Liquidity Crises and Discount Window Lending: Theory and Implications for the Dollarization Debate*

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Abstract

We study the consequences of a central bank providing an elastic currency through the use of discount window lending. In particular, we compare the set of equilibria generated when the interest rate is fixed in nominal terms with that generated when it is fixed in real terms. The two policies generate the same steady state equilibrium. However, fixing the nominal interest rate always generates additional, inflationary equilibria while the while fixing the real rate never does, regardless of the rate chosen. We argue that dollarization can be viewed as a mechanism for committing to having a fixed real interest rate on short-term credit, and discuss some implications of this analysis for the current debate in Mexico.

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1. Introduction

Among the many issues that arise in the discussion of optimal monetary arrangements, the availability of short-term credit to the banking system in times of high liquidity demand stands out in our minds as particularly important. Monetary history is full of instances in which a lack of liquidity has led to crises and to distortions of real allocations that are believed to have been at least somewhat avoidable.[1] These occurrences suggest that avoiding crises requires the currency supply to be at least somewhat *elastic* with respect to currency demand. In most modern economies, the ability to issue money is granted exclusively to a central bank, and with this comes the responsibility of adjusting the money supply in response to transitory changes in liquidity demand. This responsibility was important enough to merit high billing in the act establishing the Federal Reserve System in the United States, “An act to provide for the establishment of Federal Reserve Banks, to furnish an elastic currency, . . . and for other purposes.” Similarly, providing liquidity in an effort to avert crises is typically believed to be a critical function of Banco de México.[2]

One way a central bank can make the money supply elastic is through the use of short-term credit, or discount window lending. In this paper, we focus on whether the interest rate on short-term credit should be fixed in nominal or in real terms. To address this issue, we draw on the existing elastic-currency literature. Beginning with Sargent and Wallace [15] , papers including Champ, Smith, and Williamson [6] , Williamson [18] , and Freeman [8] , [9] have employed formal economic models to examine the effects of having an elastic currency supply. These papers focus on stationary equilibria and show how an elastic currency can promote a more efficient allocation

1 See, for instance, the examples described in Freeman [8] and Champ, Smith, and Williamson [6] . Additional historical information on financial crises can be found in Kindleberger [12] .

2 See, for example, the discussion in O’Dogherty [14] .

of resources in these equilibria. This provides a rigorous theoretical justification for the claim that having an elastic currency is important for achieving an efficient allocation of resources. In the stationary equilibria studied, however, real and nominal interest rates are the same and hence our issue of interest does not arise. A recent paper by Antinolfi, Huybens, and Keister [2] allows for nonstationary equilibria and demonstrates that the short-term lending used to make the currency elastic can be destabilizing in the sense that it can introduce inflationary equilibria.[3] In this paper and in its predecessors cited above, the (explicit or implicit) interest rate on short-term credit is fixed at zero, since this rate allows the economy to achieve a Pareto optimal equilibrium allocation. In the present paper, we generalize the results of Antinolfi, Huybens, and Keister [2] by allowing the central bank to set arbitrary non-negative interest rates. By allowing short-term credit to be costly, we make the model applicable to a wider range of economic situations, including dollarization (which we discuss in detail below). Fixing the interest rate in nominal terms may seem the more natural approach in this environment, since it generates non-negative revenue for the central bank regardless of how the price level moves through time. However, we show that this policy can be destabilizing. Specifically, we show that fixing the nominal interest rate generates inflationary equilibria regardless of the level at which the rate is set. In contrast, fixing the real interest rate rules out inflationary equilibria, again regardless of the level at which the rate is set.

The discussion above assumes that the monetary authority has the ability to print currency and put it into circulation. However, proposals have recently been made in a number of countries, including Mexico, to either establish a currency board or abolish the national currency altogether and dollarize. While adopting such policies may eliminate the current or potential problems caused

[3] See also Smith and Weber [16] on this issue, which shows how having an elastic currency generated by unrestricted private banknote issue can lead to indeterminacy of equilibrium.

by speculation against the domestic currency, they severely limit the ability of the central bank to provide liquidity in times of high demand. Eichengreen [7] cites this as a primary concern for Mexico, saying that the banking system must be strengthened before dollarization should be considered, so that it is able to withstand movements in liquidity demand without central bank intervention. We argue that the model we employ below provides a useful framework for analyzing these issues. When the central bank cannot print money, the economy loses the ability to provide short-term credit at a zero (nominal) interest rate. In times of high liquidity demand, however, the economy will still be able to borrow from abroad at some positive (real) interest rate. This could be achieved, for example, through the issue of public debt, with the proceeds being lent to the banking sector (as noted by Calvo [5] , this is how advanced countries typically fund large lender-of-last-resort operations). We therefore argue that dollarization can be seen as a mechanism for committing to having an interest rate on short-term credit that is fixed in real terms. Such a commitment obviously does not come without cost, but our analysis shows that the effect on the banking sector is not necessarily negative. In particular, the model predicts that liquidity crises would be less frequent under dollarization. The increased cost of obtaining short-term credit leads banks to hold “extra” liquidity for insurance purposes, which in turn makes the occurrence of a liquidity crisis less likely.

The remainder of the paper is organized as follows. In the next section, we describe the model that we employ, which is taken from Champ, Smith, and Williamson [6] and Antinolfi, Huybens, and Keister [2] . We review the physical environment and the behavior of both consumers and banks. We then analyze the bank’s portfolio allocation problem in a general structure that allows for both fixed nominal and fixed real interest rates. In section 3, we analyze equilibrium in the economy with a fixed nominal interest rate on discount window loans. We show that there exist inflationary equilibria regardless of the interest rate chosen. In section 4, we turn to equilibrium in the economy with a fixed real interest rate. We show that in this case there are no inflationary equilibria, again regardless of the rate chosen. Finally, in section 5 we discuss some implications of the model for the dollarization debate in Mexico.

