Commitment, Banks and Markets*

Gaetano Antinolfi
Department of Economics
Washington University
gaetano@wustl.edu

Suraj Prasad
Department of Economics
Washington University
srprasad@wustl.edu

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Abstract

We examine how banks and financial markets interact with one another to provide liquidity to investors. The critical assumption is that financial markets are characterized by limited enforcement of contracts, and, in the event of default only a fraction of borrowers' assets can be seized. Limited enforcement reduces the fraction of assets that count as collateral and thus individuals with liquidity shocks face borrowing constraints. We show how banks endogenously overcome these borrowing constraints by pooling resources across several depositors, and increase the liquidity provided by financial markets.

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1 Introduction

Banks engage in maturity transformation and supply liquidity with deposit contracts despite the presence of illiquid assets in their balance sheets. This activity is reflected in models of banking, which consider risk pooling as one of the main reasons for the existence of intermediation. Mutual insurance provides the economic incentive to contribute to a common pool of resources, a bank, which can exploit the high return of long-term assets while in part overcoming the risk associated with idiosyncratic consumption shocks.

In this paper, we study banking in a model with limited commitment besides idiosyncratic consumption shocks. By limited commitment we mean the inability of individuals to fully commit to repay debt.[1] The significance of this assumption is that it creates a role for collateral and the possibility for credit constraints. This allows us to consider a notion of liquidity based on the efficiency with which claims written on real assets can be traded in financial markets. We employ this notion in addition to the one typically adopted in the banking literature, which relates to the efficiency with which technology allows liquidation of real assets. Our main result is to show that with limited commitment banks pool collateral, and allocate it more efficiently than individuals while complementing the liquidity provided by market activity.

To clarify, and overview some of the arguments of the paper, consider a simple illustrative example. Imagine two individuals who may have random liquidity needs in the future, and face the decision to allocate their wealth over two real assets, a low yielding liquid asset and a high-yielding illiquid asset. They have access to a market in which they can issue claims written on their real assets. If the market were to allow them to borrow against their wealth without friction, there would be little use for the liquid asset: the market itself would generate the liquidity they need.[2] In contrast, suppose that the market does not allow individuals to borrow easily against their future returns: for example, suppose that individuals can borrow only against a (constant) fraction of their wealth. In this case, the role of the liquid asset becomes more important, because in the future an individual with only illiquid assets may be credit constrained. However, one can contemplate another possibility. Suppose that the two individuals form a coalition and pool their

[1] This assumption is like in Kehoe and Levine (2001) and, in a different context, Azariadis and Lambertini (2003).

[2] Of course, assuming that prices are aligned to eliminate arbitrage opportunities.

resources before undertaking investment. For simplicity, call this coalition a bank. Pooling resources means that the bank is able to pledge a larger collateral, simply because the market allows it to borrow against a fraction of its funds, which are now the sum of the funds of the two individuals. In as much as it will not be the case that in the future both individuals will face sudden liquidity needs, the collateral of one individual can be pledged (through the bank) to fund the liquidity needs of the other. Again, the role of the liquid asset is diminished, this time because the bank allows better access to the market. This intuition, which the example illustrated loosely, is behind an important part of our analysis, and we will model it precisely in the following sections.

The identification of intermediaries as economizers of collateral also contributes to a long standing debate concerning the coexistence of banks and markets. Diamond and Dybvig (1983) showed that deposit contracts are optimal when individuals face uncertainty over the timing of their preference for consumption. In a subsequent contribution, Jacklin (1987) noted that if individuals are allowed to issue claims on directly owned real assets upon observing their relative patience for consumption, the presence of an intermediary does not improve over the provision of liquidity supplied by contingent claims markets. In the presence of markets banks' contracts have to satisfy a modified incentive compatibility constraint that takes into account arbitrage between returns on deposits and asset price movements.

Diamond (1997) allows for the presence of markets by assuming restricted participation.[3] When only a fraction of the population is allowed to issue claims on directly held real assets and trade these claims in competitive financial markets, banks do play a role in liquidity provision. The significance of Diamond's (1997) results is to show that as long as any fraction of the population is exogenously excluded from trading in financial markets (but is allowed to deposit in banks), intermediaries play a role in liquidity provision and can coexist with markets.

We assume limited commitment to make market participation endogenous. We show that with limited commitment the risk pooling of banks has another effect which may allow for equilibria where the commitment problem is completely overcome, and banks and markets coexist. In addition, banks

[3] Wallace (1988) adopts a similar interpretation of the Diamond and Dybvig (1983) model, and considers agents as being "isolated" one from the other so that bilateral exchange of claims is prevented.

"maximize" their maturity transformation role, and hold all the long term assets, while individuals directly hold only short-term assets. By pooling resources banks pool risk, but they also in effect pool collateral capacity. Because not all depositors withdraw at the same time, banks can subsidize the ability to borrow of those depositors who have high demand for liquidity. Thus, our result highlights the importance of banks as a commitment mechanism in economies where enforcement of financial contracts is limited or costly. Banks, therefore, can allow financial markets to function better, and increase the liquidity provided by them.

We treat banks and individuals symmetrically in market transactions: banks may face the same commitment problem as individuals in trading claims. We do assume, however, that banks are able to commit to repaying their depositors. The problem of the commitment of banks towards their depositors and its influence on the structure of the deposit contract has been studied, for example, by Diamond and Rajan (2000). They show that the maturity transformation role performed by banks with deposit contracts exposes them to bankruptcy if too many depositors want to withdraw at the same time. This vulnerability of the deposit contract constitutes an effective commitment device for banks. Intuitively, it makes default towards their depositors too costly for banks. While it would not be difficult to built the structure of the model by Diamond and Rajan (2000) into our model, there is not much additional economic insight to performing such an exercise, and we simply invoke the additional structure of Diamond and Rajan (2000) to justify our assumption.

Following Jacklin (1987), there have been several studies that examine the viability of banks as liquidity providers when investors have access to competitive financial markets. A paper by von Thadden (1999) considers the presence of two assets, one with a higher short-term return but a smaller long-term return than the other, as a way of relaxing the incentive compatibility constraint imposed on banks by the presence of markets. Allen and Gale (2004) consider a market where banks can trade risk. Their analysis highlights the importance of limited market participation: Allen and Gale (2004) assume that banks have access to markets but individuals do not. An example of a different approach aimed at yielding the same results is provided by Fecht, Huang, and Martin (2004), where the population is partitioned in sophisticated and unsophisticated investors who have different incentives in participating in direct (market) investment. Finally, an extensive discussion of the issues involved in the coexistence of banks and markets in relation to liquidity provision can be found in Hellwig (1998).

2 Basic Model with Financial Markets

We begin by analyzing the model without the banking sector.

2.1 Technology and Preferences

There are three dates, indicated by $t = 0, 1, 2$. There is a single good in the economy which is used for consumption and investment purposes. There is a continuum of individuals on the interval [0,1] who are identical at date 0; each individual is endowed with one unit of the good at $t = 0$, and has no endowment in the remaining periods. There are two linear investment technologies: a short term asset which yields $R_1 > 0$ at date 1 and 0 at date 2,[4] and a long term, completely illiquid, asset which yields 0 at date 1 and $R_2 > R_1$ at date 2. All investment takes place at date 0. The following table summarizes the asset structure where one unit of the consumption good is invested at date 0.

