# Banks and Markets in a Monetary Economy

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## Abstract

We study an economy subject to aggregate real and liquidity shocks. We use this environment to study how banks, asset markets, and a central bank interact to achieve efficient allocations. Economies where one institution is missing do not, by construction, achieve efficient allocations. We analyse how interest rates and asset prices depend on the structure of the economy and the presence of active or passive policies by the central bank. We determine a simple lender of last resort policy which allows for efficient equilibrium allocations and relate the notion of liquidity which we adopted to the one used in other studies.

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# 1 Introduction

We present a model with a financial structure composed by banks, securities markets, and a central bank. We study how these institutions interact to achieve efficient allocations of risk, and how their interaction affects interest rates and asset prices. Our analysis focuses in particular on the distinction between liquidity risk and solvency shocks, and identifies the monetary policy conduct which implements a Pareto optimal outcome in the presence of these two kinds of risks.

Financial systems in modern economies include a variety of institutions. They all rely, albeit to different extent, on banks and security markets to transfer resources from savers to investors, and empirical evidence suggests that banks and markets distinctively contribute to the activity of the economy.[1] In addition, all financial systems are characterized by the presence of outside (fiat) money, and a central bank which manages the quantity of money to influence the cost of credit. Finally, all financial systems, besides transferring resources from savers to investors, provide insurance against different kinds of risks emerging from both the asset side and the liability side of the balance sheets of intermediaries.

In general, economic agents may face two kinds of risk. One is the typical *default risk* associated with financing risky projects; these projects generate an uncertain real payoff in the future. There is also another kind of risk. Given the different needs and incentives of savers and borrowers, the time profile of their demand for financial services also differs. Intermediaries then engage in maturity transformation and expose themselves to what is referred to as *liquidity risk*, which relates to the lack of certainty with respect to the distribution of resources over a certain period of time rather than the total amount of resources available at a point in time.[2] One of the main manifestations of liquidity risk is the shift of the composition of financial portfolios towards fiat money, the most commonly accepted instrument for purchasing goods and services. Thus, the institution managing the supply of fiat money plays a fundamental role in hedging aggregate liquidity risk.

The analysis of the issues discussed in the previous paragraph requires a model that presents

1 See Levine (1997) and La Porta et al. (1998).

2 We discuss this definition in Section 6. Empirically differentiating between these two types of shocks or risks is clearly very difficult. See the discussion in section 5 of Rajan (2001) and in Lindgreen et al. (1999). In a completely unrelated framework Covitz and Downing (2002) intend to differentiate these two risk concepts to explain yield spreads in the corporate bond market.

a specific description of a financial structure, which must contain both notions of liquidity and real risk. At the same time, the notion of liquidity risk must be compatible with the idea that money, once a store of value, becomes also a financial asset representative of liquidity needs and susceptible to liquidity risk. As a vehicle, this paper develops an extension to the model originally presented in Champ, Smith, and Williamson (1996), augmented with the presence of real risk. We consider an overlapping generations economy in which at each date the population is partitioned into two groups of two-period-lived agents: lenders and borrowers (or entrepreneurs). Agents are born at either of two identical locations (islands). At the end of each period, a fraction of lenders born in one island is relocated to the other island. Spatial separation and limited communication prevent trade across islands, and relocated agents *must* carry (fiat) currency.[3] This friction generates a stochastic demand for real balances (liquidity).

In this model, the role of banks is to provide insurance to depositors facing liquidity shocks that generate stochastic withdrawals at the end of each period. For this reason, banks hold precautionary reserves of real balances, which can be dominated in rate of return. Banks also make loans to borrowers, who are of two types in each island. We think of these types as residing in two different regions of the island. Each borrower is endowed with a stochastic investment project, and with given probability borrowers of each type face an unsuccessful outcome. We assume that investment projects are negatively correlated across types (and perfectly correlated within types). In addition, we assume that banks in each region can only lend to, and accept deposit from, agents located in the same region, and therefore are not allowed to diversify. Following Allen and Gale (2004) we also consider a version of this economy with a complete set of Arrow securities on the set of states of nature generated by shocks on the investment projects, in which only banks are allowed to trade.[4] Banks face *solvency* risk because they cannot perfectly diversify across the investment outcomes of the two regions. Hence, they have an incentive to trade risk with other banks in the Arrow securities market.

We study this environment in three different settings. The first version excludes both the Arrow securities markets and the central bank. In this case we show that real shocks and liquidity shocks interact by affecting banks' precautionary demand for real balances and the probability of liquidity shortages in each region. In particular, this probability is shown to be zero in the region that

[3] This part of the model is as in Townsend (1987).
[4] We also follow Allen and Gale (2004) in restricting market participation: individual depositors are not allowed to trade in contingent claims.

is affected by the negative realization of the entrepreneur's shock, while positive in the other. The reason behind this result is that cash reserves, independently of their liquidity properties, become a fundamental asset to repay all depositors, without regard to their relocation status, when entrepreneurs default.

The second version of the model introduces Arrow securities and lets banks trade these assets to share risk. We study how solvency and liquidity shocks interact in this setting. We show how banks' demand for precautionary reserves and the presence of liquidity shocks affect asset prices in equilibrium. In particular, numerical examples show that the probability distribution over the solvency shocks affects the way in which equilibrium asset prices depend on the distribution of liquidity shocks. The examples suggest that when investment outcome is more likely to be successful in one region, then the relative price of the Arrow security that pays off if the investment is successful in that region is higher the more concentrated the probability distribution over liquidity shocks is towards high demand for liquidity. Thus, the insurance provided by Arrow securities against solvency shocks is less valued when cash withdrawals are more likely.

The third version of the model introduces a central bank that makes one-period liquidity loans to (private) banks in addition to Arrow securities markets. We show that in this case the equilibrium outcome is the same that obtains in an economy with complete markets and a safe asset. Only in this setting is the equilibrium Pareto optimal. Therefore, banks, financial markets, and the central bank contribute distinctively to the implementation of a Pareto-efficient equilibrium. Commercial banks provide insurance to the idiosyncratic component of the liquidity shocks. Arrow securities allow to share risk arising from investment projects. Finally, the central bank provides insurance against the aggregate component of the liquidity shocks, given a suitable lender-of-last-resort policy. Even though the three institutions by construction make distinct contributions to risk sharing, they jointly determine the equilibrium outcome.

The literature on the role of banks and financial markets, and in general financial systems, is very extensive.[5] To our knowledge, Allen and Gale (2004) is the first paper presenting a general equilibrium model including both banks and security markets with two types of shocks (which may be interpreted as liquidity and solvency shocks). They show that financial intermediaries and complete security markets together imply constrained Pareto-efficient allocations as long as investors cannot trade in those securities. With incomplete financial markets and idiosyncratic

5 For a recent survey see, for example, Gorton and Winton (2003).

liquidity shocks, financial regulation in the form of a minimum amount of liquidity may improve efficiency in their paper, provided that the relative-risk-aversion coefficient is above unity. Unlike these results, our model suggests that, in the presence of an aggregate component of the liquidity shock, such liquidity regulation is not relevant for implementing efficient risk sharing, and states an essential need of a lender of last resort to reach a Pareto-efficient allocation.[6]

Gale (2005) presents a two-period model with financial markets but without banks. The model assumes cash-in-advance constraints not only for trading in goods but also for trading in assets. Gale (2005) demonstrates that asset prices are determined in part by the supply of liquidity. Our model, on the other hand, shows that not only money supply matters, but also the demand for liquidity influences equilibrium asset prices when there is no central bank acting as a lender of last resort. One reason for this difference is the absence of an aggregate component of the liquidity shock, which is present in our paper, in the example of section 4.4 of Gale (2005).

Other papers also study some aspects of the differential roles of banks and financial markets which are complementary to our main argument. Allen and Gale (2000, chapter 15) study how markets and banks collect information in different ways and hence provide different services. Diamond and Rajan (2001) and Diamond (1997) make a distinction about the roles of markets and banks which is different from the one present in our analysis. They point to different type of commitments that are involved in market contracts and bank based contracts (deposits), and the resulting incentives for banks, firms, and depositors.

Finally, the literature on liquidity provision is also related to our analysis. Allen and Gale (1997) show that banks produce a Pareto-optimal allocation while markets do not. A similar conclusion, albeit for different reasons, is reached by Holmstrom and Tirole (1998) when firms face idiosyncratic liquidity shocks. In the presence of an aggregate liquidity shock Holmstrom and Tirole (1998) show that government debt is needed as a tool for optimal liquidity provision. The role that public debt plays in Holmstrom and Tirole (1998) is similar to the role that the central bank plays in our model.

Unlike the papers cited above, our model introduces fiat money explicitly, with banks offering state-contingent deposit contracts that are denominated in currency. The preference shock that hits certain consumers does not take the form of an urgency to consume, but the urgency to hold a liquid

[6] The subsequent paper, Allen and Gale (2006), discusses the relationship between liquid investment and volatility of asset prices in a special case of Allen and Gale (2004) with no solvency shocks. See section 6 for further discussion.

asset, that is, fiat money. We want to study the role of banks and markets at the aggregate level, and this formulation seems more appropriate for this purpose than past models. From a historical perspective, central banks have been introduced (in part) to bear the task of moderating the risk for banks originating from the liability side of their balance sheets (mainly, liquidity shortages). Financial markets, on the other hand, help intermediaries moderating the risks originating from the asset side of their balance sheet. These are the features of the financial system that we wish to capture.

Section 2 presents the main elements of the general model. Section 3 analyzes the first version of the model in which only banks are present. Section 4 adds to the model in section 3 two Arrow securities, one for each project shock. Section 5 adds to the former model a central bank providing liquidity in fiat money against the realization of liquidity and solvency shocks. Section 6 presents the discussion of the main results of this model, specially related to the literature mentioned above. Finally section 7 concludes.

# 2 The model

We study an economy populated by a sequence of two-period lived overlapping generations and an initial old generation. There is a unique consumption good in the economy. There are two separate but identical islands, denoted $A$ and $B$, and two regions in each island, called region $1$ and region $2$.[7] In every period $t = 0, 1, 2, ...$, a new generation in each island and region is born. Each generation consists of two groups, each of a continuum of agents of unit mass. One group consists of risk neutral $entrepreneurs$, who invest when young and value consumption only when old. The second group consists of a continuum of risk averse lenders who also value consumption only when old. At time $t = 0$ there is an initial old generation of lenders each endowed with $\frac{M}{2}$ units of fiat money. We assume that goods cannot be transported between islands, and limited communication also prevents the transfer of assets. Only money is universally recognizable and can be transferred between islands.[8]

## 2.1 Entrepreneurs

Each entrepreneur has access to a (risky) technology, and has no endowment. Investment of $k$

[7] An alternative interpretation is to think about these as sectors.

[8] For a more detailed description of a related environment see Antinolfi, Huybens and Keister (2001), and Champ, Smith, and Williamson (1996).

units of the consumption good at time $t$ yields $g(k)$ units at $t+1$ with positive probability, or else $0$. We assume that $g(k)$ is strictly increasing, strictly concave, $C^2$, and satisfies the standard Inada conditions. In addition, we assume that the real shocks which affect investment projects are negatively correlated between the two regions: either entrepreneurs in region $1$ get $g(k)$ and entrepreneurs in region $2$ get $0$, or the converse is true. We indicate with $s_1$ and $s_2$ the states of nature in which the real shock is favorable to entrepreneurs in region $1$ and $2$ respectively. We let $\eta(s_j)$ be the probability that the investment is successful only in region $j=1,2$. Let $S \equiv \{s_1, s_2\}$.

Because entrepreneurs have no endowment, they need to borrow to invest. In case of successful projects, entrepreneurs in region $j$ pay back the amount they borrowed one period earlier with interest. We denote with $R_t^j$ the gross competitive interest rate that the entrepreneur in region $j=1,2$ pays in the favorable state.

An entrepreneur in region $j$ takes as given borrowing costs and solves the following expected income maximization problem:

$$\max_{k_t^j} \quad \eta(s_j)\left[g\left(k_t^j\right) - R_t^j k_t^j\right].$$

The first-order condition for this problem is

$$g'\left(k_t^j\right) = R_t^j.$$

We can express the demand for funds by an entrepreneur of region $j$ as

$$k_t^{j*} = \psi\left(R_t^j\right) \equiv \left(g'\right)^{-1}\left(R_t^j\right).$$

## 2.2 Lenders

All lenders receive an endowment vector $(\omega_1, \omega_2) = (x, 0)$, with $x > 0$. At the end of each period a fraction $\pi_t$ of young agents in each island is relocated to a different island (we will refer to them as *movers*). The remaining agents stay in the island until old, and will be referred to as *non movers.* The fraction $\pi_t$ represents the size of the aggregate liquidity shock in each island, and determines the existence of banks in a similar fashion as in Diamond and Dybvig (1983). In addition, because money is the only asset that can be transported between islands, the random liquidity shock determines a transactions role for money as in Townsend (1987). We assume that $\pi_t$ is drawn from a distribution function $F(\pi_t)$, which is assumed to be twice continuously differentiable with density $f(\pi_t)$. Moreover, we assume that lenders born in a certain region can only deposit their endowment with banks of the same region.

Lenders have preferences given by $U(c_1, c_2) = \ln c_2$, where $c_2$ represents consumption of an agent when old. Because lenders face the possibility of a liquidity shock, they deposit their endowment in a local bank. Banks promise a rate of return to depositors contingent on three factors: the state of nature prevailing in the region where the bank is located; the depositor's relocation status; and finally the fraction of total population relocated.

### 2.3 Banks

Banks take deposits, decide their portfolio of loans and reserves, announce rates of return on deposits, and trade in asset markets. We assume perfect competition in the banking sector: banks act as Nash competitors and maximize the expected utility of depositors. Both liquidity and real shocks are realized at the same instant.