2. The Model

In this section we present the Champ, Smith, and Williamson [6] model[4] with the modification that banks can borrow from the discount window at some interest rate r_t. We describe the physical environment and the solutions to the consumers' problems. The bulk of the section is devoted to deriving the solution to the bank's problem in this environment. These results are then used in the analysis of equilibrium in the sections that follow.

2.1 The Physical Environment

The economy consists of an infinite sequence of overlapping generations of consumers, each of which lives for two periods. There is a single, non-storable consumption good in each period. In each period, a continuum of agents with unit mass is born at each of two identical locations. Half of these agents have endowments $(\omega_1, \omega_2) = (x, 0)$ and are called "lenders," while the remaining half have endowments $(\omega_1, \omega_2) = (0, y)$ and are called "borrowers." All consumers have $\mathbb{R}^2_{++}$ as their consumption set and have preferences given by $u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$. It is assumed that $y < \beta x$ holds, which implies that this is a Samuelson-case economy (see Gale [10]) and hence there is a role for money as a store of value. At $t = 0$ there is a continuum of old agents with unit

4 This is actually a simplified version of the model, where seasonal effects have been removed.

mass in each location, each of whom is endowed with $M > 0$ units of fiat money.

In addition to the store of value role for money, there is a transactions role for money generated by spatial separation and limited communication.[5] To simplify matters, we assume that all transactions must be intermediated by a bank.[6] At the beginning of each period, all agents receive their endowments. At this point, agents (and banks) cannot move between or communicate across locations, and therefore each agent trades with a bank in her own location. Young lenders deposit their savings in a bank, then young borrowers contact a bank and obtain a loan. The bank also trades with old agents in order to achieve the desired allocation of cash in its portfolio. At this point, all agents consume. Next, a fraction π_t of young lenders in each location is notified that they will be moved to the other location. Lenders who will be relocated are called "movers," while the remaining lenders are called "non-movers." Goods can never be transported between locations. Limited communication prevents privately-issued liabilities, such as checks, from being verifiable in the other location. Currency, on the other hand, is universally recognizable and non-counterfeitable, and is therefore accepted in both locations. Movers are able to contact their bank and withdraw currency. Immediately afterwards, the movers are relocated and the next period begins. Agents now receive their old-age endowments, and borrowers use part of this endowment to repay their loans. With this revenue, banks make repayments to non-movers. Movers use the currency they received from the bank to buy consumption in their new location, and at this point all old agents consume and end their lifecycle. Notice that the old-age consumption of a mover will always be equal to the real value of the money that she takes with her to the new location.[7]

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[5] This is as in Townsend [17] , Mitsui and Watanabe [13] , and Hornstein and Krusell [11] .

[6] This strong assumption is not necessary, although some restrictions on who can trade with whom are required. See the discussion in Champ, Smith, and Williamson [6] .

[7] Since the consumption set is $\mathbb{R}^2_{++}$, this eliminates by assumption the non-monetary equilibrium that typically exists in overlapping generations models (see, for example, Balasko and Shell [4]).

The relocation probability π_t is a random variable in each period. Since there is a continuum of lenders, it represents both the probability of relocation for each lender and the fraction of all lenders who move. That is, π_t gives the size of the aggregate liquidity shock; high values of π_t correspond to high liquidity demand and low values to low liquidity demand. It has support $[0, 1)$ and is drawn from the twice continuously differentiable, strictly increasing distribution function F with associated density function f. It is independently and identically distributed over time.

2.2 Consumers

Borrowers face a gross market interest rate of R_t. They choose their quantity of borrowing ℓ_t to maximize utility, that is, to solve the problem

$$\max_{\ell_t} \ \ln\left(\ell_t\right) + \beta \ln\left(y - R_t \ell_t\right).$$

The demand for borrowing is therefore given by

$$\ell_t = \frac{y}{\left(1+\beta\right) R_t}. \tag{1}$$

Lenders deposit all of their savings in a bank. The return they receive on this saving depends on both whether or not they move and what fraction of all young lenders move. Specifically, they are promised a real return $r_t^m\left(\pi\right)$ if they move and $r_t^n\left(\pi\right)$ if they do not move. Lenders then choose the amount they save d_t to maximize expected utility, that is, to solve

$$\max_{d_t} \ \ln\left(x - d_t\right) + \beta \int_0^1 \pi \ln\left[r_t^m\left(\pi\right) d_t\right] f\left(\pi\right) d\pi + \beta \int_0^1 \left(1-\pi\right) \ln\left[r_t^n\left(\pi\right) d_t\right] f\left(\pi\right) d\pi.$$

The solution to this problem sets

$$d_t = d = \frac{\beta}{1+\beta} x. \tag{2}$$

Notice that the amount of saving is independent of the distribution of the rates of return. This result clearly depends on the assumptions of log utility and no old-age income for lenders, which imply that the income and substitution effects of a change in the rate of return exactly offset each other. These assumptions allows us to obtain an explicit expression defining the solution to the bank's problem below.

2.3 Banks

Banks serve two important functions in this setup: they provide intermediation between borrowers and lenders and they provide lenders with insurance against the relocation shock. It is assumed that banks behave competitively in the sense that they (i) take the real return on assets as given and (ii) choose the deposit return schedules $r_t^m(\pi)$ and $r_t^n(\pi)$ to maximize the expected utility of young lenders. A young lender deposits her entire savings d with a bank. Per unit of deposits, the bank acquires an amount γ_t of real money balances, and makes loans with a real value $1-\gamma_t$. In addition, the bank borrows a non-negative amount δ_t from the discount window. The bank faces two constraints on the return schedules it can offer. First, relocated agents must be given currency, since that is the only asset which will allow these agents to consume in their new location. This currency comes from the bank's reserve holdings and from the discount window. Let $\alpha_t(\pi)$ denote the fraction of the bank's reserves that are given to movers and p_t the general price level at time t. Then the return to holding money between time t and $t+1$ is given by $\frac{p_t}{p_{t+1}}$ and the constraint