Date	$t = 0$	$t = 1$	$t = 2$
Short Term Asset	-1	R_1	0
Long Term Asset	-1	0	R_2

At date 1, each individual is hit by a privately observable liquidity shock with probability q. This shock makes an individual value consumption at both dates $t = 1$ and $t = 2$. With probability $(1 - q)$ individuals are not hit by a liquidity shock, and only value consumption at date 2. We refer to individuals who are subject to a shock as impatient or Type 1 individuals. Likewise, individuals who do not receive a shock are referred to as patient or Type 2 individuals. An individual's ex-ante preferences, at date 0, are given

[4] Unlike Diamond and Dybvig (1983) and Diamond (1997), all investment takes place at date 0. Hence, the short term asset cannot be reinvested at date 1. The reason to introduce this assumption is to simplify the analysis. Allowing reinvestment of the short term asset would simply introduce an additional arbitrage condition, between asset prices and the return on short term assets. This would impose an additional constraint to keep track of in the solution of the model but otherwise leave the economics unchanged.

by the utility function

$$U(c_{11}, c_{12}, c_{22}) = \begin{cases} u(c_{11}) + \phi c_{12} & \textit{with prob } q \\ u(c_{22}) & \textit{with prob } (1-q) \end{cases}$$

where c_{it} denotes consumption by type i at date t. The function u is strictly increasing, twice continuously differentiable, strictly concave, and satisfies the standard Inada conditions; in addition, $\phi > 0$. The law of large numbers implies lack of aggregate uncertainty: a fraction q of the population is hit by liquidity shocks at date 1, and the remaining fraction $(1-q)$ derives utility from date 2 consumption only. Individual preferences differ from those in Diamond (1997) in that individuals with liquidity shocks also get utility out of consuming at date 2. The reason for this assumption is to create an incentive to default for individuals who become borrowers at $t = 1$. In addition, we make the following assumption:

Assumption 1: $\phi > 0$ is sufficiently small so that $\dfrac{\phi}{u'(\frac{R_1}{q})} < \dfrac{R_1}{R_2}$.

Assumption 1 states that the marginal rate of substitution between date 2 consumption and date 1 consumption for the impatient type is less than the marginal rate of transformation for all technologically feasible consumption bundles. In other words, an impatient type has preferences biased towards date 1 consumption. The next section characterizes optimal allocations for the model economy.

2.2 Optimal Liquidity

Suppose that there are no informational asymmetries at date 1. Then the optimal provision of liquidity is the solution to the following maximization problem.

$$\underset{c_{11}, c_{12}, c_{22}}{Max} \quad qu(c_{11}) + q\phi c_{12} + (1-q)u(c_{22})$$

subject to the resource constraint

$$\frac{qc_{11}}{R_1} + \frac{qc_{12}}{R_2} + \frac{(1-q)c_{22}}{R_2} = 1 \tag{1}$$

and subject to the non-negativity constraint

$$c_{12} \geq 0.$$

The first order conditions to the problem are

$$qu'(c_{11}) = \frac{\lambda_o q}{R_1}; \tag{2}$$

$$q\phi - \frac{\lambda_o q}{R_2} = -\mu_o; \tag{3}$$

$$(1-q)u'(c_{22}) = \frac{\lambda_o(1-q)}{R_2}, \tag{4}$$

where λ_o is the multiplier associated with the resource constraint and μ_o is the multiplier associated with the non-negativity constraint. Let c^*_{it} denote the optimal consumption levels for type i at date t. The following lemma characterizes these optimal consumption levels.[5]

Lemma 1 *If the coefficient of relative risk aversion is everywhere strictly greater than 1 then the first best consumption levels must satisfy* $c^*_{11} > R_1$, $c^*_{22} < R_2$ *and* $c^*_{12} = 0$.

In the following section we introduce financial markets and we examine the effect of limited enforcement on the amount of liquidity that markets provide. We will show that financial markets cannot adequately insure those subject to shocks relative to the case of perfect information analyzed in this section, and that limited enforcement compounds the problem.

2.3 Financial Markets

At date 0, individuals invest a fraction of their endowment α in the short term asset and the remaining fraction $(1-\alpha)$ in the long term asset to try to insure themselves against the possibility of liquidity shocks. At date 1, after the shock is realized, they can buy or sell claims that pay out one unit of the good at date 2. The price of each claim is denoted by p and is measured in terms of the good at date 1. At date 1, impatient types consume their returns from the short term asset and supply claims (borrow) against a fraction of their returns from the long term asset, which is used as collateral. We denote with β the fraction of long term assets which an individual sells

[5] Proofs of all Lemmas and Propositions are contained in the Appendix, with the exception of Lemmas 3, 4 and 5, the proofs of which can be found in Antinolfi and Prasad (2007), available at http://gaetano.wustl.edu.

at date 1. The remaining fraction $(1-\beta)$ of date 2 returns is consumed at date 2. Patient types, on the other hand, demand claims at date 1 (lend) and consume their returns at date 2. Consumption by type i at date t, c_{it}, is given by

$$c_{11} = \alpha R_1 + \beta(1-\alpha)R_2 p; \tag{5}$$

$$c_{12} = (1-\beta)(1-\alpha)R_2; \tag{6}$$

$$c_{22} = \frac{\alpha R_1}{p} + (1-\alpha)R_2. \tag{7}$$

The assumption of limited commitment is equivalent to postulating the existence of an external enforcement agency that can seize a fraction θ of the borrower's assets in case he chooses to default. To prevent borrowers from defaulting strategically, α and β must satisfy a debt incentive constraint given by

$$(1-\theta)(1-\alpha)R_2 \leq (1-\beta)(1-\alpha)R_2.$$

The left hand side of the constraint is what the borrower gets when he defaults on his loan at date 2, and the right hand side is what he gets if he decides to pay back his creditors. When $\alpha < 1$, the individual debt incentive constraint reduces to

$$\beta \leq \theta. \tag{8}$$

This debt incentive constraint has a simple interpretation: only a fraction θ of long term returns can be used as collateral and this therefore restricts the total number of claims that a borrower can supply.[6]

2.4 Equilibrium

An **equilibrium** in a financial market is defined as a triplet (p, α, β) such that:

1. Taking p as given, agents solve their utility maximization problem, that is $\alpha, \beta \in \text{argmax } qu(c_{11}) + q\phi c_{12} + (1-q)u(c_{22})$ subject to (5) – (8).

2. The price p clears markets: $(1-q)\alpha R_1 = qp\beta(1-\alpha)R_2$.

[6] As typical in these models, there will be no default in equilibrium.

The first order conditions for the individual maximization problem are given by the equations:

$$qu'(c_{11})(\frac{R_1}{R_2} - \beta p) - q\phi(1-\beta) + (1-q)u'(c_{22})(\frac{R_1}{R_2 p} - 1) = 0 \qquad (9)$$

$$qu'(c_{11})(1-\alpha)R_2 p - q\phi(1-\alpha)R_2 = \lambda_m \qquad (10)$$

$$\beta \leq \theta \text{ and } \lambda_m \geq 0 \text{ with complementary slackness,} \qquad (11)$$

where λ_m is the multiplier associated with the debt incentive constraint.[7]

The following lemma and proposition examine the effect of limited enforcement on the liquidity provided by financial markets.

Lemma 2 *In equilibrium we have* $\frac{R_1}{R_2} \leq p \leq \frac{R_1}{R_2\theta}$ *and* $\beta = \theta$. *The inequalities are strict if* $\theta < 1$.

The lemma establishes upper and lower bounds for asset prices, and that borrowing constraints are always binding in equilibrium. If prices of claims are too high then all agents at $t = 0$ invest only in the long term asset, and markets do not clear at $t = 1$. If prices are too low, then all agents at $t = 0$ invest in the short term asset only, and once again markets do not clear at $t = 1$. To understand why the debt incentive constraint binds let us combine the inequality $p \geq \frac{R_1}{R_2}$ with Assumption 1. This implies that the quantity of date 1 consumption that an impatient agent wants in exchange for a unit of date 2 consumption is less than what the market gives him. Therefore, impatient agents will not want to consume at all at date 2. Limited enforcement of contracts, however, allows only a fraction θ of long-term returns to be used as collateral. The rest has to be consumed at date 2.

Proposition 1 *There exists a unique equilibrium with* $\alpha \leq q$ *and* $c_{11} \leq R_1, c_{12} \geq 0$, *and* $c_{22} \leq R_2$. *Inequalities are strict if and only if* $\theta < 1$. *Moreover,* α *and* c_{11} *are strictly increasing in* θ, *and* c_{12} *is strictly decreasing in* θ.