We assume that banks in each region can lend only to entrepreneurs of the *same* region. Thus, banks are not allowed to perfectly diversify credit risk. We maintain this assumption for simplicity, but it is possible to make this feature of the model endogenous, for example by introducing a cost function for the intermediation process that would limit the number of entrepreneurs whom a bank finds profitable to lend to.[9]

We study the problem of a bank in three economies. In the first scenario, the bank faces only the problem of determining its demand for monetary reserves. In the second scenario, we open a market for contingent claims where banks in different regions (but not different islands) are allowed to trade in the contingent claim market. Finally, we add a central bank that provides an elastic currency to the economy through discount window loans.

## 3 The economy without asset markets or a central bank

In this section banks take deposits, choose their portfolio of loans and reserves, and announce rates of return on deposits. Their problem is to choose the fraction of deposits to invest in real balances to maximize the expected utility of depositors. Let $\gamma_t^j$ be the fraction of deposits invested in real balances, and let $\beta_t^j(s, \pi)$ be the fraction of cash balances that a bank of region $j$ uses to pay

[9] For analysis in which the bank size is determined endogenously see Krasa and Villamil (1992 and 1994).

relocated depositors at the end of period $t$. We denote $r_t^m(s,\pi)$ to be the real return on deposits to a mover when the aggregate state is $(s,\pi)$. Likewise, we indicate with $r_t(s,\pi)$ the real rate of return on deposits promised to a lender who does not leave the island. The first constraint that a bank faces states that it can only use real balances to satisfy the demand for withdrawals of the $\pi$ relocated depositors. Formally,

$$\pi r_t^{mj}(s,\pi) \leqq \gamma_t^j \beta_t^j(s,\pi)\frac{p_t}{p_{t+1}}, \tag{1}$$

where $p_t$ denotes the price of the consumption good in terms of fiat money. Banks use the remaining resources, possible remaining real balances and return on loans, to repay deposits and provide the promised return to the $1-\pi$ depositors who are not relocated to the other island. This constraint is given by

$$(1-\pi)\, r_t^j(s,\pi) \leqq \gamma_t^j\left(1-\beta_t^j(s,\pi)\right)\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s), \tag{2}$$

where $R_t^j(s) = R_t^j$ if $s = s_j$, and $0$ otherwise, with $j = 1, 2$. The problem of a bank is to choose $r_t^{mj}(s,\pi)$ and $r_t^j(s,\pi)$ to maximize the utility of depositors, taking the amount deposited, $x$, as given. The problem is:

$$\max_{\gamma^j,\beta^j,r^{m,j},r^j} \sum_{s\in S}\eta(s)\int_0^1 \left[\pi \ln r_t^{mj}(s,\pi)\,x + (1-\pi)\ln r_t^j(s,\pi)\,x\right] f(\pi)\,d\pi$$

subject to $(1)$ and $(2)$ in addition to the non-negativity constraints $0 \leq \beta \leq 1$ and $0 \leq \gamma \leq 1$. Substituting the constraints $(1)$ and $(2)$, which will hold with equality in equilibrium, and deleting irrelevant constants, the problem can be equivalently written as

$$\max_{\gamma^j,\beta^j(s,\pi)} \sum_{s\in S}\eta(s)\int_0^1 \Big[\pi \ln \gamma_t^j\beta_t^j(s,\pi) +$$
$$(1-\pi)\ln\left(\gamma_t^j\left(1-\beta_t^j(s,\pi)\right)\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s)\right)\Big] f(\pi)\,d\pi$$

subject to $0 \leq \beta^j(s,\pi) \leq 1$ and $0 \leq \gamma^j \leq 1$, and where $\eta(s_1) \equiv \eta$ and $\eta(s_2) \equiv 1-\eta$. Note that $\beta^j$, the fraction of real balances used to repay relocated depositors, is chosen after the shocks are observed. Therefore, the optimal value of $\beta^j$ is contingent of the choice of $\gamma^j$, the total amount of real balances available. On the other hand, the optimal amount of real balances is chosen before the observation of the realization of the shocks, and cannot be contingent on their value. The solution to the bank's problem is given by

$$\beta_t^j(s,\pi) = \begin{cases} \frac{\pi\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]}{\gamma_t^j \frac{p_t}{p_{t+1}}}; & \pi < \pi_j^*\left(\gamma_t^j, s\right) \\ 1; \quad \pi_j^*\left(\gamma_t^j, s\right) \le \pi < 1 \end{cases}$$

where

$$\pi_j^*\left(\gamma_t^j, s\right) \equiv \frac{\gamma_t^j \frac{p_t}{p_{t+1}}}{\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)}.$$

In practice, $\pi_j^*$ indicates the critical value of the liquidity shock such that a bank in region $j$ exhausts the whole amount of real balances held as reserves. For shocks higher than $\pi_j^*$ banks in region $j$ face a liquidity shortage, and movers and non-movers receive different returns on their deposits. It is immediate that, when $R_t^j(s) = 0$, then $\pi_j^*\left(\gamma_t^j, s\right) = 1$. The reason behind this property is that real balances are the only source of funds for banks when the real shock is unfavorable to borrowers in that region, and in these circumstances the bank will never give all its currency reserves to movers. Hence, fiat money shortages can only occur in banks belonging to the region with the $favorable$ realization of the solvency shock. The optimal choice of real balances is given by

$$\gamma_t^j = 1 - \eta(s_j) \int_{\pi^*\left(\gamma_t^j, s_j\right)}^1 F(\pi)\, d\pi.$$

Because of the aggregate nature of the liquidity shock, a bank provides partial liquidity insurance to its depositors. The rate of return on money is lower than the expected return on loans, and at the margin banks balance the insurance benefit of holding currency reserves and their opportunity cost due to the higher expected returns on loans to entrepreneurs. Notice that cash reserves are used by the bank to provide insurance to *both* movers and non-movers, who do not suffer the liquidity preference shock. In other words, the presence of credit risk provides an additional role for real balances. The presence of credit risk makes money an attractive asset because money has value in every future state of the world. Lack of credit risk would mean that $\eta(s_j) = 1$, and $\gamma_t^j = 1 - \int_{\pi^*\left(\gamma_t^j, s_j\right)}^1 F(\pi)\, d\pi$, which is the same result obtained in Champ, Smith and Williamson (1996) and Antinolfi, Huybens and Keister (2001) when the interest rate on bank loans is deterministic.

## 3.1 Equilibrium

In equilibrium, the money and credit markets have to clear in each island. The real money supply in each period is $\frac{M}{p_t}$, therefore the market clearing condition on the money market is given by

$$\frac{M}{p_t} = \left(\gamma_t^1 + \gamma_t^2\right) x, \tag{3}$$

where superscripts indicate the demand for real balances by banks in region 1 and 2 respectively. Equation (3) implies that

$$\frac{p_t}{p_{t+1}} = \frac{\left(\gamma_{t+1}^1 + \gamma_{t+1}^2\right)}{\left(\gamma_t^1 + \gamma_t^2\right)}.$$

Credit markets in region 1 and 2 also must clear; demand and supply of credit must be equal, that is in region $j = 1, 2$ we must have

$$\psi\left(R_t^j\right) = \left(1 - \gamma_t^j\right) x. \tag{4}$$

Under the assumption that $f(k) = k^\alpha$, $0 < \alpha < 1$ for all $j$, we have $\psi\left(R_t^j\right) = \left(\frac{\alpha}{R_t^j}\right)^{\frac{1}{1-\alpha}}$. In equilibrium:

$$\left(\frac{\alpha}{R_t^j}\right)^{\frac{1}{1-\alpha}} = \left(1 - \gamma_t^j\right) x.$$

Letting $\phi \equiv \frac{\alpha}{x^{1-\alpha}}$, it follows that $\left(1 - \gamma_t^j\right) R_t = \phi\left(1 - \gamma_t^j\right)^\alpha$. Therefore in equilibrium

$$\gamma_t^j = 1 - \eta\left(s_j\right) \int_{\frac{\frac{\gamma_t^j}{\gamma_t^1+\gamma_t^2}\left(\gamma_{t+1}^1+\gamma_{t+1}^2\right)}{\frac{\gamma_t^j}{\gamma_t^1+\gamma_t^2}\left(\gamma_{t+1}^1+\gamma_{t+1}^2\right)+\phi\left(1-\gamma_t^j\right)^\alpha}}^1 F(\pi)\, d\pi. \tag{5}$$

When both regions are considered, the resulting two-dimensional, first-order system of difference equations defines the equilibrium dynamics for the economy.

### 3.1.1 Stationary equilibrium in the economy with banks

We focus our analysis on steady-state equilibria. In steady state equation (5) becomes

$$\gamma^j = 1 - \eta\left(s_j\right) \int_{\frac{\gamma^j}{\gamma^j+\phi\left(1-\gamma^j\right)^\alpha}}^1 F(\pi)\, d\pi$$

for $j = 1, 2$. Notice that in the steady state equilibrium conditions in the two regions become independent from each other. We state the following:

**Proposition 1** *There exists a unique* $\left(\gamma^{1*}, \gamma^{2*}\right) \in (0,1)^2$ *which satisfies both equilibrium equations, hence the steady-state equilibrium is unique.*

**Proof.** See Appendix A.1. ■

It is not difficult to see that this equilibrium allocation is never Pareto optimal.[10] Intuitively, optimal risk-sharing dictates that a bank equalize the rate of return for both movers and non-movers. In fact, the economy does not face a random amount of resources relative to the liquidity shock. However, banks cannot adjust the amount of currency holdings after observing the liquidity shock. A bank must choose monetary reserves before observing the liquidity shock, even though ex-post, in the state of nature in which borrowers repay their loans with interest, a bank would be able to borrow fiat currency against its portfolio of loans, for example, from a central bank. This is the role that the central bank will play. Before introducing the central bank, however, we allow banks to trade contingent claims to trade credit risk on a given island.

# 4 The economy with asset markets

In this section we consider the problem of the bank and the equilibrium of the economy when banks can trade credit risk in asset markets. We model asset markets by opening markets for Arrow securities (depending on solvency shocks, $s$) in each island, in which banks can trade immediately after young agents make their deposits. Arrow securities are not a perfect representation of asset markets. For example, Arrow securities markets are self-financing. However, they allow like actual markets the exchange of risk through trade of goods across states of nature. Allen and Gale (2004) follow the same approach in modeling asset markets.

Let $\theta_t^j(s)$ denote the quantity of Arrow securities traded by a bank in region $j$ at the beginning of time $t$, which pay one unit of the consumption good in state $s \in S$ at time $t+1$. Arrow securities are traded before the observation of the shocks, and the determination of $\gamma' s$ and $\theta' s$ is simultaneous. As in the previous sections, the fraction of real balance reserves devoted to repay movers is determined after the observation of the shocks. The constraints that a bank faces in this case are:

$$\pi r^m(s,\pi) = \gamma_t^j \beta_t^j(s,\pi)\frac{p_t}{p_{t+1}}, \tag{6}$$

and

$$(1-\pi)\, r(s,\pi) = \gamma_t^j \left(1-\beta_t^j(s,\pi)\right)\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s) + \theta_t^j(s). \tag{7}$$

[10] Classic references are Balasko and Shell [7], [8]; see Champ, Smith and Williamson (1996) for discussion in a setting similar to ours.

The difference from the previous section is the presence of the term representing the Arrow security obligation of the bank in state $s$. Intuitively, the bank now has an additional tool that can be used to transfer consumption across states of nature for depositors who are *not* relocated to a different island. In this sense the value of monetary reserves is affected because the bank can cover some of the credit risk it is facing through asset markets. The problem of the bank in this case is

$$\max_{\gamma^j,\beta^j,r^{m,j},r^j} \ln \sum_{s\in S} \eta(s) \int_0^1 \left[\pi \ln r_t^{mj}(s,\pi)\, d + (1-\pi)\ln r_t^j(s,\pi)\, d\right] f(\pi)\, d\pi$$

subject to $(6)$ and $(7)$, the usual non-negativity constraints, and

$$q_{1t}\theta_t^j(s_1) + q_{2t}\theta_t^j(s_2) = 0,$$

where $q_1$ and $q_2$ are the prices of the Arrow securities that pay in state $s_1$ and $s_2$ respectively. Normalizing,

$$\theta_t^j(s_1) + q_t\theta_t^j(s_2) = 0, \tag{8}$$

where $q_t \equiv \frac{q_{2t}}{q_{1t}}$ is the relative price of Arrow securities. Equation $(8)$ is the self-financing constraint typical of Arrow securities trading. Conceptually, the procedure to solve bank's problem remains the same as in the previous section. We solve the problem of the bank by first determining the optimal liquidation of real balance reserves. The solution to this problem sets

$$\beta_t^j(s,\pi) = \begin{cases} \frac{\pi\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s) + \theta^j(s)\right]}{\gamma_t^j \frac{p_t}{p_{t+1}}}; & \pi < \pi_j^*\left(\gamma_t^j, s\right) \\ 1; \quad \pi_j^*\left(\gamma_t^j, s\right) \leq \pi < 1 \end{cases} \tag{9}$$

where the critical values of the liquidity shocks, depending on the state $s$, are given by

$$\pi_j^*(\gamma_t, s) = \frac{\gamma_t^j \frac{p_t}{p_{t+1}}}{\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R(s) + \theta^j(s)\right]}. \tag{10}$$

Having determined the liquidation policy of the bank once the state of the economy is realized, we need to determine the (ex-ante) choices of $\gamma_t^j$, $\theta_t^j(s_1)$, and $\theta_t^j(s_2)$.