$$\pi r_t^m(\pi) = \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi)\frac{p_t}{p_{t+1}} \tag{3}$$

must hold. The second constraint is that payments to non-movers cannot exceed the value of the bank's remaining portfolio – remaining reserves plus the returns from the bank's lending minus the repayment of the discount window loan. Since the bank's loans earn the gross real rate of return R_t, this constraint can be written as

$$(1-\pi)\, r_t^n(\pi) = [1-\alpha_t(\pi)]\,\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)\, R_t - \delta_t(\pi)\,(1+r_t)\,\frac{p_t}{p_{t+1}}, \tag{4}$$

where $r_t > 0$ is the nominal interest rate on discount window loans at time t.[8] In this section we are only interested in solving the bank's problem at a point in time, so we can treat the process r_t as being arbitrary. In Section 3, we will assume that r_t is constant through time, that is, that the central bank fixes the nominal interest rate on discount window loans. In Section 4, we will assume that r_t is set by a rule that makes the real interest rate constant through time.

Banks maximize a typical young lender's expected utility subject to these constraints. Given (2), the bank's problem is to choose $r_t^m(\pi)$ and $r_t^n(\pi)$ to maximize

$$\ln\left(\frac{x}{1+\beta}\right) + \beta \int_0^1 \left(\pi \ln\left[r_t^m(\pi)\,\frac{\beta x}{1+\beta}\right] + (1-\pi)\ln\left[r_t^n(\pi)\,\frac{\beta x}{1+\beta}\right]\right) f(\pi)\, d\pi$$

subject to the constraints (3) and (4), which will hold with equality at an optimum. Substituting in these constraints and dropping the constant terms yields the problem

$$\max_{\alpha_t(\pi),\,\delta_t(\pi),\,\gamma_t} \quad \int_0^1 \pi \ln[\alpha_t(\pi)\,\gamma_t + \delta_t(\pi)]\, f(\pi)\, d\pi \;+$$
$$\int_0^1 (1-\pi)\ln\left[[1-\alpha_t(\pi)]\,\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)\, R_t - \delta_t(\pi)\,(1+r_t)\,\frac{p_t}{p_{t+1}}\right] f(\pi)\, d\pi.$$

subject to

$$0 \le \gamma_t \le 1$$

$$0 \le \alpha_t(\pi) \le 1$$

$$\delta_t(\pi) \ge 0.$$

[8] If r_t were equal to zero, the results in Antinolfi, Huybens, and Keister [2] would apply. That case involves an indeterminacy in the bank's borrowing rule which complicates the exposition, and therefore we do not present it here. Nevertheless, the equilibrium results for that case can be obtained from ours by taking the limit as r_t approaches zero.

Both the fraction of bank reserves paid out to movers α_t and the real amount of borrowing δ_t are chosen after the realization of π, while γ_t, the fraction of reserves in the bank's asset portfolio, is chosen before the realization of π. Hence we can first solve for the optimal values of α_t and δ_t as functions of γ and π. That is, we can choose α_t and δ_t to solve the problem

$$\max_{\alpha_t,\delta_t} \quad \pi \ln\left[\alpha_t\gamma_t + \delta_t\right] + (1-\pi)\ln\left[(1-\alpha_t)\,\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)\,R_t - \delta_t\,(1+r_t)\,\frac{p_t}{p_{t+1}}\right]$$

subject to

$$0 \leq \alpha_t \leq 1$$

$$\delta_t \geq 0.$$

The solution to this problem sets

$$\alpha_t(\pi) = \left\{ \begin{array}{c} \pi\left(1+\frac{1-\gamma_t}{\gamma_t}R_t\frac{p_{t+1}}{p_t}\right) \\ 1 \\ 1 \end{array} \right\} \text{ and } \delta_t(\pi) = \left\{ \begin{array}{c} 0 \\ 0 \\ \gamma_t\left[\pi\left(1+\frac{1-\gamma_t}{\gamma_t}\left(\frac{R_t}{1+r_t}\right)\frac{p_{t+1}}{p_t}\right)-1\right] \end{array} \right\} \tag{5}$$

$$\text{for } \pi \in \left\{ \begin{array}{c} [0,\pi^*) \\ [\pi^*,\pi^{**}) \\ [\pi^{**},1) \end{array} \right\},$$

where we have

$$\pi^* = \frac{\gamma_t\frac{p_t}{p_{t+1}}}{\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)\,R_t} \tag{6}$$

and

$$\pi^{**} = \frac{\gamma_t\frac{p_t}{p_{t+1}}}{\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)\,\frac{R_t}{1+r_t}}. \tag{7}$$

When demand for liquidity is low (the relocation shock is below a critical value π^*), the bank is able to give movers and non-movers the same return by paying out only a fraction of its reserves to movers. Since the bank wants to provide lenders with insurance against the relocation shock, this is the optimal thing to do. When the realization of the relocation shock is greater than π^*, however, this is no longer feasible. In this case, there are so many movers that even if all of the bank's cash reserves are given to them, they will receive a lower return than the (relatively few) non-movers. Following Champ, Smith, and Williamson [6] , we call such an event a *liquidity crisis*. In a crisis, the bank has an incentive to borrow currency from the discount window in order to transfer resources from non-movers to movers. However, such borrowing is costly and, as a result, the bank waits until the number of movers is above a second critical level π^{**} before obtaining a loan from the discount window.

Some intuition for the range of inaction $[\pi^{*}, \pi^{**}]$ can be gained from looking at figure 1. The horizontal axis measures total resources given to movers, while the vertical axis measures total resources given to non-movers (both per unit of deposits). The figure presents the bank's set of feasible alternatives in this space once γ_t has been chosen. One action that is always feasible is to give all cash reserves (which will be worth $\gamma_t \frac{p_t}{p_{t+1}}$ next period) to movers and the returns from all real lending (worth $(1-\gamma_t)\, R_t$) to non-movers. This reference point is labelled on both axes in the figure. If instead the bank wants to give fewer resources to movers and more to non-movers (perhaps because there are very few movers this period), it can do so on a one-for-one basis. That is, for every unit of future consumption (in the form of currency) that is taken away from movers, exactly one unit is given to non-movers. For this reason, the slope of the boundary is (-1) above the reference point. Now suppose that instead the bank wants to give more resources to movers and fewer to non-movers. In this case the bank must obtain a loan from the discount window, so that for every unit of additional consumption given to movers, non-movers must give up $(1 + r_t)$ units. This change in the rates of substitution is what leads to the kink in the constraint set presented in the figure.