[7] The subscripts for the multipliers denote the specific problem that we are dealing with, in this case, markets.

Proposition 1 illustrates how consumption levels at date 1 of those hit by liquidity shocks are positively related to the limited enforcement parameter θ.[8] Therefore, financial markets may not be able to provide adequate insurance against these shocks. To gain some insight into the economics behind this result, note that a fall in θ has two contrasting effects. Ex ante, before shocks are realized, long-term assets become less attractive for individuals, because it is more difficult to borrow against them when long-term assets provide less collateral capacity. This effect tends to increase α, the share of endowment invested in short-term assets. However, imagine what would happen with a decrease in θ with investment positions unchanged: the supply of claims would contract as individuals issuing them face higher borrowing constraints. This effect tends to drive the price of claims up until the demand for claims equals its supply. Proposition 1 states that the general equilibrium price effect dominates. Even if a decline in θ makes issuing claims against long-term assets more difficult, the investment in long-term assets increases. Therefore, the equilibrium value of α is increasing in θ. As a result, overall consumption by impatient types at date 1 falls.[9] Lemma 1 and Proposition 1 allow us to endogenize the liquidity provided by a financial market: consumption levels of both types now depend on the limited enforcement parameter θ. These results also indicate two potential roles that a bank can play. First, banks can improve liquidity and, second, banks can commit to repay loans by pooling collateral.

[8] To Pareto rank equilibria as θ changes one needs to make the additional assumption that the marginal utility of consumption in period 2 is relatively small for impatient types. Specifically, a sufficient condition is $u'(R_2) > \phi$. When this condition holds, equilibrium allocations with higher θ Pareto dominate those with lower θ.

[9] Another way to think about what is happening is to consider that when θ decreases, after preference shocks are realized, it becomes more difficult to borrow, but lending is not affected (it is equally easy to purchase claims, what has become harder is to sell claims). Thus, after shocks are realized only one side of the market is affected; specifically only a fraction q of the population is affected. This direct effect would seem to indicate that it is advantageous, ex ante, to increase α. Ex ante, however, there is another effect, a price effect. Because borrowing contraints are tighter the supply of claims goes down, and the price of claims goes up. This effect involves borrowers and lenders, the entire population. Because of the different shares of population affected the price effect dominates, and it is actually more advantageous to invest less in the short term asset.

3 Banks and Financial Markets

In this section we introduce banks. Banks are modeled as depository institutions like in Diamond and Dybvig (1983). Competition in the banking sector ensures that banks maximize the expected utility of depositors. Taking as given the decisions of individuals, banks offer a deposit contract that pays d_1 units of consumption per unit deposited to individuals who withdraw at $t = 1$, and d_2 units of consumption per unit deposited to individuals who withdraw at $t = 2$. Because banks cannot observe depositor types, contracts must be incentive compatible.

Individuals at $t = 0$ have three choices: they can invest a fraction of their endowment, denoted by α_1, in the short term asset, another fraction, α_2, in the long term asset, and finally deposit the remaining fraction, α_3, with a bank.

Unlike Diamond (1997) and Allen and Gale (2004), we treat banks and individuals symmetrically with respect market participation. Hence, the assumption of limited commitment applies to banks as well. Banks can default in markets transactions, and in this case only a fraction of the loan can be recovered. The reason for allowing for this symmetry is to understand the role of banks in overcoming debt constraints in equilibrium, without giving banks an advantage in market transactions. We do, however, assume that banks can commit to repay their depositors. The role of deposit contracts as commitment mechanisms has been studied by Diamond and Rajan (2000), and introducing their construct into our model would not provide additional insight into the problem we are considering.[10]

We prove two results. First, prices have to line up with the marginal rate of transformation. If not, then all investment is either in long or short term assets, and markets do not clear. Second, we prove that when the fraction of individuals with liquidity shocks is not too high relative to the loan recovery rate, then the presence of banks allows the economy to completely overcome the commitment problem in equilibrium. This result shows how banks increase liquidity, and how the presence of banks allows to reduce transaction costs associated with financial markets.

The explanation of this result, which is behind the main message of the

[10]Diamond and Rajan (2000) show that the failure of banks to honor their deposit contracts will result in a run, which is costly for a bank as it loses rents on its loans when they are liquidated inefficiently. In their framework the fragility of the deposit contract allows banks to commit to repay depositors.

paper, is that banks pool resources across several depositors and in so doing build up their collateral capacity, which they can use to borrow and fund depositors who withdraw early. In other words, the same mechanism that allows banks to subsidize the consumption of impatient depositors with the resources of patient depositors, risk pooling, also allows banks to subsidize the borrowing capacity of impatient consumers with the collateral capacity of patient depositors, who do not need it. As long as the combined collateral is large enough to finance early withdrawals, banks do not face any debt constraints and can commit to repay their loans. We now formulate the maximization problems of individuals and banks.

The consumption of an individual depends on the fraction of his endowment deposited with a bank and his investment in long and short term assets. Consumption for type i at date t, c_{it}, is given by

$$c_{11} = \alpha_1 R_1 + \beta\alpha_2 R_2 p + \alpha_3 d_1; \tag{12}$$

$$c_{12} = (1-\beta)\alpha_2 R_2; \tag{13}$$

$$c_{22} = \frac{\alpha_1 R_1}{p} + \alpha_2 R_2 + \alpha_3 d_2. \tag{14}$$

The fractions invested in all three assets have to satisfy non-negativity and feasibility constraints:

$$0 \leq \alpha_1 \leq 1; \tag{15}$$

$$0 \leq \alpha_2 \leq 1; \tag{16}$$

$$0 \leq \alpha_3 \leq 1; \tag{17}$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1, \tag{18}$$

and the debt incentive constraint

$$\beta \leq \theta. \tag{19}$$

Deposit contracts offered by banks must satisfy resource constraints and incentive compatibility conditions. Let δ denote the fraction of the per capita endowment invested in the short term asset, γ_1 the fraction of short term returns paid to those who make early withdrawals, and $(1-\gamma_2)$ the fraction of long term returns paid directly to those who make late withdrawals. The resource constraints of a bank are given by

$$d_1 = \delta \frac{\gamma_1 R_1}{q} + (1-\delta)\frac{p\gamma_2 R_2}{q}; \tag{20}$$

$$d_2 = \delta \frac{(1-\gamma_1)R_1}{(1-q)p} + (1-\delta)\frac{(1-\gamma_2)R_2}{(1-q)}, \tag{21}$$

and incentive compatibility constraints by

$$u(c_{11}) \geq u(\alpha_1 R_1 + \beta p \alpha_2 R_2 + \beta p \alpha_3 d_2) + \phi(1-\beta)\alpha_3 d_2; \tag{22}$$

$$d_2 \geq \frac{d_1}{p}. \tag{23}$$

Equations (20) and (21) clearly demonstrate the effects of pooling resources. First, notice that $(1-\delta)R_2$ is a pool of collateral against which a bank can borrow to finance the consumption of impatient types. Second, notice that the fraction of long term assets $(1-\gamma_2)$ which is not used to finance impatient types need not be consumed inefficiently at date 2 by those hit by liquidity shocks.

The fraction of assets that can be recovered by the lender in the event of default is denoted by θ_B .[11] As for the case of individuals, the debt incentive compatibility constraint of banks can be written as

$$(1-\gamma_2)(1-\delta)R_2 \geq (1-\theta_B)(1-\delta)R_2.$$

When $\delta < 1$ this reduces to

$$\gamma_2 \leq \theta_B. \tag{24}$$

Finally, in solving the bank's problem,[12] it is easier to combine the resource constraints (20) and (21) into one resource constraint for the bank:

$$d_2 = \delta\left(\frac{R_1}{(1-q)p} - \frac{qd_1}{(1-q)p}\right) + (1-\delta)\left(\frac{R_2}{(1-q)} - \frac{qd_1}{(1-q)p}\right). \tag{25}$$

[11] θ_B may be different from θ.