Let us solve the case of bank $j = 1$ (the case of bank 2 is symmetric). Recall that $\eta(s_1) = \eta$

and $\eta(s_2) = 1 - \eta$. Using the optimal values for $\beta' s$ we can formulate this problem as

$$\max_{\gamma_t^1, \theta_1, \theta_2} \eta \int_0^{\pi^*(s_1)} \log\left[\gamma_t^1 \frac{p_t}{p_{t+1}} + \left(1 - \gamma_t^1\right) R(s_1) + \theta_t^1(s_1)\right] f(\pi)\, d\pi +$$

$$+\eta \int_{\pi^*(s_1)}^{1} \left\{\pi \log \gamma_t^1 + (1-\pi) \log\left[\left(1 - \gamma_t^1\right) R(s_1) + \theta_t^1(s_1)\right]\right\} f(\pi)\, d\pi +$$

$$+(1-\eta) \int_0^{\pi^*(s_2)} \log\left[\gamma_t^1 \frac{p_t}{p_{t+1}} + \theta_t^1(s_2)\right] f(\pi)\, d\pi +$$

$$+(1-\eta) \int_{\pi^*(s_2)}^{1} \left[\pi \log \gamma_t^1 + (1-\pi) \log \theta_t^1(s_2)\right] f(\pi)\, d\pi$$

subject to:

$$\theta_{1t}^1 + q_t \theta_{2t}^1 = 0,$$

where $\theta_{it}^1 \equiv \theta_t^1(s_i)$, $i = 1, 2$. The solution to the problem of the bank in region 1 gives

$$\gamma_t^1 = 1 - \eta \int_{\pi_1^*(s_1)}^{1} F(\pi)\, d\pi - (1-\eta) \int_{\pi_1^*(s_2)}^{1} F(\pi)\, d\pi,$$

and

$$\theta_t^1(s_2) = \frac{R_t^1(s_1)}{q_t} (1-\eta) \int_{\pi_1^*(s_2)}^{1} F(\pi)\, d\pi.$$

The symmetric solution to the problem of the bank in region 2 gives

$$\gamma_t^2 = 1 - \eta \int_{\pi_2^*(s_1)}^{1} F(\pi)\, d\pi - (1-\eta) \int_{\pi_2^*(s_2)}^{1} F(\pi)\, d\pi$$

and

$$\theta_t^2(s_1) = q_t R^2(s_2) \left(\eta \int_{\pi^*_{(s_1)}}^{1} F(\pi)\, d\pi\right).$$

Two observations are important about the solution to the bank's problem. First, the presence of asset markets affects the demand for real balance reserves of the bank. Asset markets give the bank a new tool for transferring real risk (that is, the credit risk generated by real shocks on investment). The bank still insures relocated depositors against liquidity risk, for which it needs currency, but now it has another tool that provides resources to repay non-relocated depositors when borrowers do not pay back their loans. In general, asset markets will in part substitute for real balances. However, the presence of the liquidity shock will affect the demands (and prices) for Arrow securities. Specifically, the amount of Arrow security that pays off in state $s_2$ for banks in region $1$ (that is, when entrepreneurs default) depends on the ratio of the interest rate on loans and the price of the Arrow security, which is a measure of the relative cost of obtaining consumption in state of nature $s_2$, multiplied by the likelihood of the event that $s_2$ will occur *and* the bank will suffer a shortage of liquidity. The form that the dependence of the demand for assets on interest rates and prices takes is intuitively clear: interest rates represent the opportunity cost of holding real balances, but real balances play a dual role as they are the only alternative asset available to insure depositors who are *not* relocated.

The second observation concerns the efficiency of the equilibrium. As it will be shown below, the equilibrium is never Pareto optimal for the case analyzed in this section.

## 4.1 Steady state equilibrium

In equilibrium, the money market, loans markets, and Arrow securities market must clear. The market clearing conditions are the same as in the previous section, with the addition of the Arrow securities market. In order, money markets clearing requires

$$\left(\gamma_t^1 + \gamma_t^2\right) x = \frac{M}{p_t},$$

which implies that

$$\frac{\left(\gamma_{t+1}^1 + \gamma_{t+1}^2\right)}{\left(\gamma_t^1 + \gamma_t^2\right)} = \frac{p_t}{p_{t+1}}.$$

Loan markets clearing conditions imply that

$$\left(1 - \gamma_t^1\right) x = \left(\frac{\alpha}{R_1}\right)^{\frac{1}{1-\alpha}},$$

$$\left(1 - \gamma_t^2\right) x = \left(\frac{\alpha}{R_2}\right)^{\frac{1}{1-\alpha}}.$$

The Arrow securities market clears when

$$\theta_{jt}^{1} = -\theta_{jt}^{2}; \quad j = 1, 2.$$

It is easy to show that equilibria always exist. In particular, there is always an equilibrium where $\pi_{1,1}^{*} = \pi_{2,2}^{*} = 1$ and $\pi_{ij}^{*} \in (0, 1)$, $i \neq j$.[11] This means that a bank in region $j$ never exhausts the amount of real balances held as reserves when the real shock in region $j$ is favorable. Therefore, the bank optimally sets the consumption of movers and non-movers to be the same as long as the realization of the real shock is favorable. In this case, there is complete risk sharing with respect to the liquidity shock region-wise, and so no liquidity shortage occurs in banks of the region with the favorable solvency shock. This is exactly the opposite to what happens when there are no Arrow securities. Note, however, that in the region where the real shock is not favorable depositors do not get full insurance. Clearly risk sharing is not complete island-wise.

One remaining question is whether there are other equilibria in the economy analyzed in this section. We do not have a proof of global uniqueness of the steady state equilibrium, even though in all the examples we produced only one steady-state equilibrium exists. It is important to notice, however, that equilibria are never Pareto optimal in this case as well. Depositors who are relocated and depositors who are not relocated still face risk about their consumption when old, even though ex-ante a bank no longer faces risk about the availability of resources in different states of nature. When there is an excess demand for liquidity movers pay a cost in terms of lower return on their deposits.

### 4.1.1 Cash-position of banks and asset pricing: some examples

An interesting question raised by the analysis in this section is whether the steady-state-equilibrium demands for real balances in an economy with Arrow securities are greater or smaller than their corresponding quantities in the stationary equilibrium of the economy without Arrow securities. Arrow securities provide an additional means to insure non-movers and, in principle, a higher portion of cash reserves could be devoted to the return offered to movers, possibly even with a larger portfolio of loans. This intuition, in general, is not correct, as the interaction between the

[11] There is a technical issue that must be clarified. The objective function for bank 1 includes an expression that, in equilibrium, implies an indeterminancy of the form $0 \cdot \infty$ when evaluated at $\pi_{1,1}^{*} = 1$. However, it can be shown that this indeterminacy can be resolved using L'Hopital's rule (see Appendix A.2 for details). Hence, the equilibrium exists as long as we define the equilibrium value of the objective function equal to its limit.

demand for real balances and asset markets depends on the cost of transferring resources across states of nature as well as on other parameters in a bank's optimal risk sharing problem. In other words, while asset trading widens the scope for risk sharing, it also has an ambiguous effect on the demands for real balances, and little can be said in general about their properties.[12]

To gain insight into this issue, let $(\gamma_A^1, \gamma_A^2)$ denote the steady-state equilibrium vector of fractions of real balances in the bank portfolio for the economy with Arrow securities[13] presented in the previous section. It is the solution to the system given by the two equations

$$\begin{cases} \gamma_A^1 = 1 - (1-\eta) \displaystyle\int_{\frac{\gamma_A^1}{\gamma_A^1 + \phi\left(1-\gamma_A^2\right)^\alpha}}^{1} F(\pi)\, d\pi \\[2em] \gamma_A^2 = 1 - \eta \displaystyle\int_{\frac{\gamma_A^2}{\gamma_A^2 + \phi\left(1-\gamma_A^1\right)^\alpha}}^{1} F(\pi)\, d\pi \end{cases} \tag{11}$$

Likewise, let $(\gamma_B^1, \gamma_B^2)$ denote the steady-state equilibrium fractions of real balances for the economy with only banks. It is the solution to the system given by

$$\begin{cases} \gamma_B^1 = 1 - \eta \displaystyle\int_{\frac{\gamma_B^1}{\gamma_B^1 + \phi\left(1-\gamma_B^1\right)^\alpha}}^{1} F(\pi)\, d\pi \\[2em] \gamma_B^2 = 1 - (1-\eta) \displaystyle\int_{\frac{\gamma_B^2}{\gamma_B^2 + \phi\left(1-\gamma_B^2\right)^\alpha}}^{1} F(\pi)\, d\pi \end{cases} \tag{12}$$

Note that both system $(11)$ and $(12)$ are symmetric around $\eta = \frac{1}{2}$, and that at $\eta = \frac{1}{2}$ the solution implies $\gamma_A^1 = \gamma_A^2 = \gamma_B^1 = \gamma_B^2$. It is not difficult to show that

$$\lim_{\eta \to 0} \gamma_A^j = \lim_{\eta \to 1} \gamma_A^j = 1$$

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12 This is not surprising, because when both liquidity and real shocks are considered the economy still has an incomplete set of markets.

13 That is, when $\pi_{i,i}^* = 1$, $i = 1, 2$.

for $j = 1, 2$. In addition,

$$\lim_{\eta \to 0} \gamma_B^1 = \lim_{\eta \to 1} \gamma_B^2 = 1,$$

and

$$\lim_{\eta \to 1} \gamma_B^1 = \lim_{\eta \to 0} \gamma_B^2 = 0.$$

Finally, direct computation reveals that $\frac{\partial \gamma_B^1}{\partial \eta} < 0$ and $\frac{\partial \gamma_B^2}{\partial \eta} > 0$ always hold. The implication of the properties displayed above is that for extreme values of $\eta$, while the demands for real balance reserves by banks tend to diverge across different regions in the economy without asset markets, they tend to converge in the economy with Arrow securities. The same property holds near $\eta = \frac{1}{2}$, where direct computation shows that

$$\left.\frac{d\gamma_B^2}{d\eta}\right|_{\eta=\frac{1}{2}} > \left.\frac{d\gamma_A^1}{d\eta}\right|_{\eta=\frac{1}{2}} > 0 > \left.\frac{d\gamma_A^1}{d\eta}\right|_{\eta=\frac{1}{2}} > \left.\frac{d\gamma_B^1}{d\eta}\right|_{\eta=\frac{1}{2}}$$

Figure 1 provides an example for the special case of a uniform distribution for $\pi$, and parameters values $\alpha = 0.65$ and $\phi = 0.9$. This example illustrates the sense in which the demands for reserves are less "reactive" to changes in real (solvency) risk in the economy with asset markets. Therefore, while it is not true in general that the introduction of asset markets reduces the total demand for real balances when both regions are considered, the introduction of asset markets may decrease the "volatility" of the demand for reserves. The reaction of real balance reserves in response to a change in real risk is milder when asset markets are present.

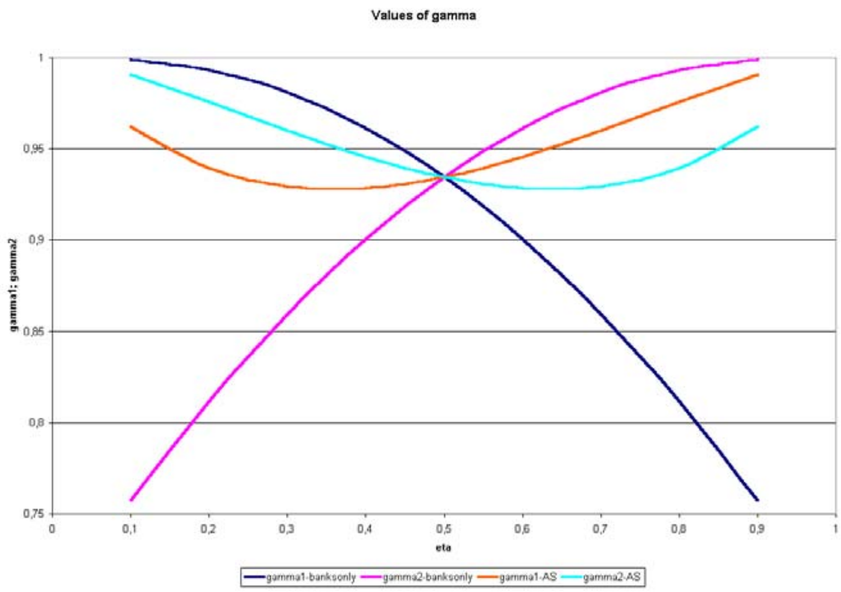


Fig 1: The demands for real balance reserves: an example.

With these equilibrium values for $\gamma_A^{j*}$, it is easy to compute the equilibrium value of $q^*$, the relative price of Arrow securities, for each $\eta$. This is clearly strictly decreasing in $\eta$. The higher $\eta$ the higher the probability that projects in region 1 are successful, while the opposite happens with the projects in region 2. These different likelihoods must be reflected in the prices of the corresponding Arrow securities.

Another interesting issue is how the cash-position and asset prices vary with the distribution of the $liquidity$ shocks. To gain some insight, we parameterize the cumulative distribution function $F(\pi)$ assuming that $F(\pi) = \pi^\sigma$, with $\sigma > 0$. Note that when $\sigma = 1$ we get the uniform distribution function as a special case. When $\sigma$ is low (less than one) it is more likely that the number of movers is low, while when $\sigma$ is high (greater than one) it is more likely that the number of movers is high. We compute the equilibrium cash position of the bank of each region for two different values of $\eta$, $\frac{1}{4}$ and $\frac{3}{4}$. We just report the results for $\eta = \frac{1}{4}$ since the figure for the case of $\eta = \frac{3}{4}$ is completely symmetric. We consider four values for $\sigma$ : $\frac{1}{3}$, $\frac{1}{2}$, 2 and 3.