The solution presented in (5) gives (implicitly) the bank's optimal choice from this set for each value of π. When there are very few movers, the optimal action is to give almost all of the resources to non-movers and hence choose a point near the vertical axis. As we examine larger and larger realizations of π, the optimal point moves to the southeast along this boundary. That is, as more lenders are relocated, it is optimal to give more total resources to movers as a group. At $\pi = \pi^*$, the optimal action reaches the kink in the constraint set. This point remains the optimal choice for a range of values of π; only when the realization is greater than π^{**} is it optimal to move to the steeper-sloped part of the boundary. Hence this kink in the bank's opportunity set generates the range of inaction $[\pi^*, \pi^{**}]$ in the optimal levels of α_t and δ_t. In conjunction with equation (7), this also demonstrates how the interest rate r_t determines the potential severity of crises. The more expensive borrowing is, the larger π_t must be (and therefore the larger the gap between the returns of movers and non-movers will be) before a bank resorts to a loan to ease the crisis.

We now proceed to solve for the optimal value of γ_t. To do so, we substitute the optimal values of α_t and δ_t into the bank's objective function so that the only remaining variable to be determined is γ_t. This generates the problem

$$\begin{aligned}\max_{0\leq\gamma_t\leq 1} \quad & \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi \\ & + \int_{\pi^*}^{\pi^{**}} \pi \ln(\gamma_t) + (1-\pi)\ln((1-\gamma_t) R_t) f(\pi)\, d\pi \\ & + \int_{\pi^{**}}^{1} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)\frac{R_t}{1+r_t}\right] f(\pi)\, d\pi\end{aligned}$$

Because borrowing is costly, the solution to this problem will always be interior. The first-order condition is given by

$$\frac{R_t - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t} F(\pi^*) + \frac{\frac{R_t}{1+r_t} - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) \frac{R_t}{1+r_t}} \left[1 - F(\pi^{**})\right]$$

$$= \frac{1}{\gamma_t} \int_{\pi^*}^{\pi^{**}} \pi f(\pi) \, d\pi - \frac{1}{1-\gamma_t} \int_{\pi^*}^{\pi^{**}} (1-\pi) f(\pi) \, d\pi,$$

which can be reduced to

$$\gamma_t = \pi^{**} - \int_{\pi^*}^{\pi^{**}} F(\pi) \, d\pi. \tag{8}$$

This implicitly defines the solution to the bank's portfolio allocation problem (recall that γ_t appears in the expressions for π^* and π^{**} above). In the next two sections we use this solution to study equilibrium under two different policies: a fixed nominal interest rate and a fixed real interest rate.

3. Fixing the Nominal Interest Rate

We first analyze the regime in which the central bank fixes the interest rate on short-term credit in nominal terms, that is, sets $r_t = r$ for all t. This policy has the nice feature that it is always feasible; it requires no real resources from the central bank no matter how the price level changes through time. If a bank demands a loan of δ (in real terms, per unit of deposits), it goes to the discount window and receives δp_t pesos. In the following period, the bank must pay back $\delta p_t (1+r)$ pesos. We assume that the central bank destroys δp_t of these pesos and uses the remaining $\delta r p_t$ to purchase goods. In this way, the stock of currency in circulation remains fixed at M. We assume that agents derive no utility from the revenue earned by the central bank. If instead the revenue were rebated to banks as a state-contingent, lump-sum payment, the qualitative properties of the results would not change. Such rebates complicate the derivations substantially, so we present the simpler case here. In addition, having the revenue exit the economy will be the appropriate assumption in Section 5, where we apply the model to the issue of dollarization.

The only change to the analysis of the bank's problem above is that the time subscript is dropped from interest rate. An equilibrium of this economy is characterized by the market clearing conditions for real balances and loans. Since the supply of real balances is equal to $\frac{M}{p_t}$ and the demand for real balances is given by $\gamma_t d$, market clearing for real balances and (2) require that we have

$$\frac{M}{p_t} = \gamma_t \frac{\beta}{1+\beta} x.$$

Similarly, the demand for loans is given in (1), while the supply of loans is given by $(1-\gamma_t)\, d$. Together these yield the market clearing condition for loans,

$$\frac{y}{(1+\beta)\, R_t} = (1-\gamma_t) \frac{\beta}{1+\beta} x.$$

These equations imply that in equilibrium we must have both

$$\gamma_t \frac{p_t}{p_{t+1}} = \gamma_{t+1} \tag{9}$$

and

$$R_t (1-\gamma_t) = \frac{y}{\beta x} \equiv \psi. \tag{10}$$

We define the parameter ψ to save on notation in what follows. Recall that the assumption that our economy falls in the Samuelson case implies that ψ is less than unity. Substituting (9) and (10) into the expressions in (6) and (7) yields

$$\pi^* = \frac{\gamma_{t+1}}{\gamma_{t+1} + \psi} \quad \text{and} \quad \pi^{**} = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{\psi}{1+r}} \tag{11}$$

Substituting these into (8), we obtain an implicit difference equation for γ_t,

$$\gamma_t = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{\psi}{1+r}} - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1}+\psi}}^{\frac{\gamma_{t+1}}{\gamma_{t+1}+\frac{\psi}{1+r}}} F(\pi)\, d\pi. \tag{12}$$

The following proposition gives the properties of equilibrium that follow from this equation. This is a generalization of Proposition 2 in Antinolfi, Huybens, and Keister [2] , which covers the case of r equal to zero.