[12] Details about the solution to the problem of the bank are provided in the appendix. The strategy which we have adopted is to solve the problem of the bank subject to (25), and then verify that there exist γ_1 and γ_2 such that (20) and (21) are satisfied.

3.1 Equilibrium

An equilibrium with banks and financial markets consists of a price p, an individuals' portfolio $(\alpha_1, \alpha_2, \alpha_3, \beta)$ a deposit contract (d_1, d_2), and a bank's portfolio $(\delta, \gamma_1, \gamma_2)$ such that:

1. Banks take the price p and individual portfolios $(\alpha_1, \alpha_2, \alpha_3, \beta)$ as given, and choose (d_1, d_2) and $(\delta, \gamma_1, \gamma_2)$ to maximize the expected utility of depositors.
2. Individuals take the price p and the actions of banks as given and choose $(\alpha_1, \alpha_2, \alpha_3, \beta)$ to maximize their expected utility.
3. Financial markets clear.

The market clearing condition is given by

$$q\alpha_2 R_2 \beta p + \alpha_3(1-\delta)\gamma_2 R_2 p = (1-q)\alpha_1 R_1 + \alpha_3 \delta(1-\gamma_1)R_1. \qquad (26)$$

Substituting $\alpha_2 = 1 - \alpha_1 - \alpha_3$ into the set of feasibility constraints, the individual problem can be equivalently written as:

$$\max_{\alpha_1, \alpha_3, \beta} \quad q(u(c_{11}) + \phi c_{12}) + (1-q)u(c_{22})$$

subject to the constraints specified in equations $(12)-(14)$, (19), and

$$\alpha_1 \geq 0;$$

$$\alpha_3 \geq 0;$$

$$\alpha_1 + \alpha_3 \leq 1.$$

Differentiating with respect to α_1, α_3, and β gives the first order conditions of the individuals' problem:

$$qu'(c_{11})(R_1 - \beta p R_2) + (1-q)u'(c_{22})(\frac{R_1}{p} - R_2) - q\phi(1-\beta)R_2 = \lambda_{b2} - \lambda_{b1}; \quad (27)$$

$$qu'(c_{11})(d_1 - \beta p R_2) + (1-q)u'(c_{22})(d_2 - R_2) - q\phi(1-\beta)R_2 = \lambda_{b2} - \lambda_{b3}; \quad (28)$$

$$q\alpha_2 R_2(u'(c_{11})p - \phi) = \lambda_{b0}, \tag{29}$$

where λ_{b0} is the multiplier associated with the debt incentive constraint, and λ_{b1}, λ_{b2}, and λ_{b3} are the multipliers associates with the non-negativity constraints for α_1, α_2, and α_3 respectively.

We focus our attention on equilibria with $\alpha_3 > 0$, namely equilibria in which the banking sector is active. Equilibria with $\alpha_3 = 0$ would be identical to the equilibria studied in the previous section.

The bank's problem is characterized as

$$\max_{d_1, d_2, \delta, \gamma_1, \gamma 2} \quad q(u(c_{11}) + \phi c_{12}) + (1-q)u(c_{22})$$

subject to the constraints given by equations $(12) - (24)$. There is not much immediate insight to be gained from the first-order conditions to the bank's problem. In the appendix, we provide details about its solution as we need them for the proofs of our results. Here, we state a series of lemmas that simplify our analysis and concentrate on the features of the equilibrium that are relevant for our purposes. First, these lemmas place lower and upper bounds on asset prices. Second, they show that the non-linear incentive compatibility constraint (22) is superfluous, which makes the problem more tractable.

Lemma 3 *In equilibrium we must have* $p \geq \dfrac{R_1}{R_2}$, *and hence* $\beta = \theta$ *whenever* $\alpha_2 > 0$.

The intuition for the lemma is exactly the same as Lemma 2. If prices of claims were too low then both types of individuals and banks would want to invest in the short term asset and markets would never clear. Moreover, when prices of claims are high enough, individuals want to use all of their long term assets as collateral and hence the debt constraint binds.

The following lemma shows that in solving the model we can ignore the incentive compatibility condition for impatient agents.

Lemma 4 *Suppose* $p \geq \dfrac{R_1}{R_2}$ *and* $c_{11} \leq \dfrac{R_1}{q}$. *Then, if the incentive compatibility condition for Type 2 binds, the incentive compatibility condition for Type 1 is satisfied.*

This result allows us to simplify the equilibrium analysis and ignore the incentive compatibility condition (22). The intuition underlying this lemma is that impatient types do not benefit from withdrawing late, because they face debt constraints and have to inefficiently consume at date 2.

The following lemma and proposition provide formal statements of the results described earlier in this section.

Lemma 5 *Suppose $q \leq \theta_B$. Then in equilibrium $p = \dfrac{R_1}{R_2}$.*

Intuitively, if prices differ from the marginal rate of transformation, then all the investment in the economy is either only in long-term assets or only in short-term assets, and the market clearing condition is violated. When prices are below the marginal rate of transformation then all agents, including banks, invest in the short term assets. When prices are above the marginal rate of transformation, banks and individuals will only invest in the long term asset and the market clearing condition again cannot hold.

We now want to determine under what conditions we can find equilibria such that both banks and markets coexist. In this situation, asset trades take place between banks and individuals at prices that equal the marginal rate of transformation. We characterize the equilibrium when both banks and markets coexist and banks trade in markets in the next proposition.

Proposition 2 *Let $q \leq \theta_B$. In an equilibrium with banks and markets $c_{11} = R_1$, $c_{12} = 0$, and $c_{22} = R_2$. When $\theta < 1$ all long term assets are held by banks and the debt incentive constraint of banks does not bind.*

Proposition 2 states two results. First banks and markets together provide more liquidity relative to markets alone (higher c_{11} and lower c_{12}), and consumption allocations do not vary with θ. Second, when $\theta < 1$ all long term assets in the economy are held by banks. The reason for this is that at prices $p = \dfrac{R_1}{R_2}$ no individual will be willing to hold long term assets because of their debt constraints. Banks, on the other hand, are indifferent between holding short and long term assets because their debt incentive constraints do not bind.

Even though the consumption allocation in equilibrium is unique, there are multiple portfolios that support them. We point out two important ones. In the first case $\alpha_3 = 1$, $\gamma_1 = 1$, $\gamma_2 = 0$, and $\delta = q$. In this case all of the endowment is deposited in the banks and banks do not trade in markets. In the second case banks rely entirely on markets to finance early withdrawals by impatient types. This is given by $\delta = 0$, $\gamma_2 = q$, $\alpha_1 = q$, $\alpha_3 = (1 - q)$. Banks debt constraints do not bind because of resource pooling. Pooling resources allows a bank to pool collateral and accurately predict the fraction of impatient agents (through the law of large numbers). In this case banks use this large pool of collateral to finance the smaller fraction of individuals who need to consume early. Even though banks and markets provide more liquidity relative to markets alone, banks face an upper limit on the liquidity they can provide because asset prices cannot rise above the marginal rate of transformation.

4 Conclusion

We studied the interaction between banks and markets when the main function of the financial system is to provide liquidity. We showed that individual limited commitment of the type studied in Kehoe and Levine (2001) constitutes an additional motive for intermediation, and allows markets and banks to become complements in the supply of liquidity.

The assumption of limited enforcement is a way to endogenize restricted market participation. Limited enforcement imposes debt constraints on borrowers and reduces the liquidity that financial markets can provide. Banks can increase liquidity if and only if enforcement is limited, and in this case mixed financial systems are sustained in equilibrium.

The existing literature that studies banks and markets in the Diamond and Dybvig (1983) framework imposes exogenous restrictions to market participation to allow for a mixed financial system. Previous literature also treats banks and markets asymmetrically with respect to market participation. We treat individuals and banks symmetrically: both have access to markets in the same fashion, and, in particular, both banks and individuals have to satisfy debt incentive constraints that prevent strategic default. We show that in an equilibrium where banks trade in markets, the debt incentive constraint does not bind for the bank, but that it binds for individuals. This is because banks build collateral capacity which they can allocate more effectively than individuals can, much like collecting deposits allows them to allocate risk more efficiently than individuals can. Another way to express this idea is to note that from a social welfare perspective it is more costly for banks to default than it is for an individual, as banks pool collateral and borrowing capacity.