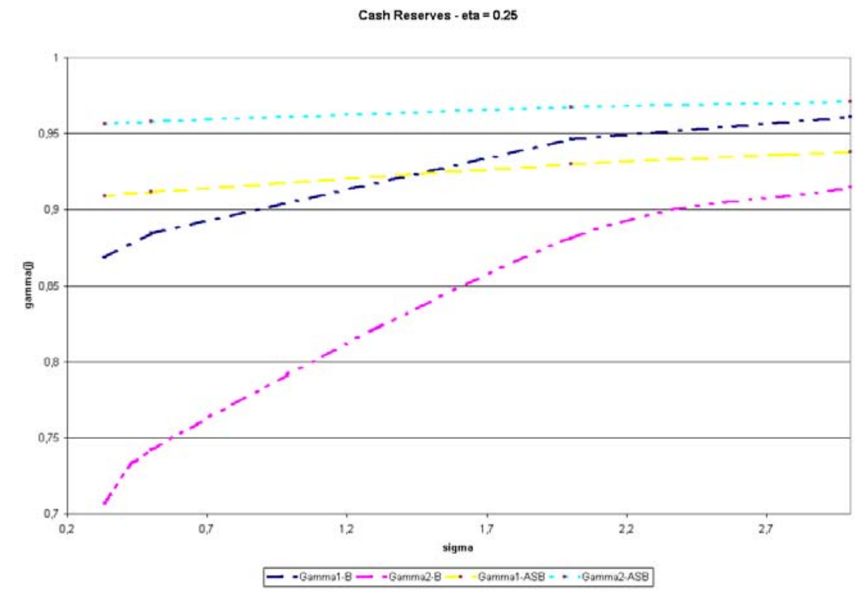


Figure 2: cash-position and liquidity shocks

Figure 2 confirms the intuition that the cash position is increasing in $\sigma$, since a higher $\sigma$ implies a higher expected number of movers and so a higher need for reserves to satisfy cash withdrawals. However, it is apparent from this figure that the equilibrium cash reserves are less sensitive to $\sigma$ with Arrow security markets than without them. The intuition for this result is that Arrow securities seem to be partially hedging liquidity risk, in the sense that whether agents expect a large or a small number of movers tend to become less of an issue given than banks can trade in these securities.

Perhaps more interestingly, cash reserves in region 2 are always higher when Arrow securities are traded than without them. Thus, the presence of Arrow securities implies a reversion in the demands for cash reserves: with no Arrow securities, the cash position of the region with higher probability of failure is higher than that of the other region, while the opposite occurs with Arrow securities. Thus, the introduction of such markets clearly reduces liquidity in the region with lower probability of success in its projects. The same is true for region 1 for sufficiently low values of $\sigma$ (i.e., at least for $\sigma \leq \frac{1}{2}$), even though it is more likely that region 2's projects are successful than region 1's projects. This effect may be related to the higher return on deposits for each consumer when Arrow security markets are open, which requires higher cash reserves (and expected marginal product of real investment), provided that risk sharing is not complete. This does not happen in region 1 when $\sigma$ is large enough (in which case $\gamma_B^1 > \gamma_A^1$). One possible explanation relies on the same argument used in the paragraph above: Arrow securities may contribute to the hedging of liquidity risk, which becomes important for a highly right-skewed distribution of such a shock.

We also compute the equilibrium relative price $q^*$ for the same parameter values. Figures 3 and 4 summarize the findings for the cases of $\eta = \frac{1}{4}$ and $\eta = \frac{3}{4}$.

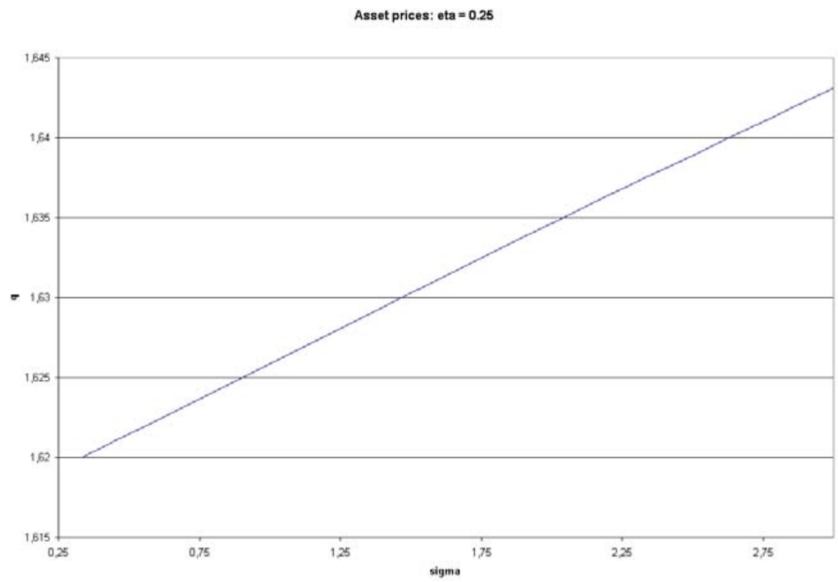


Figure 3: Arrow-security relative price when $\eta = \frac{1}{4}$

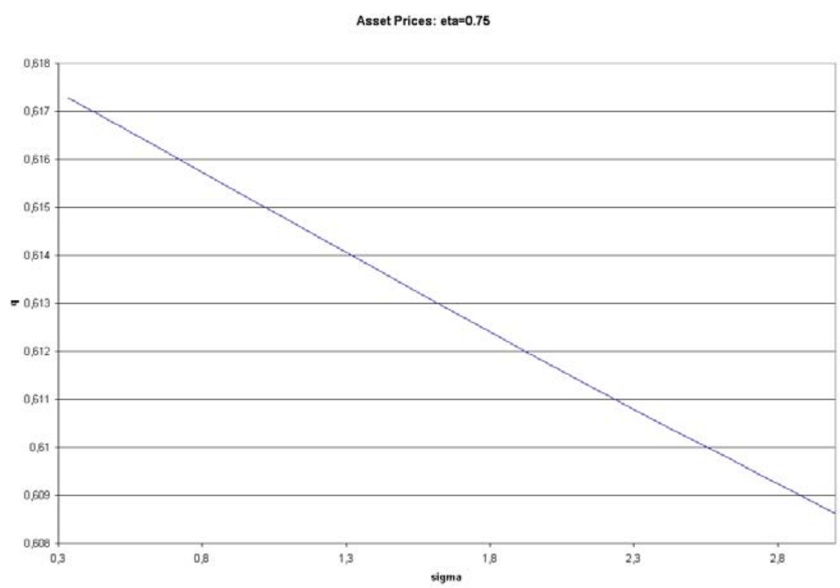


Figure 4: Arrow-security relative price when $\eta = \frac{3}{4}$.

The slope of the curve relating $\sigma$ with $q^*$ is positive when $\eta = \frac{1}{4}$, whereas it is negative when $\eta = \frac{3}{4}$. Figures 3 and 4 suggest that higher ex-ante liquidity needs (larger values of $\sigma$) make insurance less valuable in the region where it is more likely that projects fail and more valuable in the other region. This is a natural consequence of having that, in equilibrium, $q^* = \left(\frac{1-\gamma_A^{1*}}{1-\gamma_A^{2*}}\right)^\alpha$. The cash position of the region with higher probability of success in projects is always larger than that of the other region, for any $\sigma$, but the difference between the two decreases with $\sigma$. This means that higher ex-ante liquidity needs make the equilibrium amounts of cash reserves of the two regions closer to each other. The mirror argument is that real investment decreases more in the region with lower probability of success when $\sigma$ gets higher, which drives the results in figures 3 and 4.

This analysis describes explicitly the interaction between Arrow securities and liquidity shocks, even in the case where such securities are not intended to hedge liquidity risk. This result emphasizes that the value that markets give to Arrow securities for insuring against solvency shocks may depend on some features of the probability distribution of liquidity shocks. In the present example, skewness constitutes such key feature. The more skewed is such distribution towards high values of $\pi$, the more markets tend to value the Arrow security that pays off in the more likely (solvency) state. In other words, the higher the probability of high amounts of cash withdrawals in the economy, the less lending banks make available to entrepreneurs. Thus, the amount of insurance provided by Arrow securities is smaller. The reason is that banks facing higher probability of default can get insurance from cash holdings. A higher $\sigma$ implies higher cash reserves, and banks in the region with lower probability of success can finance consumption in the $default$ state with cash reserves and with payments from Arrow securities. With strictly concave preferences, the higher the amount of cash reserves, the lower the marginal benefit in terms of future utility of one additional unit of the corresponding Arrow security.

# 5 The economy with asset markets and a central bank

In this section, we complete the financial structure of our simple economy and add a central bank. The central bank operates a discount window to provide one-period loans of currency to banks facing an excessively high amount of withdrawals at the end of period $t$. These loans are made after shocks are realized. Therefore, they constitute pure liquidity loans: a bank will be solvent in period $t+1$, when borrowers repay their loans and Arrow securities trades clear. The difference with the previous case is that a bank knows before shocks are realized that it will be able to take contingent loans from the central bank.

We assume that the central bank charges a zero net nominal (and real in steady state) interest rate on discount window loans. We let $\delta_t^j(s,\pi)$ denote the amount of real balances that a bank in region $j$ borrows from the central bank at date $t$. This amount of currency is used to pay movers in period $t$ and will be repaid in period $t+1$ to the central bank. The budget constraints of a bank are given by the following equations:

$$\begin{aligned} \pi r_t^{mj}(s,\pi) &= \gamma_t^j \beta_t^j(s,\pi)\frac{p_t}{p_{t+1}} + \delta_t^j(s,\pi)\frac{p_t}{p_{t+1}}, \\ (1-\pi) r_t^j(s,\pi) &= \gamma_t^j\left[1-\beta_t^j(s,\pi)\right]\frac{p_t}{p_{t+1}} - \delta_t^j(s,\pi)\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s) + \theta_t^j(s). \end{aligned}$$

In this case, the problem of the bank is to choose optimally a liquidation policy $\beta_t^j(s,\pi)$, a borrowing policy $\delta_t^j(s,\pi)$, the amounts of Arrow securities $\theta_t^j(s)$ to trade, and the fraction of deposits $\gamma_t$ to hold as reserves. A bank chooses $\beta_t^j(s,\pi)$ and $\delta_t^j(s,\pi)$ after observing the shocks, and, as in the previous sections, solves the problem:

$$\begin{aligned} &\max_{0\leqq\beta_t^j(s,\pi)\leqq 1,\, \delta_t^j(s,\pi)} \pi \ln\left(\gamma_t^j\beta_t^j(s,\pi)\frac{p_t}{p_{t+1}} + \delta_t^j(s,\pi)\frac{p_t}{p_{t+1}}\right) + \\ &(1-\pi)\ln\left(\gamma_t^j\left[1-\beta_t^j(s,\pi)\right]\frac{p_t}{p_{t+1}} - \delta_t^j(s,\pi)\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s) + \theta_t^j(s)\right). \end{aligned}$$

In Appendix A.3 we show that one solution to this problem sets

$$\beta_t^j(s,\pi) = \begin{cases} \frac{\pi\left[\gamma_t^j\frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right)R_t^j(s) + \theta_t^j(s)\right]}{\gamma_t^j\frac{p_t}{p_{t+1}}}; & \pi \leqq \pi_j^*\left(\gamma_t^j, s\right) \\ 1; \quad \pi > \pi_j^*\left(\gamma_t^j, s\right) \end{cases}$$

and

$$\delta_t^j(s,\pi) = \begin{cases} 0; \quad \pi \leqq \pi_j^*\left(\gamma_t^j, s\right) \\ \frac{p_{t+1}}{p_t}\left[\pi\left[\left(1-\gamma_t^j\right)R_t^j(s) + \theta_t^j(s)\right] - \gamma_t^j\frac{p_t}{p_{t+1}}(1-\pi)\right]; & \pi > \pi_j^*\left(\gamma_t^j, s\right) \end{cases}$$

where $\pi_j^*$ is given by $(10)$. A bank holds a certain amount of reserves, and uses them to pay movers as long as $\pi \leqq \pi_j^*\left(\gamma_t^j, s\right)$. For larger values of the relocation shock, the bank borrows currency for one period from the discount window. This is not the only solution to a bank's problem. In fact, the liquidation and borrowing policies depend on the total amount of currency reserves a bank decided to acquire before observing the liquidity and productivity shocks, and this amount is indeterminate with zero-nominal-rate discount window lending.[14] If it were not, the bank would hold only currency when currency's rate of return dominated other rates of return. Vice versa, the demand for currency reserves would be zero if money were dominated in rate of return by other portfolios of assets.

We can make these statements because in this section, with a central bank operating a discount window, we essentially have constructed a complete set of markets.[15] The easiest way to note this equivalence with the complete market case is by rewriting the maximization problem of the bank subject to a single budget constraint. Solving the first constraint of the bank's problem for $\delta_t^j(s,\pi)$, and substituting in the second constraint we obtain:

$$\pi r_t^{mj}(s,\pi) + (1-\pi)\, r_t^j(s,\pi) = \gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s) + \theta_t^j(s),$$

which implies

$$\theta_t^j(s) = \pi r_t^{mj}(s,\pi) + (1-\pi)\, r_t^j(s,\pi) - \left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]. \qquad (13)$$

Recall that the self-financing condition at the beginning of date $t$ is given by:

$$\theta_t^j(s_1) + q_t \theta_t^j(s_2) = 0.$$

Replacing in this equation the expression for $\theta_t^j(s)$ gives the sole budget constraint for maximization problem of the bank:

$$\sum_{s\in S} q_{st}\left[\pi r_t^{mj}(s,\pi) + (1-\pi)\, r_t^j(s,\pi)\right] = \sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right] \qquad (14)$$

which holds for every $\pi$. It states that the weighted sum over states of nature of the promised returns to depositors is equal to the weighted sum of the returns on the bank's portfolio, with the weights given by the prices of the Arrow securities. The problem of the bank can then be written as

$$\max \sum_{s\in S} \eta(s)\left[\int_0^1 \left(\pi \ln\left(r_t^{mj}(s,\pi)\right) + (1-\pi)\ln\left(r_t^j(s,\pi)\right)\right) f(\pi)\, d\pi\right]$$

[14] The indeterminacy is present only for an individual bank, not for the aggregate economy, where the price level must be well defined in equilibrium.

