supported by the National Science Foundation under agreement No. DMS-1055897. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

[2] Dmitry Kozlov. *Combinatorial Algebraic Topology*, volume 21 of *Algorithms and Computation in Mathematics*. Springer, 2008.

[3] Jiří Matoušek. *Using the Borsuk-Ulam Thoerem*. Springer, 2003.

[4] J. P. May. *A Concise Course in Algebraic Topology*. Chicago Lectures in Mathematics Series. University of Chicago Press, 1999.

[5] Rick Miranda. *Algebraic Curves and Riemann Surfaces*, volume 5 of *Graduate Studies in Mathematics*. American Mathematical Society, 1995.

[6] Jean-Pierre Serre. *Trees*. Springer-Verlag, 1980.

[7] Helmut Völklein. *Groups as Galois Groups*, volume 53 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 1996.