One important issue that we do not address is the analysis of bank-runs equilibria. As noted by, for example, Diamond and Rajan (2000), there is an important relation between the possibility of bank runs and the ability of banks to commit to repay their depositors. Bank runs, however, are also potentially important for the coexistence of banks and financial markets, because they essentially represent a (social) cost in the provision of liquidity. Therefore, an important extension of our analysis is the consideration of bank runs in a model where both banks and markets are present.

Appendix

Proof of Lemma 1: Substituting λ_o from (2) into (3) we have $q(\phi - \frac{u'(c_{11})R_1}{R_2}) = -\mu_O$. From Assumption 1 we have $\mu_o > 0$ which implies $c_{12}^* = 0$. Combining (2) and (4) we have,

$$\frac{u'(c_{22}^*)}{u'(c_{11}^*)} = \frac{R_1}{R_2} \tag{30}$$

From the fundamental theorem of calculus we know that

$$u'(R_2)R_2 = u'(R_1)R_1 + \int_{R_1}^{R_2} \frac{dtu'(t)}{dt} dt \tag{31}$$

Since the coefficient of relative risk aversion is strictly above unity everywhere we know that

$$\int_{R_1}^{R_2} \frac{dtu'(t)}{dt} dt = \int_{R_1}^{R_2} (tu''(t) + u'(t))dt < 0 \tag{32}$$

Combining (31) and (32) we have $u'(R_2)R_2 < u'(R_1)R_1$, which implies

$$\frac{u'(R_2)}{u'(R_1)} < \frac{R_1}{R_2} \tag{33}$$

Since u is strictly concave, at the optimum we must have either $c_{11}^* > R_1$ or $c_{22}^* < R_2$. Suppose $c_{11}^* > R_1$, then from the resource constraint (1) it must be the case that $c_{22}^* < R_2$. Like wise if $c_{22}^* < R_2$ it follows from the resource constraint (1) that $c_{11}^* > R_1$. Thus at the optimum we must have $c_{11}^* > R_1$ and $c_{22}^* < R_2$.■

Proof of Lemma 2: Suppose $p = (\frac{R_1}{R_2} - \epsilon)$ in equilibrium where $\epsilon > 0$. Define the left hand side of (9) as $MU(\alpha)$. Then

$$MU(\alpha) = qu'(c_{11})\left[\frac{R_1}{R_2} - \beta\left(\frac{R_1}{R_2} - \epsilon\right)\right] - q\phi(1-\beta) + (1-q)u'(c_{22})(\frac{R_1}{R_2 p} - 1)$$

which can be rewritten as

$$MU(\alpha) = qu'(c_{11})\frac{R_1}{R_2}(1-\beta) - q\phi(1-\beta) + K$$

where $K = qu'(c_{11})\beta\epsilon + (1-q)u'(c_{22})(\frac{R_1}{R_2 p} - 1) > 0$ since $u'(.) > 0$.

From Assumption 1 we have $qu'(c_{11})\frac{R_1}{R_2}(1-\beta) - q\phi(1-\beta) \geq 0$. Thus $MU(\alpha)$ is strictly positive implying $\alpha = 1$. Hence markets do not clear contradicting the fact that $p < \frac{R_1}{R_2}$ is an equilibrium price. So in equilibrium $p \geq \frac{R_1}{R_2}$.

Since we have $\alpha < 1$ and $p \geq \frac{R_1}{R_2}$ in equilibrium, it follows from Assumption 1 that $q(1-\alpha)R_2(u'(c_{11})p - \phi) = \lambda_m > 0$.

The complementary slackness conditions from equation (11) imply $\beta = \theta$. Now suppose $p = (\frac{R_1}{R_2\theta} + \epsilon)$ in equilibrium where $\epsilon > 0$. Since $\beta = \theta$ we can write down the left hand side of equation (9) as

$$MU(\alpha) = qu'(c_{11})\left[\frac{R_1}{R_2} - \theta\left(\frac{R_1}{R_2\theta} + \epsilon\right)\right] - q\phi(1-\theta) + (1-q)u'(c_{22})\left(\frac{\theta R_1}{(R_1 + \theta R_2\epsilon)} - 1\right)$$

which is strictly negative. This implies $\alpha = 0$ and markets do not clear again resulting in a contradiction. Hence in equilibrium $\frac{R_1}{R_2} \leq p \leq \frac{R_1}{R_2\theta}$. Similar arguments show that the inequalities are strict if $\theta < 1$.■

Proof of Proposition 1: To show existence substitute the market clearing condition into (5), (7) and (9). Since we know from Lemma 1 that $\beta = \theta$ in equilibrium the left hand side of (9) can be defined in the following way

$$g(\alpha,\theta) = q\frac{R_1(q-\alpha)}{R_2q(1-\alpha)}u'(\frac{\alpha R_1}{q}) - q\phi(1-\theta)$$

$$+(1-q)\left(\frac{q\theta(1-\alpha)}{(1-q)\alpha}-1\right)u'\left((1-\alpha)R_2\left(\frac{q\theta}{(1-q)}+1\right)\right)$$

From the inada conditions on u we have $\underset{lim\ \alpha\to 0}{g(\alpha,\theta)} = \infty$ and $\underset{lim\ \alpha\to 1}{g(\alpha,\theta)} = -\infty$. Since u' is differentiable it follows that g is continuous in α over $[0,1]$. Hence from the intermediate value theorem we know that there exists $\alpha \in (0,1)$ such that $g(\alpha,\theta) = 0$. Since $p \leq \frac{R_1}{R_2\theta}$ in equilibrium it follows from the market clearing condition that $\alpha \leq q$.

To show uniqueness notice that

$$g_\alpha(\alpha,\theta) = \frac{R_1}{R_2}\left(u'(c_{11})\frac{(q-1)}{(1-\alpha)^2} + \frac{(q-\alpha)}{(1-\alpha)}u''(c_{11})\frac{R_1}{q}\right)$$

$$+(1-q)\left[u'(c_{22})\frac{(-q\theta)}{(1-q)\alpha^2} - \left(\frac{q\theta(1-\alpha)}{(1-q)\alpha}-1\right)u''(c_{22})\left(\frac{q\theta}{(1-q)}+1\right)R_2\right]$$

Since $p \geq \frac{R_1}{R_2}$ we have $\frac{q\theta(1-\alpha)}{(1-q)\alpha} - 1 \leq 0$. Since u is strictly increasing and strictly concave we have $g_\alpha(\alpha,\theta) < 0$ for all $\alpha \in (0,q]$. Hence in equilibrium α is unique.

Substitute p from the market clearing condition into equation (5). We then have

$$c_{11} = \alpha R_1 + \theta(1-\alpha)R_2\left(\frac{(1-q)\alpha R_1}{q\theta(1-\alpha)R_2}\right)$$

which can be reduced to $c_{11} = \frac{\alpha R_1}{q}$. Since $\alpha \leq q$, it follows that $c_{11} \leq R_1$. Also since $p \geq \frac{R_1}{R_2}$ we have from (7) that $c_{22} \leq R_2$. Using Lemma 2 and the individual consumption constraints it is possible to verify that $c_{11} = R_1$, $c_{22} = R_2$, and $c_{12} = 0$ if and only if $\theta = 1$.