Proposition 1 When the interest rate on discount window loans is fixed in nominal terms, there is a continuum of equilibria. One of these equilibria is stationary, with $\gamma_t = \gamma^*$ for all t, where we have $\gamma^* \in \left[1-\psi, 1-\frac{\psi}{(1+r)}\right]$. All other equilibria exhibit sustained inflation with γ_t asymptotically approaching zero.

The proof of Proposition 1 is presented in Appendix A. The graph of the implicit function defined in (12) is presented in figure 2. The curve crosses the 45-degree line exactly once; this corresponds to the unique steady-state equilibrium.

The interesting feature of this law of motion is that it generates inflationary equilibria. In these equilibria, the real value of the stock of base money goes to zero, and therefore bank reserve holdings must also go to zero. Because of the logarithmic preferences and the fact that movers need money in order to consume, there is a lower bound below which the total real demand for money (reserves plus short-term credit) in this model never falls. The only way reserve holdings can go to zero is if they are being replaced by an expanding stock of short-term credit. Hence, inflation is necessarily accompanied by an expansion of the real value of discount window lending in this model.

As reserve holdings decrease, real lending increases and therefore the market interest rate R_t falls. In the long run, this rate approaches ψ. At the same time, the rate of return to holding money

$\frac{p_t}{p_{t+1}}$ approaches $\frac{\psi}{1+r}$ (this follows from (9) and (16)). These two facts combine to show that the long-run nominal interest rate is given by $(1+r)$, which is exactly the rate at which the discount window is lending. The fact that the central bank is lending at the market rate of interest in the long run is crucial for understanding why inflationary equilibria arise here. In the stationary equilibrium, the central bank is charging a “penalty rate” at the discount window. The interest rate on discount window loans is higher than the market rate, so that a bank whose reserve holdings turn out to be too low (relative to the realization of π_t) is penalized. This gives banks an incentive to hold precautionary reserves, and thereby generates a strong demand for money. This is initially true in an inflationary equilibrium as well. As the inflation rate increases, however, the market nominal interest rate is rising and therefore this penalty is decreasing. As the penalty falls, the strong incentive for banks to hold reserves falls with it. Asymptotically, banks are indifferent between holding reserves and borrowing, since the discount window is charging exactly the market rate. This undermines the strong demand for base money in the model and thereby permits inflation as an equilibrium outcome.

In the next section, we suppose instead that the interest rate on discount window loans is fixed in real terms. We show that the structure of the penalty faced by banks when their reserves turn out to be too low is substantially different in this case, and that for this reason there are no inflationary equilibria.

4. Fixing the Real Interest Rate

For the interest rate on short-term credit to be fixed in *real* terms, we must have

$$1 + r_t = \frac{p_{t+1}}{p_t}(1 + r)$$

for every t, where r is the (fixed) real interest rate. One can think of this case as being implemented in either of two ways. The first is straightforward: the interest rate at the discount window is posted in real terms and is always equal to r. Another implementation is to post the interest rate in nominal terms, but adjust it each period so that the implied real rate is always equal to r.

As before, a bank observes the realization of π and determines the real amount $\delta \geq 0$ that it would like to borrow . It then obtains δp_t pesos from the discount window, and in the next period it must return $(1+r)\,\delta p_{t+1}$ pesos. We again assume that the central bank engages in purchases or sales of goods so that the stock of money remains unchanged at M. We will show below that in equilibrium, the central bank will always earn a positive amount of revenue with this policy.[9]

In solving the bank's problem above, we assumed that nominal interest rate r_t is positive. In the present setting, this implies that we must have

$$\frac{p_t}{p_{t+1}} < 1 + r, \tag{13}$$

or that the return to holding money is not too high. In other words, the solution to the bank's problem given in section 2 does not apply in the face of a very strong deflation. If the inequality in (13) were reversed, the bank would have to repay fewer pesos than it borrowed. Since pesos can be costlessly stored, demand for borrowing would be unbounded and the bank's problem would have no solution.

When (13) holds, the bank's optimal portfolio is still given by (8) and π^* is still given by (6). The expression for π^{**} is now given by

$$\pi^{**} = \frac{\gamma_t}{\gamma_t + (1-\gamma_t)\frac{R_t}{1+r}}. \tag{14}$$

[9] If there were a strong enough deflation, the central bank would earn negative revenue and need a way to finance this policy. However, deflation will not be an equilibrium outcome in this model regardless of the finance method used.

The market-clearing equations are the same as in Section 3, and hence (9) and (10) continue to hold. We proceed by adding the law of motion for this case to the phase diagram in figure 2. First, note that in the region of the phase plane where $\gamma_{t+1} > (1+r)\,\gamma_t$ holds, (13) is violated and the bank's problem has no solution. Therefore the equilibrium law of motion is not defined in this region. For the remainder of the plane, we substitute (9) and (10) into the expression for π^* in (6) and the expression for π^{**} in (14) to get

$$\pi^* = \frac{\gamma_{t+1}}{\gamma_{t+1} + \psi} \qquad \text{and} \qquad \pi^{**} = \frac{\gamma_t}{\gamma_t + \frac{\psi}{1+r}}.$$

Substituting these into (8), we obtain the graph of the law of motion for γ_t that applies in this lower region,

$$\gamma_t = \frac{\gamma_t}{\gamma_t + \frac{\psi}{1+r}} - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1}+\psi}}^{\frac{\gamma_t}{\gamma_t+\frac{\psi}{1+r}}} F(\pi)\, d\pi. \tag{15}$$

The properties of this equation give us the following proposition.[10]

Proposition 2 When the interest rate on discount window loans is fixed in real terms, there is a unique equilibrium. This equilibrium is stationary, with $\gamma_t = \gamma^* \in \left[1-\psi, 1-\frac{\psi}{(1+r)}\right]$ for all t.