[15] The analogy with complete markets makes sense in a steady-state equilibrium, because in this case the economy is essentially a sequence of two-period economies with two states of nature with respect to the real shock. Note that for other equilibria, in an overlapping-generations setting, this interpretation is not correct and the definition of complete markets more complex.

subject to $(14)$. In Appendix A.3 we show that the first order conditions to this problem imply that

$$r_t^{mj}(s,\pi) = r_t^j(s,\pi)$$

for every $s$ and $\pi$. The bank in this case is able to offer movers and non-movers the same rate of return. Note that the rate of return offered is not random, as it would be if there were not asset markets. It is now evident what role banks, the central bank, and asset markets play in this model. Banks provide liquidity insurance to depositors and the central bank allows for the existence of complete insurance against liquidity shocks. Asset markets allow banks to trade credit risk. Note that credit risk is not "intertemporal" but "cross-sectional": that is, risk for which asset markets are used does not concern the intertemporal distribution of resources, but total amount of resources available in a certain period.

We show in Appendix A.3 that the rates of return offered to movers and non-movers must be equal to

$$\frac{\eta(s)}{q_{st}}\left\{\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]\right\}. \tag{15}$$

Equation $(15)$ states that the return that a bank promises to movers and non-movers in state $s$ is equal to the total present value of goods received in period $t+1$ weighted by the ratio of the probability of $s$ relative to the price of the Arrow security that pays off in $s$. This is natural given the completeness of Arrow securities markets.

In Appendix A.3, we also show that to insure an interior solution for $\gamma_t$ the following condition must hold:

$$(q_{1t}+q_{2t})\frac{p_t}{p_{t+1}} = q_{1t}R_t^j(s_1) + q_{2t}R_t^j(s_2). \tag{16}$$

Equation $(16)$ is a no-arbitrage condition stating that the return on money (the inverse of the gross inflation rate) must be a weighted average of the promised returns from entrepreneurs of both regions, where the weights depend upon the prices of Arrow securities. Using the normalization adopted so far for the prices of Arrow securities we let

$$q_t \equiv \frac{q_{2t}}{q_{1t}}.$$

Therefore the no-arbitrage condition can be expressed as

$$\frac{p_t}{p_{t+1}} - R_t^j(s_1) = -q_t \left( \frac{p_t}{p_{t+1}} - R_t^j(s_2) \right)$$

for every $j$. To analyze the equilibrium of the economy, it is first essential to get the optimal net demand for Arrow securities by each bank type.

We use the constraint (13), and substitute optimal rates of return (15) and the arbitrage condition (16) to obtain

$$\theta_t^j(s) = \left( \frac{\eta(s)}{q_{st}} \right) \left( q_{1t} R_t^j(s_1) + q_{2t} R_t^j(s_2) \right) - \left[ \gamma_t^j \frac{p_t}{p_{t+1}} + \left(1 - \gamma_t^j\right) R_t^j(s) \right].$$

We show in Appendix A.3 that the optimal net demand functions for these securities by each bank type are given by

$$\begin{aligned} \theta_t^1(s_1) &= \left( \frac{p_t}{p_{t+1}} \right) \left[ q_t \gamma_t^1 - (1-\eta)(1+q_t) \right], \\ \theta_t^1(s_2) &= \left( \frac{p_t}{q_t p_{t+1}} \right) \left[ (1-\eta)(1+q_t) - q_t \gamma_t^1 \right], \end{aligned}$$

and

$$\begin{aligned} \theta_t^2(s_1) &= \left( \frac{p_t}{p_{t+1}} \right) \left[ \eta(1+q_t) - \gamma_t^2 \right], \\ \theta_t^2(s_2) &= \frac{1}{q_t} \left( \frac{p_t}{p_{t+1}} \right) \left[ \gamma_t^2 - \eta(1+q_t) \right]. \end{aligned}$$

## 5.1 Equilibrium

In equilibrium $\theta_t^1(s) + \theta_t^2(s) = 0$ for every $s$, so that the asset market clears. This condition implies that

$$q_t = \frac{\gamma_t^2 + 1 - 2\eta}{\gamma_t^1 + 2\eta - 1}. \tag{17}$$

Thus, the relative price of the Arrow securities, which is the relative cost of transferring resources from one solvency state of nature to the other, must be equal in equilibrium to the ratio of two expressions which depend on the fraction of deposits maintained in cash, $\gamma_t^j$. Equation (17) can also be rewritten as

$$q_t = \frac{2(1-\eta) - (1-\gamma_t^2)}{2\eta - (1-\gamma_t^1)}.$$

This expression shows that the relative cost of transferring goods across states of nature must be related to the probabilities of success for every entrepreneur type and to the fraction of deposits that banks are willing to lend.

Also, note that in equilibrium

$$(1+q_t)\frac{p_t}{p_{t+1}} = R_t^1(s_1) + q_t R_t^1(s_2) = R_t^2(s_1) + q_t R_t^2(s_2)$$

holds. Because we assumed that $R_t^1(s_2) = R_t^2(s_1) = 0$, the last equation is equivalent to

$$R_t^1(s_1) = q_t R_t^2(s_2).$$

Hence, the relative cost of transferring goods between states $s_1$ and $s_2$ must be equal to the relative returns that banks obtain from entrepreneur types when their projects are successful.

In equilibrium, the money market and the loans markets must also clear. These market clearing conditions remain unchanged:

$$\frac{M}{p_t} = \left(\gamma_t^1 + \gamma_t^2\right)x,$$

and

$$R_t^j = \frac{\alpha}{x^{1-\alpha}\left(1-\gamma_t^j\right)^{1-\alpha}}.$$

## 5.2 Steady state analysis

In Appendix A.4, we prove the  that the steady state equilibrium is unique and we show that there are no other equilibria. Specifically, we show the following:

**Proposition 2** *Under the condition* $\frac{\alpha^{\frac{1}{1-\alpha}}}{x} < \min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\}$ *there exists a unique steady state* $(\gamma^1, \gamma^2) \in \Re^2_{++}$*; the steady state is locally unstable. Hence, it is the unique equilibrium for this economy.*

Interestingly, unlike in the analysis of Antinolfi, Huybens and Keister (2001), this model does not allow for the presence of inflationary equilibrium trajectories when the central bank acts as a lender of last resort and lends at a zero nominal rate.[16]

[16] The different equilibrium behavior is due to the curvature present in the investment technology available to banks as well as the presence of real shocks and Arrow securities.

It is important to remark that the return that each lender type obtains is equal not only across types but also across regions. To see this, and dropping the subscript $t$ for simplicity, note that in steady state $r^j(\pi, s) = \frac{\eta(s)}{q_s}\left\{\sum_{s\in S} q_s\left[\gamma^j + \left(1-\gamma^j\right)R^j(s)\right]\right\}$. Given the definition of $q$ we have that

$$r^j(\pi, s_1) = \eta\left[\gamma^j + \left(1-\gamma^j\right)R^j(s_1) + q\left(\gamma^j + \left(1-\gamma^j\right)R^j(s_2)\right)\right],$$

$$r^j(\pi, s_2) = (1-\eta)\left[\left(\gamma^j + \left(1-\gamma^j\right)R^j(s_1)\right)\frac{1}{q} + \gamma^j + \left(1-\gamma^j\right)R^j(s_2)\right].$$

From the no-arbitrage conditions we know that $(1+q) = R^1 = q\,R^2$, or $\left(1+\frac{1}{q}\right) = R^2$, $1+q = R^1$, $\frac{R^1}{q} = R^2$ and $qR^2 = R^1$. Replacing these equalities in the expressions for $r^j(s, \pi)$ above gives

$$r^j(\pi, s_1) = \eta\left[\gamma^j R^1 + \left(1-\gamma^j\right)R^1\right] = \eta R^1; \quad j = 1, 2$$

$$r^j(\pi, s_2) = (1-\eta)\left[\left(1-\gamma^j\right)R^2 + \gamma^j R^2\right] = (1-\eta)R^2; \quad j = 1, 2.$$

Thus, $r^j(\pi, s) = \eta(s)R^s$, for $j = 1, 2$ and $s \in \{s_1, s_2\}$. In words, Arrow securities and the central bank acting together imply an allocation where *all* lenders obtain the same consumption quantity and consumption only depends on the realization of the real shock $s$.

It is not difficult to show that the equilibrium analyzed in this section is Pareto optimal. Intuitively, this is easy to see from the characteristics of the equilibrium allocation. First, borrowers are risk neutral. In addition, independently of the state of nature realized, all lenders have the same marginal rate of substitution. Specifically, all lenders consume, per unit deposited, $\eta R^1$ when $s_1$ is realized, and $(1-\eta)R^2$ when $s_2$ is realized. The marginal rate of substitution of consumption in the two states is simply $\frac{\eta R^1}{(1-\eta)R^2}$, where recall that $R^j = \alpha\left[\left(1-\gamma^j\right)x\right]^{\alpha-1}$. Finally, the marginal rate of substitution between consumption in state $s_1$ and $s_2$ is equal to the marginal rate of transformation between the same states of nature.[17] This is easily seen by noting that the social return to investing $k^j$ units of the consumption good in technology $j$ is, because of the law of large numbers, $\eta(s_j)g'(k^j)$, the probability of success multiplied by marginal product of the technology employed. In equilibrium, $R^j = g'(k^j)$ and $k^j = \alpha\left[\left(1-\gamma^j\right)x\right]^{\alpha-1}$. Therefore, the marginal rate of transformation is equal to the marginal rate of substitution. In Appendix A.5, we prove formally

[17] Recall that an arbitrage condition insures that fiat money and real investments are seen as equally good means to transfer resources across states of nature, given the presence of Arrow securities, and that in steady state the return on money is set to unity.

that the equilibrium allocation is Pareto optimal by analyzing the problem of a central planner, and show that the equilibrium obtained in this section decentralizes the allocation chosen by the central planner.

# 6 Discussion

## 6.1 The concept of *liquidity*

After having presented the main results, it is useful to get a deeper look at some of the concepts involved in the model. Unlike risk that relates to uncertainty over the total amount of resources available at a future date, *liquidity risk* relates to uncertainty over the distribution of a given amount of resources over a certain period of time. Liquidity, described as above, is the property of an asset, and relates to the readiness with which resources committed to a certain utilization can be transformed into consumption without loss.[18]

An important feature which links these two kinds of risk is that in both cases the result of an (aggregate) *unfavorable* shock is a relative scarcity of consumption at a point in time: in the case of someone who faces a liquidity shock which takes the form of an impending need to consume, even if it is known that resources will be available, this knowledge does not help satisfying present consumption needs. In this sense, even though liquidity and real risk pertain to different economic phenomena, their result is the same. However, at the level of the individual investor, the role that *fiat money* plays in this notion of liquidity should become clear. Money is the most liquid asset in the sense that its value (in money terms) is certain and equal to unity. Thus, for an individual investor money is a safe asset in the sense that it can hedge from both liquidity and real risk, provided that its purchasing power is stable over time, at the (opportunity) cost represented by the return on alternative investments. At the aggregate level, however, fiat money cannot play this role. Money does not, in the aggregate, allow the economy to transfer consumption over time, because being an outside asset it is liquid (readily transformable into consumption) only in as much as consumption goods are available.[19] Fiat money does not increase the aggregate consumption possibilities at a point in time.

It would seem then that money does not provide liquidity insurance in the aggregate. However,

[18] For an account of the early conceptual history of the idea of liquidity see Hicks' (1962) presidential address delivered at the end of his mandate as president of the Royal Economic Society.

[19] This is the notion of liquidity used by the search literature (see, e.g., Lagos, 2006).

there is another kind of demand for liquidity for which fiat money does provide a fundamental role. This occurs when, in the portfolio of individual investors, liabilities issued by private borrowers are perceived to be undesirable and there is a shift of portfolio investment towards liquid assets, money in particular (of course assuming that money has a known and stable purchasing power). In these cases, there is no need to inefficiently liquidate fixed investment to satisfy consumption needs, but the inefficient liquidation of investment goes to finance a portfolio composition shift in which money has a larger weight. In other words, the manifestation of liquidity risk in this case resides into a shift from certain types of liabilities into fiat money. It is with respect to this kind risk the function of the central bank becomes fundamental, and this is the role that the central bank plays in our analysis of section 5.

Our notion of liquidity is not always identical to the one adopted elsewhere. The following subsection explores more in depth the relationship that this paper has with the relevant literature.

### 6.2 A short overview of the literature.

As briefly described in the introduction, Allen and Gale (2004) model a real economy in which it is possible to identify the same two types of shocks as in this paper. They relate the existence of different intermediary contracts to their ability to complete markets. Their way of modelling financial intermediaries and (complete) security markets is the same as in this paper, although the nature of the shocks is somewhat different. They prove that the presence of financial intermediaries and complete Arrow-security markets together imply constrained Pareto-efficient allocations (as long as investors cannot trade in those securities). They also consider particular examples of incomplete security markets to analyze the role of financial regulation in improving efficiency in the absence of the aggregate component of liquidity shocks. One way to compare our results with theirs is to emphasize that we actually start from an economy in which markets are *incomplete*, both relative to liquidity risk and real risk. In this formulation *both* private banks *and* the central bank complete the market with respect to liquidity shocks, while securities complete markets with respect to "real" risk.