To show that α and c_{11} are strictly increasing in θ notice that

$$g_\theta(\alpha,\theta) = q\phi + (1-q)\left[\left(\frac{q\theta(1-\alpha)}{(1-q)\alpha}-1\right)u''(c_{22})\frac{(1-\alpha)R_2q}{(1-q)} + u'(c_{22})\frac{q(1-\alpha)}{(1-q)\alpha}\right]$$

Since u is strictly increasing and strictly concave, and since $\dfrac{q\theta(1-\alpha)}{(1-q)\alpha} - 1 \leq 0$ it follows that $g_\theta(\alpha,\theta) > 0$. Using the implicit function theorem we have $\dfrac{d\alpha}{d\theta} = -\dfrac{g_\theta(\alpha,\theta)}{g_\alpha(\alpha,\theta)} > 0$. Also $\dfrac{dc_{11}}{d\theta} = \dfrac{R_1}{q}\dfrac{d\alpha}{d\theta}$. Thus α and c_{11} are strictly increasing in θ. To show that c_{12} is strictly decreasing in θ note that $\dfrac{dc_{12}}{d\theta} = -(1-\alpha)R_2 - (1-\theta)R_2\dfrac{d\alpha}{d\theta} < 0$.■

Proof of Lemma 3: Let $(\alpha_1,\alpha_2,\alpha_3,\beta)$, (d_1,d_2), (γ_1,γ_2), δ, p be an equilibrium with $p < \frac{R_1}{R_2}$. The proof is split into a series of claims to establish a contradiction. The first claim shows that individuals do not invest anything in the long term asset. For future reference, denote the marginal utilities in the left hand side of (27) and (28) as $MU(\alpha_1)$ and $MU(\alpha_3)$ respectively.

Claim 1 $\alpha_2 = 0$.

To prove Claim 1 notice that when $p < \dfrac{R_1}{R_2}$, from Assumption 1 we have $MU(\alpha_1)$ is strictly positive. This implies $\lambda_{b2} > 0$. From the complementary slackness conditions it follows that $\alpha_2 = 0$.

The next claim characterizes the set of contracts that satisfy incentive compatibility and the resource constraint.

Claim 2 $d_1 \leq R_1$.

To see why the above claim is true, suppose $d_1 > R_1$. Since $p < \dfrac{R_1}{R_2}$ and $d_1 > R_1$ we have

$$\frac{R_1}{p} = \frac{R_1}{(1-q)p} - \frac{R_1 q}{(1-q)p} > \frac{R_1}{(1-q)p} - \frac{d_1 q}{(1-q)p} \tag{34}$$

and

$$\frac{R_1}{p} > R_2 > \frac{R_2}{(1-q)} - \frac{R_1 q}{(1-q)p} > \frac{R_2}{(1-q)} - \frac{d_1 q}{(1-q)p} \tag{35}$$

Using equations (34) and (35) it follows from the resource constraint (25) that

$$d_2 = \delta\left(\frac{R_1}{(1-q)p} - \frac{d_1 q}{(1-q)p}\right) + (1-\delta)\left(\frac{R_2}{(1-q)} - \frac{d_1 q}{(1-q)p}\right) < \delta\frac{R_1}{p} + (1-\delta)\frac{R_1}{p} = \frac{R_1}{p} \tag{36}$$

This implies that $\frac{d_1}{d_2} > p$ which is a contradiction because the incentive constraint (23) is violated.

Claim 3 $\delta = 1$ *or* $\alpha_3 = 0$.

To prove this claim consider two possible cases.

CASE 1: $d_2 \geq \frac{R_1}{p}$.

Suppose $\alpha_3 > 0$ and $\delta < 1$. Consider a new deposit contract (d_1', d_2') with $d_1' = \frac{R_1}{q} - \frac{(1-q)pd_2}{q}$, $d_2' = d_2$ and $\delta' = 1$. Since $p < \frac{R_1}{R_2}$, we have

$$\frac{R_1}{q} - \frac{(1-q)pd_2}{q} > \frac{R_2 p}{q} - \frac{(1-q)pd_2}{q} \tag{37}$$

which implies

$$d_1' = \frac{R_1}{q} - \frac{(1-q)pd_2}{q} > \delta\left(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}\right) + (1-\delta)\left(\frac{R_2 p}{q} - \frac{(1-q)pd_2}{q}\right) = d_1 \tag{38}$$

Since $d_2 \geq \frac{R_1}{p}$ it follows from (38) that $d_1' \leq R_1$. The new contract (d_1', d_2') satisfies the incentive constraint (23) since $\frac{d_1'}{d_2'} = \frac{d_1'}{d_2} \leq \frac{R_1}{d_2} \leq p$. Also since $d_1' > d_1$ and $d_2' = d_2$ the contract satisfies the incentive constraint (22) as well. Finally set $\gamma_1' = \frac{qd_1'}{R_1}$ and $\gamma_2' = 0$. Since $0 \leq d_1' \leq R_1$ we have $0 \leq \gamma_1' \leq 1$. It can be checked that (d_1', d_2'), (γ_1', γ_2'), and δ' satisfy both resource constraints (20) and (21) , and the debt incentive constraint (24). Since $\alpha_3 > 0$, the new contract increases c_{11} while holding c_{12} and c_{22} fixed and thus increases the expected utility of an individual leading to a contradiction. Thus whenever the optimal deposit contract sets $d_2 \geq \frac{R_1}{p}$ it must be the case that $\delta = 1$ or $\alpha_3 = 0$.

CASE 2: $d_2 < \frac{R_1}{p}$.

From Claim 2 we know that $d_1 \leq R_1$. Since $d_2 < \frac{R_1}{p}$ we have $MU(\alpha_1) > MU(\alpha_3)$. Thus the multipliers in (27) and (28) have to satisfy $\lambda_{b3} > \lambda_{b1} \geq 0$ which in turn implies $\alpha_3 = 0$.

Combining Claim 1 and Claim 3 it follows that markets never clear resulting in a contradiction. Thus in equilibrium it must be the case that $p \geq \frac{R_1}{R_2}$. From (29) and Assumption 1 it follows that $\beta = \theta$ whenever $\alpha_2 > 0$.■

Proof of Lemma 4: Let $p \geq \frac{R_1}{R_2}$ and suppose the incentive constraint (23) binds with $d_1 = d_2 p$. Define $c'_{11} = \alpha_1 R_1 + \beta p \alpha_2 R_2 + \beta p \alpha_3 d_2$. Then the incentive constraint (22) holds iff

$$u(c_{11}) - u(c'_{11}) \geq \phi(1-\beta)\alpha_3 d_2$$

If $\beta = 1$ then the expression above holds with equality and we are done. Now suppose $\beta < 1$.

Then dividing both sides by $\alpha_3 d_2 p(1-\beta)$ we get

$$\frac{u(c_{11}) - u(c'_{11})}{\alpha_3 d_2 p(1-\beta)} \geq \frac{\phi(1-\beta)\alpha_3 d_2}{\alpha_3 d_2 p(1-\beta)}$$

which reduces to

$$\frac{\phi(c_{11} - c'_{11})}{u(c_{11}) - u(c'_{11})} \leq p$$

Since $d_1 = d_2 p$ and $\beta < 1$ we have $c'_{11} < c_{11}$. Since u is strictly concave and since $c'_{11} < c_{11}$, it follows that

$$\frac{\phi}{u'(c_{11})} > \frac{\phi(c_{11} - c'_{11})}{u(c_{11}) - u(c'_{11})}$$

Since $c_{11} < \frac{R_1}{q}$ it follows that

$$\frac{\phi(c_{11} - c'_{11})}{u(c_{11}) - u(c'_{11})} < \frac{\phi}{u'(c_{11})} < \frac{R_1}{R_2} \leq p$$

Hence the incentive constraint (22) is satisfied.■

Proof of Lemma 5: The proof of Lemma 5 again is split into a series of claims. Figure 1 allows us to understand each step of the claim more clearly. On the horizontal axis we have d_1 and on the vertical axis we have d_2. The ray through the origin given by $d_1 = d_2 p$ represents those points where the incentive compatibility condition of type 2 binds. The two parallel lines consist of all contracts that satisfy the resource constraint when $\delta = 1$ and $\delta = 0$ respectively.