The proof of Proposition 2 is presented in Appendix B. Notice that the unique equilibrium in this case is identical to the stationary equilibrium discussed in Proposition 1. Hence fixing the real interest rate "selects" the steady state by making it the unique equilibrium of the economy.

In the previous section, we saw that inflationary equilibria exist because, during an inflation, the penalty that a bank pays when its reserves turn out to be too low falls to zero. In other words, in the long run the discount window in that case is lending at exactly the market rate of interest.

[10] This is a generalization of Proposition 4 in Antinolfi, Huybens, and Keister [2] , which covers the case of r equal to zero. That paper handles the problem of deflation differently, by having the lender of last resort switch to a fixed nominal rate policy so that feasibility is maintained. Because inflation is not an equilibrium outcome in either case, both approaches yield the same result.

With a fixed real interest rate on discount window loans, that does not happen. In this case, if there were an inflation the real return on lending would fall to ψ, exactly as before. However, the real rate on discount window loans remains fixed at $(1 + r)$. Therefore, the real penalty faced by a bank that goes to the discount window would *increase* over time. This generates a lower bound on the demand for reserves (and hence for base money) that is absent in the previous case. To see why, suppose that the economy were to follow an inflationary trajectory, where γ_t asymptotically approaches zero. When γ has become very close to zero, π^* and π^{**} are very small. This implies that in practically every period, the bank will be borrowing currency at a cost of $(1 + r)$, while its loans are earning a rate slightly above $\psi < 1$. Hence, regardless of the rate of return on money, the bank would be better off holding more reserves and engaging in less lending, simply because borrowing is so expensive. This means there is a lower bound on the demand for base money, even as the rate of return to holding money goes to zero, and for this reason there cannot be a sustained inflation. It is interesting to note that this policy is effective in eliminating inflationary equilibria because it is always in line with the recommendation of Bagehot [3] that "in a crisis, the lender of last resort should lend freely, at a penalty rate."

5. Implications for the Dollarization Debate

In this section, we discuss some interpretations of our analysis that we believe are relevant to the current debate over dollarization in Mexico. We begin with a clarifying remark. We have assumed in this analysis that the central bank's policy is fixed at the beginning of time and cannot be changed in response to observed price levels. This is obviously not true for a central bank's announced policies. If an inflation were to start, the central bank is free to change the interest rate that it charges at the discount window in response to this. Hence the policies in our model are best thought of as *implicit* policies (or policy rules) that include the central bank's reactions to possible future contingencies. An example of this is discussed at the beginning of the previous section; the interest rate could be posted in nominal terms but the central bank could follow a rule that adjusts the rate in each period in such a way that the real rate remains constant. Alternatively, the central bank may announce that the real interest rate will be kept constant but may, under some circumstances, be forced to abandon this policy and switch to a fixed nominal rate. The key assumption in our analysis is that the implicit policies or rules are correctly perceived by private agents, that is, that consumers and banks have rational expectations.

The results in the previous two sections indicate that the banking system should face a fixed real interest rate on short-term credit and not a fixed nominal rate, because the former policy contains an anti-inflationary mechanism that the latter lacks. When the real interest rate on discount window loans is fixed, the size of the penalty faced by a bank whose reserves turn out to be too low increases during an inflation. This gives banks a strong incentive to hold cash reserves, which increases the demand for money, which in turn implies that the inflation is inconsistent with market clearing. In contrast, when the interest rate at the discount window is fixed in nominal terms, the penalty faced by banks with insufficient reserves decreases during an inflation. This gradually erodes the demand for reserves, and hence the demand for money. As a result, inflationary beliefs can be self-fulfilling.

Why, then, would a central bank ever fix (either explicitly or implicitly) the nominal interest rate on short-term credit? There are many potential answers to this question, but we conjecture that most (if not all) of them would fall under the heading of "political pressure," or an inability to commit to the desired policy. In a situation where the currency starts to lose its value, as in the inflationary equilibria in our model, short-term credit serves two purposes: (i) by allowing banks to transfer resources to those in need of liquidity, it spreads the burden of the inflation more evenly (and thereby increases *ex ante* welfare) and (ii) it allows banks to hold fewer cash reserves and thereby pay a lower inflation tax (on average). To maintain a constant real interest rate on these loans, the central bank must increase the nominal rate in such a way that the penalty – the difference between the discount window rate and the market rate – increases. That is, in a period where lenders are becoming worse off and are most in need of a discount window, the central bank must increase the cost of using the window. Political pressure may make it very difficult for the central bank to maintain this policy.[11] If the central bank bows to pressure and decreases the penalty, it is effectively fixing the nominal rate and inflation becomes an equilibrium outcome. The irony in this is that if the central bank could commit to raising the penalty in this situation, the situation would not arise in the first place.

How, then, could a central bank commit to a fixed real rate on short-term credit? If the institutional framework fails to isolate the central bank from current political pressure, this may be difficult to do as long as the bank is capable of decreasing the penalty rate during an inflation. The bank is able to do this as long as it is free to print money. Hence, we see dollarization (giving up the ability to print money) as a mechanism for committing to a fixed real interest rate on short-term credit by forcing this credit to come at a cost that is determined outside the economy. In terms of the model, we see Mexico as potentially falling in an inflationary equilibrium under a fixed nominal

[11] If the central bank is completely free of current political concerns, this is not an issue. Of course, more independent central banks tend to have better records on inflation (see, for example, Alesina [1] and the references therein), so that if the central bank were completely free of political concerns there may be no need to consider dollarization in the first place.

interest rate. Dollarization would entail a move to a fixed real interest rate, where the economy can only be in the stationary equilibrium. This is the benefit of dollarization as captured in the model: the elimination of inflation as an equilibrium outcome.