Allen and Gale (1994) present a model of a barter economy with incomplete asset markets (and no banks) where investors are subject to preference shocks like those described in Allen and Gale (2004), but face entry costs in asset markets. Liquidity is defined as consumption goods for immediate delivery, with no possible reference to cash (given absence of fiat money in the model).

They can show the existence of an equilibrium with limited participation and highly volatile asset prices even with small aggregate component of the preference shock. Such equilibrium is shown to be Pareto-inferior to the full participation equilibrium (with low volatility of asset prices). In fact, the amount of liquidity in goods is key in determining the volatility of such prices. The results of our model in section 4 state, on the other hand, that the $level$ of the relative price of assets in equilibrium is affected by the distribution of liquidity $demand$ shocks, which also determine (endogenously) the amount of liquidity in equilibrium. Results in section 5 also state that, given a $right$ amount of liquidity provided in fiat money by the central bank, there is no interaction between asset prices and liquidity shocks. This result could be interpreted as the analog to Proposition 6 in Allen and Gale (1994).

The paper by Allen and Gale (2006), on the other hand, discusses the role of liquidity shocks in increasing the volatility of asset prices relative to fundamentals in a special case of the model in Allen and Gale (2004). They show that even when the aggregate component of the liquidity shock becomes negligible, in equilibrium asset prices are non-trivially volatile, implying the presence of robust sunspot equilibria, given that Arrow securities are completely absent from the analysis. The model in this paper, in turn, shows that if we introduce Arrow securities whose payoffs are contingent on the real shocks only, their equilibrium prices also interact with the liquidity shock in the absence of a central bank: the insurance that such securities provide against solvency shocks is less valuable the more the probability distribution of liquidity shocks is skewed towards high values.[20]

Gale (2005) presents a two-period economy where consumers face cash-in-advance constraints not only to purchase goods but also to buy securities. He investigates the effects of monetary policy decisions on the volatility of equilibrium (real) asset prices, among other topics. One implication of his model is that, if the central bank provides the $right$ amount of liquidity in fiat money, $wild$ fluctuations in interest rates and asset prices can be eliminated. Our model, in turn, reaches a similar conclusion in terms of the $level$ of (relative) asset prices, although providing a slightly stronger result. With the $right$ amount of liquidity in fiat money, the central bank, by operating in an economy with (active) Arrow security markets and private banks, allows the economy to achieve Pareto efficiency. One important reason is that, while in Gale (2005) there is a trade-off

[20] Lagos (2006) using a search model shows that equilibrium asset returns may contain a liquidity premium that reflects that ability, which is a result that is related to ours about the dependence of Arrow security prices on the distribution of preference shocks.

between stabilizing asset prices and collecting seignorage, this is not the case in our model, given that the price level our analysis does *not* depend on the aggregate state, a property that is absent in Gale (2005).

As discussed in the introduction, there is already a related literature on liquidity provision in barter economies. Allen and Gale (1997) study the different reaction of consumption in response to real shocks in an overlapping generations economy, and show that the volatility of consumption is different in bank-based and market-based financial systems. In their model banks induce a Pareto-efficient equilibrium allocation while markets do not. On the other hand, Holmstrom and Tirole (1998) study the optimal provision of liquidity in a real model with moral hazard in which banks supply liquidity. They also show that banks are Pareto-superior to markets in this role when no aggregate uncertainty is present. In their framework, unlike the others mentioned so far, firms face liquidity shocks. In the absence of aggregate risk a commitment problem needs the coordinating role of banks for the efficient allocation of liquidity (which markets are not able to provide). In one case, when there is only aggregate risk, Holmstrom and Tirole (1998) show that government debt is indeed the tool for optimal liquidity provision. Clearly this result resembles that of the present paper, where the central bank plays a key role in providing aggregate liquidity in our model.

The mechanisms in this paper and in Holmstrom and Tirole (1998) which allow for efficient equilibria are similar, although with different implications. Holmstrom and Tirole (1998) rely on the ability to commit to future taxation. This paper, instead, relies on a lender of last resort: the central bank issues fiat money today against resources that will be available in the future. The enforcement assumptions in each case are quite different. Without intending to analyze this issue in depth, it is apparent that the nature of the commitment that a central bank (as a lender of last resort) may have on this type of contingent credits is different from the commitment to future taxation.

# 7 Conclusions

We presented a model to study the different roles of different financial institutions (in the presence of restricted participation by consumers in securities markets). In this setting, banks have the role of providing efficient idiosyncratic liquidity-risk sharing among depositors. For the aggregate component of liquidity risk, central bank short-term loans are also necessary to supply an asset (fiat money) that completes the market against this intertemporal type of risk. Securities markets allow society to diversify solvency risk, that is, the uncertainty about the amount of resources available in the economy at a certain future date. When Arrow securities are written only on solvency contingencies, markets are complete only with the three institutions simultaneously present in the economy. This model (with those three institutions) produces an equilibrium allocation which is the same that would obtain in an economy with complete markets (in the traditional sense of a sufficient number of contingent claims to span the space generated by the number of states of nature) and a safe asset, and where the liquidity shock is not present.

Note that this last result has a direct implication for monetary policy. When this policy is conducted in order to achieve the best (i.e., Pareto-efficient) risk sharing allocation, the result states that the amount of fiat money lent to banks must take into account the long run return on illiquid loans (which in our model is assumed to be *known* to the central bank as well as the rest of the economy) vis-a-vis the amount of cash available to banks. This policy plays the same role as the tax-and-transfer policy in Holmstrom and Tirole (1998), that is, this mechanism allows banks to $bring$ forward goods (purchasing power of money) from the future to the present, but without recurring to any type of future taxation. The reason for this difference is the notion of liquidity adopted.

The model also generates interesting implications of risk sharing and asset pricing in incomplete markets. When banks are the only financial institution in the economy, in equilibrium liquidity shortages only occur in the $region$ where the loans of the banks are performing, but it does not occur where such loans are non-performing. When Arrow securities are added to this economy, the equilibrium prices of such assets interact with the liquidity distribution in a non-trivial fashion. The reason is that the evaluation of assets interact with the amount of cash reserves held by banks, and these reserves depend crucially on the ex-ante probability distribution of cash withdrawals.

There are several special features of the model economy that allow banks, asset markets and banks to coexist and deliver an efficient equilibrium allocation. First, as it is common in this literature, we restrict market participation:[21] only intermediaries trade in asset markets. Even though one could think of individual participation costs to justify this assumption, it is a fundamental assumption in generating a separation of roles between asset markets and banks.

Second, we assumed that banks cannot perfectly diversify credit risk directly, which is essen-

[21] See Allen and Gale (2004), and Diamond (1997).

tially another form of limited market participation. Letting banks perfectly diversify credit risk would subtract the fundamental rationale for markets in the model, but in general would not generate the same equilibrium allocation obtained with a model with markets. This is a common feature of all model relying on the Diamond and Dybvig (1983) framework, in which the coefficient of relative risk aversion plays an important role.

Finally, the central bank acts in the model as a perfect lender of last resort. The lender of last resort role of the central bank stems from the (aggregate) nature of the liquidity shock. The central bank can allow the economy to have complete risk insurance against liquidity shocks for two reasons. First, money is a perfect vehicle to transfer consumption across periods for an individual bank, that is it is not a risky asset and it is made freely (that is, at a zero net interest rate) available to the banking system. Second, banks know exactly that the central bank will do always the "right thing," and the central bank can always deliver on that expectation, because it is always possible to distinguish precisely between real and nominal shocks.

Adding some of these features, in particular adding uncertainty over the central bank behavior and removing the ability to perfectly identify liquidity and real shocks would further the study of the interaction among asset prices, interest rates, central banking, and intermediation.

# Appendix A. Proofs and Derivations

This appendix collects most of the formal analysis. The paper is long, and the appendix is not meant for publication but to provide a reference for the refereeing process.

## A.1 Proof of proposition 1

We need to show the existence of a unique pair of values $(\gamma^1, \gamma^2)$ each of which satisfies

$$\gamma^j = 1 - \eta(s_j) \int_{\frac{\gamma^j}{\gamma^j + \phi(1-\gamma^j)^\alpha}}^1 F(\pi)\, d\pi,$$

which is equivalent to show

$$\frac{1-\gamma^j}{\eta(s_j)} = \int_{\frac{\gamma^j}{\gamma^j + \phi(1-\gamma^j)^\alpha}}^1 F(\pi)\, d\pi.$$

The left hand side is a strictly decreasing line in $\gamma^j$, with slope $-\frac{1}{\eta(s_j)}$. At $\gamma^j = 0$ the left hand side is equal to $\frac{1}{\eta(s_j)} > 1$. Define $\Phi(\gamma^j) \equiv \int_{\frac{\gamma^j}{\gamma^j + \phi(1-\gamma^j)^\alpha}}^1 F(\pi)\, d\pi$. We then have that $\Phi(0) = \int_0^1 F(\pi)$ $d\pi = 1 - E[\pi] < 1$. We also have

$$\Phi'(\gamma^j) = -F\left[\frac{\gamma^j}{\gamma^j + \phi(1-\gamma^j)^\alpha}\right]\left[\frac{\gamma^j + \phi(1-\gamma^j)^\alpha - \gamma^j + \gamma^j \alpha\phi(1-\gamma^j)^{\alpha-1}}{[\gamma^j + \phi(1-\gamma^j)^\alpha]^2}\right]$$

Recalling that $\pi_j^*(\gamma^j, s_j) = \frac{\gamma^j}{\gamma^j + \phi(1-\gamma^j)^\alpha}$ then it is possible to show that

$$\Phi'(\gamma^j) = -\frac{F\left[\pi_j^*(\gamma^j, s_j)\right]\phi}{[\gamma^j + \phi(1-\gamma^j)^\alpha]^2 [1-\gamma^j]^{1-\alpha}}\left[1 - \gamma^j(1-\alpha)\right]$$

Thus, $\gamma^j > 0$ and $(1-\alpha) > 0$ imply $\Phi'(\gamma^j) < 0$. Note that $\lim_{\gamma^j \to 1} \Phi'(\gamma^j) = \infty$, since $\pi_j^*(1, s_j) = 1$,$F(1) = 1$, and $\alpha < 1$. Therefore, the curve $\Phi(\gamma^j)$ must intersect the line from above. By continuity, this implies that there exists at least one $\gamma^{j*} \in (0,1)$ such that $\Phi(\gamma^{j*}) = \frac{1-\gamma^{j*}}{\eta(s_j)}$. Now we show that this value is unique. It is enough to show that $|\Phi'(\gamma^j)| < \frac{1}{\eta(s_j)}$ at $\gamma^{j*}$. If this is the case, the existence of a second $\gamma^{j*}$ implies $|\Phi'(\gamma^j)| > 1$, a contradiction. Note that $|\Phi'(\gamma^j)|$ is equal to (after some tedious algebra):

$$\frac{F\left[\pi_j^*(\gamma^j, s_j)\right]\left[1 - \pi_j^*(\gamma^j, s_j)\right]\pi_j^*(\gamma^j, s_j)}{\gamma^j(1-\gamma^j)}\left[1 - \gamma^j + \alpha\gamma^j\right].$$

But since $\alpha < 1$ then this expression is strictly less than $\frac{F\left[\pi_j^*(\gamma^j, s_j)\right]\left[1-\pi_j^*(\gamma^j, s_j)\right]\pi_j^*(\gamma^j, s_j)}{\gamma^j(1-\gamma^j)}$. Hence it is enough to show that $\frac{F\left[\pi_j^*\left(\gamma^j,s_j\right)\right]\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right]\pi_j^*\left(\gamma^j,s_j\right)}{\gamma^j\left(1-\gamma^j\right)} < \frac{1}{\eta(s_j)}$, or equivalently

$$\eta\left(s_j\right) F\left[\pi_j^*\left(\gamma^j, s_j\right)\right]\left[1-\pi_j^*\left(\gamma^j, s_j\right)\right] \pi_j^*\left(\gamma^j, s_j\right) < \gamma^j\left(1-\gamma^j\right)$$

In equilibrium,

$$\gamma^j\left(1-\gamma^j\right) = \left[1-\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi\right]\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi$$

Since $F(\pi)$ is strictly increasing,

$$\gamma^j\left(1-\gamma^j\right) > \left[1-\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi\right]\eta\left(s_j\right)\left(\int_{\pi_j^*(\gamma^j,s_j)}^1 F\left(\pi_j^*\left(\gamma^j,s_j\right)\right)d\pi\right)$$

But $\int_{\pi_j^*(\gamma^j,s_j)}^1 F\left(\pi_j^*\left(\gamma^j,s_j\right)\right)d\pi = \left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right)$. Given that $F(\pi) < 1$ for all $\pi < 1$, then $\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi < 1-\pi_j^*\left(\gamma^j,s_j\right)$. Since $\eta\left(s_j\right) > 0$ then $\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi$ $< \eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] \Leftrightarrow -\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi > -\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] \Leftrightarrow 1-\eta\left(s_j\right)$ $\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi > 1-\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right]$. Therefore

$$\begin{aligned} & \left[1-\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi\right]\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right) \\ > \; & \left[1-\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right]\right]\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right) \end{aligned}$$

But since $\eta\left(s_j\right) < 1$, then $1-\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] > 1-\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] = \pi_j^*\left(\gamma^j,s_j\right)$. Thus,

$$\begin{aligned} & \left[1-\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right]\right]\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right) \\ > \; & \pi_j^*\left(\gamma^j,s_j\right)\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right) \end{aligned}$$