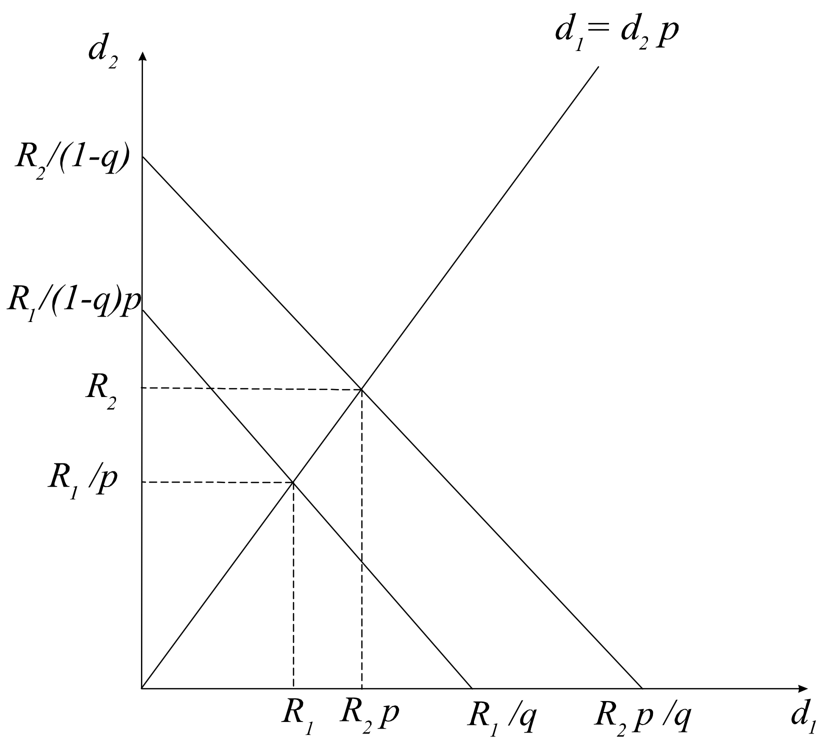

Figure 1: Graphic description of deposit contracts when $p > \dfrac{R_1}{R_2}$

The proof is by contradiction. Let $(\alpha_1, \alpha_2, \alpha_3, \beta)$, (d_1, d_2), (γ_1, γ_2), δ, p be an equilibrium with $p > \dfrac{R_1}{R_2}$.

Claim 1 $d_1 \le R_2 p$.

Suppose $d_1 > R_2 p$. Since $p > \dfrac{R_1}{R_2}$ and $d_1 > R_2 p$ we have

$$R_2 > \frac{R_1}{p} > \frac{R_1}{p}\left(\frac{1}{(1-q)} - \frac{qR_2p}{(1-q)R_1}\right) = \frac{R_1 - qR_2p}{(1-q)p} > \frac{R_1}{(1-q)p} - \frac{qd_1}{(1-q)p} \tag{39}$$

and

$$R_2 = \frac{R_2}{(1-q)} - \frac{R_2pq}{(1-q)p} > \frac{R_2}{(1-q)} - \frac{d_1q}{(1-q)p} \tag{40}$$

Using equations (39) and (40) it follows from the resource constraint (25) that

$$d_2 = \delta(\frac{R_1}{(1-q)p} - \frac{d_1 q}{(1-q)p}) + (1-\delta)(\frac{R_2}{(1-q)} - \frac{d_1 q}{(1-q)p}) < \delta R_2 + (1-\delta)R_2 = R_2 \tag{41}$$

This implies that $\frac{d_1}{d_2} > p$ which is a contradiction because the incentive constraint (23) is violated.

Claim 2 *Consider any* (d_1, d_2) *and* δ *such that the incentive constraint* (23) *and the resource constraint* (25) *hold. Then there exist* γ_1, γ_2 *and* δ' *with* $0 \leq \gamma_1 \leq 1$, $0 \leq \gamma_2 \leq 1$ *and* $0 \leq \delta' \leq 1$ *such that* (γ_1, γ_2), (d_1, d_2) *and* δ' *satisfy the resource constraints* (20) *and* (21) *and the debt incentive constraint.*

To prove this claim pick any (d_1, d_2) and any δ such that the resource constraint (25) and the incentive constraint (23) hold. Choose $\gamma_1 = \frac{qd_1}{(1-q)pd_2 + qd_1} = \gamma_2$. To see how γ_1 and γ_2 are chosen, draw a line through the origin that passes through (d_1, d_2). The intersection of this line with (25) when $\delta = 1$ determines γ_1 whereas the intersection of this line with (25) when $\delta = 0$ determines γ_2. It is clear that γ_1 and γ_2 are non negative and since the incentive constraint (23) holds it follows that $d_1 \leq (1-q)pd_2 + qd_1$ which implies $\gamma_2 \leq q$. Since $q \leq \theta_B$, it follows that the banks debt incentive constraint holds.

To show that the resource constraints (20) and (21) are satisfied, pick

$$\delta' = \frac{R_2 p - (qd_1 + (1-q)d_2 p)}{(R_2 p - R_1)}.$$

Since $p > \frac{R_1}{R_2}$ from (25) we have

$$d_1 = \delta(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}) + (1-\delta)(\frac{R_2 p}{q} - \frac{(1-q)pd_2}{q}) > \frac{R_1}{q} - \frac{(1-q)pd_2}{q}$$

and

$$d_2 = \delta(\frac{R_1}{(1-q)p} - \frac{d_1 q}{(1-q)p}) + (1-\delta)(\frac{R_2}{(1-q)} - \frac{d_1 q}{(1-q)p}) \leq \frac{R_2}{(1-q)} - \frac{d_1 q}{(1-q)p}$$

which implies $qd_1 + (1-q)d_2p > R_1$ and $R_2p - (qd_1 + (1-q)d_2p) \geq 0$. Thus $0 \leq \delta' \leq 1$.To check that γ_1, γ_2, δ' and (d_1, d_2) satisfy (20) substitute γ_1, γ_2, δ' in the right hand side of (20). The right hand side of (20) is given by

$$\frac{qd_1}{(1-q)pd_2 + qd_1}\left(\frac{(R_2p - (qd_1 + (1-q)d_2p))}{(R_2p - R_1)}\frac{R_1}{q} + (1 - \frac{(R_2p - (qd_1 + (1-q)d_2p))}{(R_2p - R_1)})\frac{R_2p}{q}\right)$$

The expression above reduces to d_1 and thus (20) is satisfied. A similar exercise can be performed for (21).

Claim 3 *$d_2 \geq R_2$. Also the deposit contract that pays R_2p at date 1 and R_2 at date 2 has to satisfy the incentive constraint* (22)*.*

Suppose the optimal deposit contract sets $d_2 < R_2$. Then it must be the case that $\delta > 0$ and $d_1 < R_2p$. To see this notice that if $\delta = 0$, then $d_2 < R_2$ implies

$$d_1 = \frac{R_2p}{q} - \frac{(1-q)pd_2}{q} > R_2p$$

which violates Claim 1. On the other hand if $d_1 \geq R_2p$ then the incentive constraint (23) is violated.

Also (23) must bind. If not banks can choose a new deposit contract (d_1', d_2') and δ' such that $d_2 = d_2'$, $\delta' = \delta - \epsilon$ where $\epsilon > 0$ is sufficiently small, and $d_1' = \delta'(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}) + (1-\delta')(\frac{R_2p}{q} - \frac{(1-q)pd_2}{q})$. Since $p > \frac{R_1}{R_2}$, it follows that $(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}) < (\frac{R_2p}{q} - \frac{(1-q)pd_2}{q})$ and thus

$$d_1' = \delta'(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}) + (1-\delta')(\frac{R_2p}{q} - \frac{(1-q)pd_2}{q})$$

$$> \delta(\frac{R_1}{q} - \frac{(1-q)pd_2}{q}) + (1-\delta)(\frac{R_2p}{q} - \frac{(1-q)pd_2}{q}) = d_1$$

Since ϵ is sufficiently small it follows that (23) still holds and because $d_1' > d_1$ and $d_2 = d_2'$, the incentive constraint (22) also holds. Since $d_1' < R_2p$ when ϵ is sufficiently small, using Claim 2 we can find γ_1' and γ_2' such that the resource constraints and debt incentive constraint holds. This results in a contradiction. Thus when the optimal deposit contract sets $d_2 < R_2$, (23) must bind.