Concerning the costs of dollarizing, the loss of revenue from liquidity lending is an obvious cost that is captured in the model. At a somewhat deeper level, some authors have expressed concern that a potential for financial instability would arise because a unilateral dollarization entails the loss of the ability to make the currency elastic, or the ability to provide lender-of-last-resort services. Eichengreen [7] , for example, writes that "The banking system must first be strengthened, so that the authorities' more limited capacity to provide lender-of-last-resort services does not expose the country to financial instability." There is undoubtedly reason for concern here, but our analysis above points out two reasons why this problem may not be as severe as it at first seems. The first point is that liquidity crises should occur less frequently in a dollarized economy. In terms of the model, in an inflationary equilibrium γ_t is low and as a result π^* is low (see (11)). Therefore shocks large enough to lead to a crisis will occur relatively often. In contrast, in the dollarized (stationary) equilibrium γ_t and π^* are higher, so that larger shocks will be required to induce a crisis. In other words, dollarization leads the banking system to hold liquidity for insurance purposes, which makes the occurrence of liquidity crises less likely. The second point is that even when a crisis does occur, it is not the case that a dollarized economy has no way of obtaining liquidity or providing lender-of-last-resort services. As Calvo [5] points out, "in advanced countries the lender of last resort does not issue money to finance the operation, it issues bonds, public debt." This can also be done in a dollarized economy. In the model, this corresponds to the central bank being forced to borrow and hence lend at some fixed real rate. The severity of crises will clearly depend on this rate (which will in turn depend on international markets), but by no means would they be as severe as if there were no liquidity available $(r = \infty)$. Hence a combination of changing bank behavior and public borrowing in international markets may lead to a more stable banking environment under dollarization.

We conclude by mentioning briefly an important issue that we have considered only implicitly in our discussion. Institutional arrangements aimed at providing a dollarized economy with an elastic currency in times of high liquidity demand (which can also be thought of as lender-of-last-resort services) would most likely rely on some form of public borrowing. This is important in our reasoning above, and implies that under dollarization monetary issues would become more closely tied to fiscal policy. For this reason it may be important to look at fiscal discipline as a crucial element of a successful dollarization program. We leave the integration of this issue into a formal banking model for future research.

APPENDIX A: PROOF OF PROPOSITION 1

Equation (12) defines γ_{t+1} as an implicit function of γ_t over some range. Where this function is defined, its slope is given by

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \left(\frac{\frac{\psi}{1+r}}{\left(\gamma_{t+1} + \frac{\psi}{1+r}\right)^2} \left[1 - F\left(\pi^{**}\right)\right] + \frac{\psi}{\left(\gamma_t + \psi\right)^2} \left[1 - F\left(\pi^{*}\right)\right] \right)^{-1} > 0.$$

Taking the limit as γ_{t+1} goes to zero in (12) shows γ_t must also go to zero and hence the implicit function starts at the origin. Evaluating the slope at this point yields

$$\lim_{\left(\gamma_t, \gamma_{t+1}\right) \to 0} \frac{d\gamma_{t+1}}{d\gamma_t} = \frac{\psi}{(1+r)} < 1. \tag{16}$$

Therefore the law of motion starts out below the 45-degree line. This and continuity of the implicit function imply that initial values for γ that are low enough will generate equilibria where γ_t converges to zero and hence there is inflation. Clearly there is a continuum of such values.

Taking the limit as γ_{t+1} goes to one in (12) shows that γ_t approaches a value less than one, and therefore the law of motion eventually lies above the 45-degree line. By continuity, there exists at least one steady state. Any steady state value of γ, which we denote γ^*, must satisfy

$$\gamma^* = \pi^{**} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi \geq \pi^{**} - \int_{\pi^*}^{\pi^{**}} d\pi = \pi^*,$$

so that we have

$$\gamma^* \geq 1 - \psi.$$

We also have

$$\gamma^* = \pi^{**} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi \leq \pi^{**},$$

which implies that we have

$$\gamma^* \leq 1 - \frac{\psi}{(1+r)}.$$

These bounds on γ^* are precisely those stated in the proposition. Using the fact that r is positive, it can be shown that the slope of the law of motion at any steady state must be greater than one, and therefore the steady state is unique. ■

APPENDIX B: PROOF OF PROPOSITION 2

When $\gamma_{t+1} > (1+r)\,\gamma_t$, we know that the law of motion is not defined. Elsewhere, the slope of the implicit function defined in (15) is given by

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{1 - \frac{\frac{\psi}{1+r}}{\left(\gamma_t + \frac{\psi}{1+r}\right)^2}\left[1 - F\left(\pi^{**}\right)\right]}{\frac{\psi}{\left(\gamma_{t+1}+\psi\right)^2} F\left(\pi^*\right)}.$$

The implicit function is defined and continuous in a neighborhood of any point where the denominator of this expression is non-zero, that is, where γ_{t+1} is non-zero. Such critical points are given by the solutions to

$$\gamma = \frac{\gamma}{\gamma + \frac{\psi}{1+r}} - \int_0^{\frac{\gamma}{\gamma+\frac{\psi}{1+r}}} F\left(\pi\right) d\pi \equiv g\left(\gamma\right). \qquad (17)$$

The function g is defined for all $\gamma \geq 0$ and is C^2. We have

$$\begin{aligned} g'\left(\gamma\right) &= \frac{\frac{\psi}{1+r}}{\left(\gamma + \frac{\psi}{1+r}\right)^2}\left(1 - F\left(\frac{\gamma}{\gamma + \frac{\psi}{1+r}}\right)\right) > 0 \\ g''\left(\gamma\right) &= -2\frac{\frac{\psi}{1+r}}{\left(\gamma + \frac{\psi}{1+r}\right)^3}\left(1 - F\left(\frac{\gamma}{\gamma + \frac{\psi}{1+r}}\right)\right) - \frac{\left(\frac{\psi}{1+r}\right)^2}{\left(\gamma + \frac{\psi}{1+r}\right)^3} f\left(\frac{\gamma}{\gamma + \frac{\psi}{1+r}}\right) < 0, \end{aligned}$$

so that for positive values of γ, the function g is strictly increasing and strictly concave. We also have

$$\lim_{\gamma \to 0} g\left(\gamma\right) = 0 \text{ and } \lim_{\gamma \to 0} g'\left(\gamma\right) = \frac{(1+r)}{\psi} > 1.$$