We have shown that

$$\begin{aligned} \gamma^j\left(1-\gamma^j\right) &= \left[1-\eta\left(s_j\right)\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi\right]\eta\left(s_j\right)\left(\int_{\pi_j^*(\gamma^j,s_j)}^1 F(\pi)\,d\pi\right) \\ &> \pi_j^*\left(\gamma^j,s_j\right)\eta\left(s_j\right)\left[1-\pi_j^*\left(\gamma^j,s_j\right)\right] F\left(\pi_j^*\left(\gamma^j,s_j\right)\right) \end{aligned}$$

which is what we wanted to demonstrate. ■

### A.2 Existence of equilibrium in the economy with Arrow securities and without a central bank.

In the economy with only Arrow securities the objective function for bank 1 includes an expression equal to

$$\int_{\pi_{11}^*}^{1} \left\{ \pi \log \gamma^1 + (1-\pi) \log \left[ \left(1-\gamma^1\right) R_1 - \frac{q_1}{q_2} \theta_1^2 \right] \right\} f(\pi)\, d\pi,$$

which, after replacing $\theta_2^1$, $R_1$ and $\frac{q_1}{q_2}$ with the expressions obtained in the text, becomes

$$\int_{\pi_{11}^*}^{1} \left\{ \pi \log \gamma^1 + (1-\pi) \log \left[ \phi \left(1-\gamma^1\right)^{\alpha-1} \left( 1 - \gamma^1 - (1-\eta) \int_{\pi_{1,2}^*}^{1} F(\pi)\, d\pi \right) \right] \right\} f(\pi)\, d\pi.$$

Note that $\pi_{1,1}^* = 1$ obtains because $1 - \gamma^1 = (1-\eta) \int_{\pi_{1,2}^*}^{1} F(\pi)\, d\pi$. This expression would equal an indeterminate expression, since $\int_{1}^{1} (1-\pi) f(\pi)\, d\pi = 0$ and $\log[0] = -\infty$. However, it can be shown that when $\gamma^1$ converges (from above) to $1 - (1-\eta) \int_{\pi_{1,2}^*}^{1} F(\pi)\, d\pi$, the expression

$$\log \left[ 1 - \gamma^1 - (1-\eta) \int_{\pi_{1,2}^*}^{1} F(\pi)\, d\pi \right] \int_{\pi_{11}^*}^{1} (1-\pi) f(\pi)\, d\pi$$

converges to 0 for $\gamma^1$ sufficiently close to $1 - (1-\eta) \int_{\pi_{1,2}^*}^{1} F(\pi)\, d\pi$. Hence the equilibrium with $\pi_{jj}^* = 1$ exists as long as we define the equilibrium value of the expression in the objective function mentioned above equal to its limit (equal to 0).

### A.3 Determination of the demand for Arrow securities when the central bank is present

The problem of the bank can be written as

$$\max \sum_{s \in S} \eta(s) \left[ \int_0^1 \left( \pi \ln \left( r_t^{mj}(s,\pi) \right) + (1-\pi) \ln \left( r_t^j(s,\pi) \right) \right) f(\pi)\, d\pi \right]$$

subject to

$$\sum_{s \in S} q_s \left[ \pi r_t^{mj}(s,\pi) + (1-\pi) r_t^j(s,\pi) \right] = \sum_{s \in S} q_s \left[ \gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s) \right]$$

Let $\lambda(\pi)$ be the multiplier associated with this constraint. We solve the problem backwards. We first take $\gamma_t^j$ as given and also we take every possible realization of $\pi$ as given. The Lagrangian for this problem is

$$\begin{aligned}\mathcal{L}(\pi) &= \sum_{s\in S}\eta(s)\left(\pi\ln\left(r_t^{mj}(s,\pi)\right)+(1-\pi)\ln\left(r_t^j(s,\pi)\right)\right) \\ &\quad+\lambda(\pi)\sum_{s\in S}q_{st}\left[\gamma_t^j\frac{p_t}{p_{t+1}}+\left(1-\gamma_t^j\right)R_t^j(s)-\left[\pi r_t^{mj}(s,\pi)+(1-\pi)r_t^j(s,\pi)\right]\right]\end{aligned}$$

The first order conditions are

$$\begin{aligned}\frac{\partial\mathcal{L}(\pi)}{\partial r_t^{jm}(s,\pi)} &= 0\Leftrightarrow\frac{\eta(s)}{r_t^{mj}(s,\pi)}=\lambda(\pi)q_{st} \\ \frac{\partial\mathcal{L}(\pi)}{\partial r_t^{j}(s,\pi)} &= 0\Leftrightarrow\frac{\eta(s)}{r_t^{j}(s,\pi)}=\lambda(\pi)q_{st}\end{aligned}$$

Therefore $r_t^{mj}(s,\pi)=r_t^j(s,\pi)$. Replacing this on the left hand side of the constraint yields

$$\sum_{s\in S}q_{st}\left[\pi r_t^{mj}(s,\pi)+(1-\pi)r_t^j(s,\pi)\right]=\frac{\pi}{\lambda(\pi)}+\frac{(1-\pi)}{\lambda(\pi)}=\frac{1}{\lambda(\pi)}$$

Then, the constraint can be rewritten as

$$\frac{1}{\lambda(\pi)}=\sum_{s\in S}q_{st}\left[\gamma_t^j\frac{p_t}{p_{t+1}}+\left(1-\gamma_t^j\right)R_t^j(s)\right]$$

and we know that $q_{st}r_t^{mj}(s,\pi)=\frac{\eta(s)}{\lambda(\pi)}$, which implies

$$r_t^{mj}(s,\pi)=\frac{\eta(s)}{q_{st}}\left\{\sum_{s\in S}q_{st}\left[\gamma_t^j\frac{p_t}{p_{t+1}}+\left(1-\gamma_t^j\right)R_t^j(s)\right]\right\}.$$

We also had $r_t^{mj}(s,\pi)=r_t^j(s,\pi)$. Therefore, replacing this in the objective function, for every $\pi$ and $s$ we get that $\left[\pi\ln\left(r_t^{mj}(s,\pi)\right)+(1-\pi)\ln\left(r_t^j(s,\pi)\right)\right]$ is equal to

$$\ln\left\{\frac{\eta(s)}{q_{st}}\left\{\sum_{s\in S}q_{st}\left[\gamma_t^j\frac{p_t}{p_{t+1}}+\left(1-\gamma_t^j\right)R_t^j(s)\right]\right\}\right\}.$$

Therefore, the ex-ante utility is

$$\sum_{s\in S}\eta(s)\left[\int_0^1\left(\pi\ln\left(r_t^{mj}(s,\pi)\right)+(1-\pi)\ln\left(r_t^j(s,\pi)\right)\right)f(\pi)\,d\pi\right]=$$

$$= \sum_{s\in S} \eta(s) \ln\left(\frac{\eta(s)}{q_{st}}\right) + \ln\left[\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]\right].$$

Then, choosing $\gamma_t^j$ to maximize $\sum_{s\in S} \eta(s) \ln\left(\frac{\eta(s)}{q_s}\right) + \ln\left[\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]\right]$ is the same as choosing $\gamma_t^j$ that maximizes $\ln\left[\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]\right]$, subject to $\gamma_t^j \in [0,1]$. Therefore, we will have that $\gamma_t^j \in (0,1)$ if

$$(q_{1t}+q_{2t}) \frac{p_t}{p_{t+1}} = q_{1t} R_t^j(s_1) + q_{2t} R_t^j(s_2)$$

for every $j$. Otherwise, we have a corner solution. For the case of an interior solution, we have

$$\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right] = q_{1t} R_t^j(s_1) + q_{2t} R_t^j(s_2)$$

since $\left[\frac{p_t}{p_{t+1}}(q_{1t}+q_{2t}) - \left(q_{1t} R_t^j(s_1) + q_{2t} R_t^j(s_2)\right)\right] = 0$. Now let $q_t \equiv \frac{q_{2t}}{q_{1t}}$. The no-arbitrage condition can be expressed as

$$(1+q_t) \frac{p_t}{p_{t+1}} = R_t^j(s_1) + q_t R_t^j(s_2).$$

Let us consider the problem of the bank for each region.

- **Region 1 Bank**

According to the derivation above, in this case we have

$$\sum_{s\in S} q_{st}\left[\gamma_t^1 \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^1\right) R_t^1(s)\right] = q_{1t} R_t^1 + 0$$

Recall that for every $j$, $\theta_t^j(s) = \pi\, r_t^{mj}(s,\pi) + (1-\pi)\, r_t^j(s,\pi) - \left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]$, that $r_t^{mj}(s,\pi) = r_t^j(s,\pi)$, and that:

$$r_t^{mj}(s,\pi) = \frac{\eta(s)}{q_{st}}\left\{\sum_{s\in S} q_{st}\left[\gamma_t^j \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^j\right) R_t^j(s)\right]\right\}.$$

Thus, for $j=1$:

$$\theta_t^1(s) = \left[\frac{\eta(s)}{q_{st}}\right] q_{1t} R_t^1 - \left[\gamma_t^1 \frac{p_t}{p_{t+1}} + \left(1-\gamma_t^1\right) R_t^1(s)\right].$$

Then, $\theta_t^1(s_1) = \eta R_t^1 - \gamma_t^1 \frac{p_t}{p_{t+1}} - \left(1-\gamma_t^1\right) R_t^1$. But from the no-arbitrage condition $(1+q_t)\frac{p_t}{p_{t+1}} = R_t^1(s_1) + q_t\, R_t^1(s_2) = R_t^1$, so $R_t^1 = (1+q_t)\frac{p_t}{p_{t+1}}$. Replacing this in the expression for $\theta_t^1(s_1)$

above we get:

$$\theta_t^1(s_1) = \left(\frac{p_t}{p_{t+1}}\right)\left[q_t\gamma_t^1 - (1-\eta)(1+q_t)\right].$$

We can now solve for $\theta_t^1(s_2)$ using identical arguments. Recalling that $R_t^1 = (1+q_t)\frac{p_t}{p_{t+1}}$ and using the equation for $\theta_t^1(s)$ at $s = s_2$ then:

$$\theta_t^1(s_2) = \left(\frac{p_t}{q_t p_{t+1}}\right)\left[(1-\eta)(1+q_t) - q_t\gamma_t^1\right]$$

- **Region 2 Bank**

  Following the same steps as above we obtain:

$$\theta_t^2(s_1) = \left(\frac{p_t}{p_{t+1}}\right)\left[\eta(1+q_t) - \gamma_t^2\right]$$

and

$$\theta_t^2(s_2) = \frac{1}{q_t}\left(\frac{p_t}{p_{t+1}}\right)\left[\gamma_t^2 - \eta(1+q_t)\right].$$

### A.4 Proof of proposition 2

Before starting the proof, we show an implication of the statement in this proposition. Let $\phi$ be defined as above. We show that the condition $\phi < \min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\}$ implies that $\phi < 2^{1-\alpha}$ $\left\{\eta^{1-\alpha} + (1-\eta)^{1-\alpha}\right\}$. To see this, note that $\min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\} = \frac{1}{1-\eta}$ when $\eta < \frac{1}{2}$ and $\min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\}$ $= \frac{1}{\eta}$ for $\eta > \frac{1}{2}$. For $\eta < \frac{1}{2}$ define

$$\varphi_1(\eta) \equiv 2^{1-\alpha}\left\{\eta^{1-\alpha} + (1-\eta)^{1-\alpha}\right\} - \frac{1}{1-\eta}.$$

Note that $\varphi_1(0) = 2^{1-\alpha} - 1 > 0$ (since $2^{1-\alpha} > 1^{1-\alpha} = 1$). Also, $\varphi_1\left(\frac{1}{2}\right) = 0$, and we have that

$$\varphi_1'(\eta) = 2^{1-\alpha}(1-\alpha)\left[\eta^{-\alpha} - (1-\eta)^{-\alpha}\right] - \frac{1}{(1-\eta)^2},$$

$$\varphi_1''(\eta) = 2^{1-\alpha}(1-\alpha)(-\alpha)\left[\eta^{-(1+\alpha)} + (1-\eta)^{-(1+\alpha)}\right] - \frac{2}{(1-\eta)^3} < 0.$$

The first expression is zero for some $\eta_1 \in \left(0, \frac{1}{2}\right)$. Clearly $\varphi_1(\eta_1) \geq 2^{1-\alpha} > 0$. Hence, the function $\varphi_1(\eta)$ attains a strictly positive value at its maximum in $\left(0, \frac{1}{2}\right)$ and the function is strictly concave in the whole interval. This implies that $\varphi_1(\eta) > 0$ for all $\eta \in \left(0, \frac{1}{2}\right)$. For $\eta \in [0, \frac{1}{2})$ then,

$2^{1-\alpha}\left\{\eta^{1-\alpha}+(1-\eta)^{1-\alpha}\right\}>\frac{1}{1-\eta}$ holds. For $\eta>\frac{1}{2}$, consider

$$\varphi_2(\eta)\equiv 2^{1-\alpha}\left\{\eta^{1-\alpha}+(1-\eta)^{1-\alpha}\right\}-\frac{1}{\eta}.$$

Clearly $\varphi_2\left(\frac{1}{2}\right)=0$ and $\varphi_2(1)=2^{1-\alpha}-1>0$. Computing the derivatives, it easy to show that $\varphi_2'(\eta)=0$ for some $\eta=\eta_2\in\left(\frac{1}{2},1\right)$. Since $\varphi_2$ is strictly concave, the function $\varphi_2$ attains a strictly positive value at $\eta_2$ and therefore $\varphi_2(\eta)>0$ for all $\eta\in(\frac{1}{2},1]$. These two arguments state then that $\min\left\{\frac{1}{\eta},\frac{1}{1-\eta}\right\}\leqq 2^{1-\alpha}\left[\eta^{1-\alpha}+(1-\eta)^{1-\alpha}\right]$, where the equality only holds at $\eta=\frac{1}{2}$. This will be used for future reference.