Now to show that $d_2 < R_2$ results in a contradiction, suppose banks choose a new contract $(d_1^{''}, d_2^{''})$ with $d_1^{''} = d_1 + \epsilon'$ and $d_2^{''} = d_2 + \epsilon^{''}$ where $\epsilon' > 0$ and $\epsilon^{''} > 0$, such that $\dfrac{d_1^{''}}{d_2^{''}} = \dfrac{d_1}{d_2} = p$. It can be shown that $(d_1^{''}, d_2^{''})$ satisfies the resource constraint (25) when ϵ' is sufficiently small. Let c_{11} be the equilibrium consumption level at date 1 for type 1. Since the equilibrium consumption levels must be feasible we have $c_{11} \leq \dfrac{R_1}{q}$. Let $c_{11}^{''}$ denote the new consumption at date 1 for the new deposit contract $(d_1^{''}, d_2^{''})$. Then for ϵ' sufficiently small it follows from Assumption 1 and the continuity of u' that $\dfrac{\phi}{u'(c_{11}^{''})} < \dfrac{R_1}{R_2}$. Using exactly the same reasoning as in Lemma 4 it follows that the new contract satisfies (22). Once again from Claim 2 we know that banks can find γ_1' and γ_2' such that the resource constraints and debt incentive constraint holds. Since the new deposit contract offers a higher return for both types it strictly dominates the old contract leading to a contradiction. Thus in equilibrium $d_2 \geq R_2$.

Now suppose the deposit contract given by $(R_2 p, R_2)$ does not satisfy (22). From Claim 1 we know that $d_1 \leq R_2 p$. This implies that $d_2 < R_2$ to satisfy (22). But this is a contradiction. Thus the deposit contract given by $(R_2 p, R_2)$ must satisfy (22).

Claim 4 *Banks choose* $\delta = 0$.

Suppose $\delta > 0$. Then by reallocating investments across the long and short term assets banks can choose $\delta' = 0$, and offer a new contract (d_1', d_2') with

$$d_1' = \frac{R_2 p}{q} - \frac{d_2(1-q)p}{q} > \delta(\frac{R_1}{q} - \frac{d_2(1-q)p}{q}) + (1-\delta)(\frac{R_2 p}{q} - \frac{d_2(1-q)p}{q}) = d_1 \tag{42}$$

and

$$d_2' = d_2 \tag{43}$$

Since $d_2 \geq R_2$ it follows that $d_1' \leq R_2 p$, and thus the new contract satisfies (23). Also since $d_1' > d_1$ and $d_2' = d_2$ the contract satisfies (22) as well. Finally using Claim 2 we can find γ_1' and γ_2' such that the resource and debt incentive constraints hold. This contradicts the fact that $(d_{1,}d_2)$ is the optimal contract. Thus $\delta = 0$.

Claim 5 *The incentive constraint* (23) *binds and* $(d_1, d_2) = (R_2p, R_2)$.

To solve the banks problem when $\delta = 0$, we leave the incentive compatibility condition of type 1 (22) and the debt incentive constraint (24) out and check that the solution satisfies it later. We also substitute the resource constraints (20) and (21) with (25) and check that those two constraints hold later. The bank's problem is

$$\underset{d_1,d_2}{Max} \;\; qu(c_{11}) + q\phi c_{12} + (1-q)u(c_{22})$$

subject to the incentive constraint (23) and the resource constraint (25). The first order conditions to the problem are given by

$$q\alpha_3(u'(c_{11}) - \frac{u'(c_{22})}{p}) = \lambda_b' \frac{1}{(1-q)}, \tag{44}$$

where λ_b' is the multiplier associated with (23).

Next consider the consumption constraints. We have $\alpha_1 R_1 \leq \alpha_1 \dfrac{R_1}{p} p$, $\alpha_2 \theta R_2 p \leq \alpha_2 R_2 p$, and from (23) we have $\alpha_3 d_1 \leq \alpha_3 d_2 p$. Combining these inequalities we have

$$\alpha_1 R_1 + \alpha_2 \theta R_2 p + \alpha_3 d_1 \leq \alpha_1 \frac{R_1}{p} p + \alpha_2 R_2 p + \alpha_3 d_2 p \tag{45}$$

which, using (12) and (14) can be rewritten as

$$\frac{c_{11}}{c_{22}} = \frac{\alpha_1 R_1 + \alpha_2 \theta R_2 p + \alpha_3 d_1}{\alpha_1 \frac{R_1}{p} + \alpha_2 R_2 + \alpha_3 d_2} \leq p \tag{46}$$

Since the coefficient of relative risk aversion is strictly greater than 1 everywhere it follows that $\dfrac{u'(c_{22})}{u'(c_{11})} < \dfrac{c_{11}}{c_{22}} \leq p$. Thus $u'(c_{11}) - \dfrac{u'(c_{22})}{p} > 0$ which implies $\lambda_b' > 0$. Thus (23) binds. Combining the binding incentive constraint (23) with the resource constraint (25) we have $(d_1, d_2) = (R_2p, R_2)$. From Claim 3 we know that this contract satisfies (22). From Claim 2 we know that both resource constraints (20) and (21) and the debt incentive constraint (24) are satisfied.

From the claim above it follows that $MU(\alpha_3) > MU(\alpha_1)$ which implies $\lambda_{b1} > \lambda_{b3} \geq 0$ and thus $\alpha_1 = 0$. Since banks choose $\delta = 0$ markets never clear leading to a contradiction. Thus it must be the case that $p = \dfrac{R_1}{R_2}$ in equilibrium.■

Proof of Proposition 2: To solve the banks problem we leave (22) and the debt incentive constraint out and check that the solution satisfies it later. We also substitute (20) and (21) with (25) and check that those two constraints hold later. When $p = \dfrac{R_1}{R_2}$, the banks resource constraint becomes

$$d_2 = \frac{R_2}{(1-q)} - \frac{d_1 q R_2}{(1-q)R_1} \tag{47}$$

The bank chooses (d_1, d_2) to maximize expected utility subject to (23) and (25). The first order condition to the problem is

$$q\alpha_3\left(u'(c_{11}) - u'(c_{22})\frac{R_2}{R_1}\right) = \lambda_b'\left(1 + \frac{q}{(1-q)}\right),$$

where λ_b' is the multiplier associated with (23).

Combining (47) with $\dfrac{d_1}{d_2} \leq \dfrac{R_1}{R_2}$ we have $d_1 \leq R_1$ and $d_2 \geq R_2$. From the consumption constraints it follows that $c_{11} \leq R_1$, whereas $c_{22} \geq R_2$. Since the coefficient of relative risk aversion is greater than 1 we have

$$\frac{u'(c_{22})}{u'(c_{11})} < \frac{c_{11}}{c_{22}} \leq \frac{R_1}{R_2}$$

This implies that $\lambda_b' > 0$ which in turn implies that (23) binds. Combining (23) and (25) we have $d_1 = R_1$ and $d_2 = R_2$. Choosing $\delta = q$, $\gamma_1 = 1$, $\gamma_2 = 0$ we see that (20), (21) and the debt incentive constraint are satisfied. Also from Lemma 4, (22) also holds. Now consider two possible cases. First suppose $\theta = 1$. From (29) and Assumption 1 we know that $\beta = \theta$ whenever $\alpha_2 > 0$. Then from the consumption constraints (12) and (14) it follows that $c_{11} = R_1$ and $c_{22} = R_2$. Now consider the other case where $\theta < 1$. Then the left-hand sides of (27) and (28) are strictly positive. Thus $\alpha_2 = 0$ and all long term assets are held by banks and we again have $c_{11} = R_1$ and $c_{22} = R_2$.■

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