This implies that there is a unique solution to (17), which we denote by $\overline{\gamma}$. For values of γ below $\overline{\gamma}$, we have $\gamma < g\left(\gamma\right)$ and therefore there does not exist a value of γ_{t+1} satisfying (15). In this region the law of motion is not defined. For values of γ above $\overline{\gamma}$, we have $g'\left(\gamma\right) < 1$, which implies that the slope of the law of motion is positive. Hence, whenever the law of motion is defined, it is strictly increasing. Steady state values of γ are given by solutions to

$$\gamma = \frac{\gamma}{\gamma + \frac{\psi}{1+r}} - \int_{\frac{\gamma}{\gamma+\psi}}^{\frac{\gamma}{\gamma+\frac{\psi}{1+r}}} F\left(\pi\right) d\pi,$$

which is exactly the same equation that defines steady states when r is fixed in nominal terms. By Proposition 1, then, there is a unique steady-state value of γ, and this value satisfies

$$\gamma^* \in \left[1 - \psi, 1 - \frac{\psi}{(1+r)}\right].$$

This analysis demonstrates that the law of motion looks like the curve in figure 2. Any trajectory with $\gamma_{t+1} < \gamma_t$ for some t will pass below $\overline{\gamma}$ and thereby leave the feasible region in finite time. Such a trajectory cannot be an equilibrium. As γ_t approaches $1 - \frac{\psi}{(1+r)}$, the value of γ_{t+1} satisfying (12) approaches $(1+r)\,\gamma_t$, which is an edge of the feasible region. This implies that any trajectory with $\gamma_{t+1} > \gamma_t$ for some t will also leave the feasible region in finite time and therefore cannot be an equilibrium. These last two observations together imply that the steady state is the unique equilibrium of this economy. ■

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Figure 1: The Bank's Opportunity Set (for fixed γ_t)

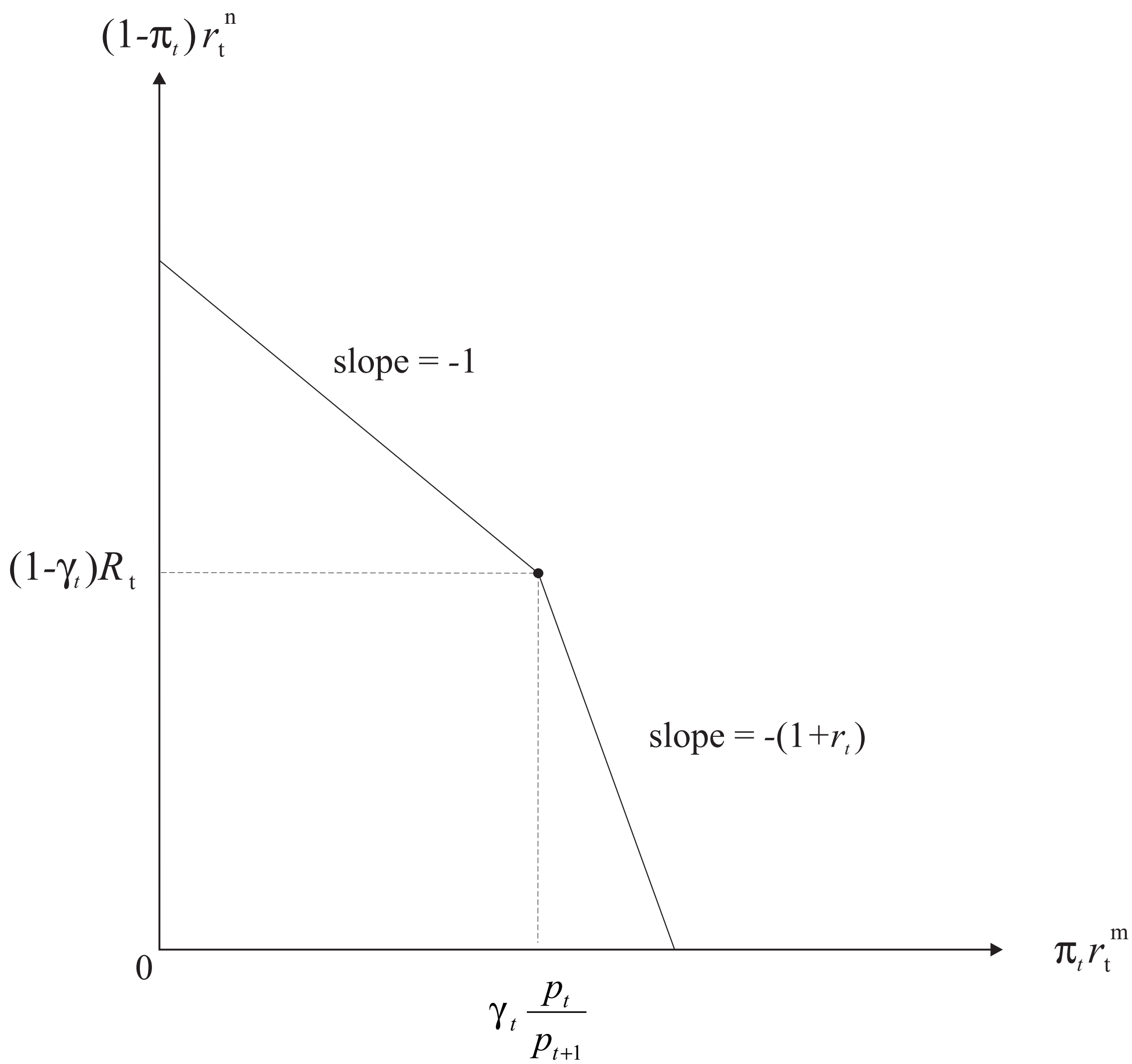

Figure 2: The Equilibrium Laws of Motion