We first show that there exists a unique pair $(R^1,R^2)$ satisfying the stationary equilibrium equations. Note that the first condition can be written as $\frac{R^1+R^2}{R^2}=R^1$, which simplifies to

$$R^2=\frac{R^1}{R^1-1}. \tag{18}$$

The second equation can be rewritten as

$$2\eta R^1-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}}=2(1-\eta)R^2-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}}. \tag{19}$$

Hence, we have two equations in two unknowns. The first equation defines a curve on the plane $(R^1,R^2)$ which is strictly decreasing with asymptotes at $(1,1)$. The second equation also defines implicitly a curve on the plane $(R^1,R^2)$. To get the derivative we apply the implicit function theorem to the map:

$$\Lambda\left(R^1,R^2\right)\equiv-\left(2(1-\eta)R^2-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}}\right)+2\eta R^1-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}}$$

at the point where $\Lambda(R^1,R^2)=0$. In general, we have that $\frac{dR^2}{dR^1}=-\frac{\Lambda_{R^1}\left(R^1,R^2\right)}{\Lambda_{R^2}(R^1,R^2)}$. In this case we have

$$\Lambda_{R^1}\left(R^1,R^2\right)=2\eta+\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(\frac{\alpha}{1-\alpha}\right)\left(R^1\right)^{\frac{-\alpha}{1-\alpha}-1}$$

$$\Lambda_{R^2}\left(R^1,R^2\right)=-\left(2(1-\eta)+\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\frac{\alpha}{1-\alpha}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}-1}\right)$$

so that

$$\frac{dR^2}{dR^1} = \frac{2\eta + \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(\frac{\alpha}{1-\alpha}\right)\left(R^1\right)^{\frac{-\alpha}{1-\alpha}-1}}{\left(2\left(1-\eta\right) + \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\frac{\alpha}{1-\alpha}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}-1}\right)} > 0$$

It remains to show that for low $R^1$ the curve defined by $(18)$ is above the curve defined by $(19)$, and for large $R^1$ the reverse is true. Clearly, according to $(18)$, $R^2$ approaches infinity when $R^1 \downarrow 1$. According to $(18)$ it is also true that as $R^1 \uparrow \infty$ then $R^2 \downarrow 1$. To understand the behavior of $(19)$ it is convenient to write down the condition

$$2\left(1-\eta\right)R^2 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}} = 2\eta R^1 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}}.$$

Suppose first that $R^1 \downarrow 0$. Therefore $2\eta R^1 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}} \downarrow -\infty$ since $\frac{-\alpha}{1-\alpha} < 0$. Hence 2 $(1-\eta)$ $R^2 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}} \downarrow -\infty$ whenever $R^1 \downarrow 0$. Therefore it must happen that $R^2 \downarrow 0$. Suppose this is not the case. Then we could have $R^2 \downarrow R^*$ finite and positive. But then $\left(2\left(1-\eta\right)R^2 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}}\right)$ has a finite limit and so for a sufficient small $R^1$ the equality

$$\left(2\left(1-\eta\right)R^2 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}}\right) = 2\eta R^1 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}}$$

is not satisfied. Since the curve is strictly increasing, $R^2$ cannot go to $+\infty$ when $R^1$ goes to $0$. Hence, when $R^1 \downarrow 0$ then $R^2 \downarrow 0$ and so the curve tends towards the origin. Also, at $R^1 = 1$ the value of $R^2$ is finite. Therefore, at $R^1 = 1 + \varepsilon$ with $\varepsilon > 0$ and small the first curve is strictly above the second curve. Also, when $R^1 \uparrow +\infty$ we will have that $R^2 \uparrow +\infty$. Otherwise, if $R^2 \uparrow R^{**} < +\infty$, the expression $2\left(1-\eta\right)R^2 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}}$ goes towards a finite number, but the expression $2\eta R^1 - \frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}} \uparrow +\infty$ when $R^1$ does, therefore the equality should not hold for sufficiently big $R^1$. Because the curve is strictly increasing, it cannot happen that while $R^1 \uparrow +\infty$ then $R^2 \downarrow -\infty$. Therefore, for sufficiently large $R^1$ the value of $R^2$ must also be very large. Therefore the second curve is strictly above the first curve for $R^1$ big. Because of the monotonicity properties and continuity, there exists a unique pair $(R^1, R^2)$ where the two curves intersect. Because of the properties of $(18)$, both $R^1$ and $R^2$ must be strictly greater than one.

Now we show that this pair of interest rates is a stationary equilibrium. To do this we show that there is an equivalence between this pair of interest rates and the pair $(\gamma^1, \gamma^2) \in (0,1)^2$ that solves the steady state of the dynamic system in $\gamma_t^j$.

Suppose $(R^1,R^2)$ is a steady state for the system

$$\frac{2-\left(\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\right)\left(\frac{1}{\left(R_{t+1}^1\right)^{\frac{1}{1-\alpha}}}+\frac{1}{\left(R_{t+1}^2\right)^{\frac{1}{1-\alpha}}}\right)}{2-\left(\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\right)\left(\frac{1}{\left(R_t^1\right)^{\frac{1}{1-\alpha}}}+\frac{1}{\left(R_t^2\right)^{\frac{1}{1-\alpha}}}\right)}=\frac{R_t^1R_t^2}{R_t^1+R_t^2};\quad \frac{R_t^1}{R_t^2}=\frac{2\left(1-\eta\right)-\frac{\alpha^{\frac{1}{1-\alpha}}}{\left(R_t^2\right)^{\frac{1}{1-\alpha}}x}}{2\eta-\frac{\alpha^{\frac{1}{1-\alpha}}}{\left(R_t^1\right)^{\frac{1}{1-\alpha}}(x)}}$$

and that $R^j>1$. Then, it must be true that $1=\frac{R^1R^2}{R^1+R^2}$ and

$$\frac{R^1}{R^2}=\frac{2\left(1-\eta\right)-\frac{\alpha^{\frac{1}{1-\alpha}}}{(R^2)^{\frac{1}{1-\alpha}}(x)}}{2\eta-\frac{\alpha^{\frac{1}{1-\alpha}}}{(R^1)^{\frac{1}{1-\alpha}}(x)}}.$$

Hence, given that $1-\gamma^j=\frac{\alpha^{\frac{1}{1-\alpha}}}{(R^j)^{\frac{1}{1-\alpha}}(x)}$ and since $R^j>1$ it is clear that $\gamma^j<1$. Recall $\phi\equiv\frac{\alpha}{(x)^{1-\alpha}}$. Since we assumed that $\phi<2^{1-\alpha}\left[\eta^{1-\alpha}+(1-\eta)^{1-\alpha}\right]$ we claim that in this steady state $R^1>\frac{\phi}{(2\eta)^{1-\alpha}}$ and $R^2>\frac{\phi}{(2(1-\eta))^{1-\alpha}}$. To show this, it sufficient to show that the point $\widehat{R}^1\equiv\frac{\phi}{(2\eta)^{1-\alpha}}$, $\widehat{R}^2\equiv\frac{\phi}{(2(1-\eta))^{1-\alpha}}$ is to the left of the intersection of the two curves. To prove this last fact, note that at $\left(\widehat{R}^1,\widehat{R}^2\right)$ it is true that

$$\begin{aligned}\phi\left(\widehat{R}^1,\widehat{R}^2\right) &= -\left(2\left(1-\eta\right)\widehat{R}^2-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(\widehat{R}^2\right)^{\frac{-\alpha}{1-\alpha}}\right)+2\eta\widehat{R}^1-\frac{\alpha^{\frac{1}{1-\alpha}}}{x}\left(\widehat{R}^1\right)^{\frac{-\alpha}{1-\alpha}}\\ &= -\left(\left(2\left(1-\eta\right)\right)^{\alpha}\phi-\phi\left(2\left(1-\eta\right)\right)^{\alpha}\right)+\left(\left(2\eta\right)^{\alpha}\phi-\phi\left(2\eta\right)^{\alpha}\right)=0\end{aligned}$$

That is, the point $\left(\widehat{R}^1,\widehat{R}^2\right)$ lies on the curve defined by $\phi\left(R^1,R^2\right)=0$. Now we compare $\widehat{R}^2$ with $\frac{\widehat{R}^1}{\widehat{R}^1-1}$ to see whether this point lies above or below the second, downward sloping curve. We know that $\widehat{R}^2=\frac{\phi}{(2(1-\eta))^{1-\alpha}}$ and

$$\frac{\widehat{R}^1}{\widehat{R}^1-1}=\frac{\phi}{(2\eta)^{1-\alpha}\left(\frac{\phi}{(2\eta)^{1-\alpha}}-1\right)}=\frac{\phi}{\phi-(2\eta)^{1-\alpha}}.$$

Since $\phi<(2\eta)^{1-\alpha}+(2(1-\eta))^{1-\alpha}$ then $\phi-(2\eta)^{1-\alpha}<(2(1-\eta))^{1-\alpha}$ . Thus, $\frac{\phi}{(2(1-\eta))^{1-\alpha}}<\frac{\phi}{\phi-(2\eta)^{1-\alpha}}$. Therefore $\widehat{R}^2<\frac{\widehat{R}^1}{\widehat{R}^1-1}$ and so the point $\left(\widehat{R}^1,\widehat{R}^2\right)$ is below the downward sloping curve. But then the pair $(R^1,R^2)$, where the curves intersect, must imply $R^1>\widehat{R}^1$ and $R^2>\widehat{R}^2$. This proves the claim. As a consequence, recalling that $1-\gamma^j=\frac{\alpha^{\frac{1}{1-\alpha}}}{(R^j)^{\frac{1}{1-\alpha}}x}=\frac{\phi^{\frac{1}{1-\alpha}}}{(R^j)^{\frac{1}{1-\alpha}}}$, it is clear that

$\frac{1}{(R^1)^{\frac{1}{1-\alpha}}} < \frac{1}{\left(\frac{\phi^{\frac{1}{1-\alpha}}}{2\eta}\right)}$ and $\frac{1}{(R^2)^{\frac{1}{1-\alpha}}} < \frac{1}{\left(\frac{\phi^{\frac{1}{1-\alpha}}}{2(1-\eta)}\right)}$. Thus

$$1-\gamma^1 = \frac{\phi^{\frac{1}{1-\alpha}}}{(R^1)^{\frac{1}{1-\alpha}}} < 2\eta \quad \text{and} \quad 1-\gamma^2 = \frac{\phi^{\frac{1}{1-\alpha}}}{(R^2)^{\frac{1}{1-\alpha}}} < 2\left(1-\eta\right),$$

which implies that $\gamma^1 > 1-2\eta$ and $\gamma^2 > 2\eta - 1$, and therefore $\gamma^j > 0$ for both $j$. With this in mind, recall then that

$$\frac{R^1}{R^2} = \frac{2\left(1-\eta\right) - \frac{\alpha^{\frac{1}{1-\alpha}}}{(R^2)^{\frac{1}{1-\alpha}}x}}{2\eta - \frac{\alpha^{\frac{1}{1-\alpha}}}{(R^1)^{\frac{1}{1-\alpha}}x}} = \frac{2\left(1-\eta\right) - \left(1-\gamma^2\right)}{2\eta - \left(1-\gamma^1\right)}.$$

Clearly $\frac{R^1}{R^2} > 0$. Therefore, either $2\ (1-\eta) - (1-\gamma^2) > 0$ and $2\eta - (1-\gamma^1) > 0$ or $2\ (1-\eta) - (1-\gamma^2) < 0$ and $2\eta - (1-\gamma^1) < 0$. If the first two inequalities hold, then we have at the same time that $\gamma^2 > 2\eta - 1$ and $\gamma^1 > 1-2\eta$. If the second two inequalities hold, then we have at the same time $\gamma^2 < 2\eta - 1$ and $\gamma^1 < 1-2\eta$, but this second case must be ruled out since this implies that at least one $\gamma^j$ is strictly negative. So the first set of inequalities must hold. Given the definition of $\gamma^j$ we have that $R^j = \frac{\alpha}{(1-\gamma^j)^{1-\alpha}(x)^{1-\alpha}}$ for every $j$. Hence $\frac{R^1}{R^2} = \frac{\left(1-\gamma^2\right)^{1-\alpha}}{(1-\gamma^1)^{1-\alpha}}$ and

$$\frac{\left(1-\gamma^2\right)^{1-\alpha}}{\left(1-\gamma^1\right)^{1-\alpha}} = \frac{2\left(1-\eta\right) - \left(1-\gamma^2\right)}{2\eta - \left(1-\gamma^1\right)} = \frac{1-2\eta+\gamma^2}{2\eta - 1 + \gamma^1},$$

or

$$\left(1-\gamma^2\right)^{1-\alpha}\left(2\eta - 1 + \gamma^1\right) = \left(1-2\eta+\gamma^2\right)\left(1-\gamma^1\right)^{1-\alpha},$$

which is one of the two equations of the dynamic system in $\gamma_t^j$ (in steady state).

On the other hand, we also had $R^1 + R^2 = R^1\ R^2$, equivalent to $\frac{R^1}{R^2} + 1 = R^1$. Replacing we have

$$1 + \frac{\left(1-\gamma^2\right)^{1-\alpha}}{\left(1-\gamma^1\right)^{1-\alpha}} = \frac{\alpha}{\left(1-\gamma^1\right)^{1-\alpha}\left(x\right)^{1-\alpha}}.$$

But $\frac{\left(1-\gamma^2\right)^{1-\alpha}}{(1-\gamma^1)^{1-\alpha}} = \frac{2(1-\eta)-\left(1-\gamma^2\right)}{2\eta-(1-\gamma^1)} = \frac{1-2\eta+\gamma^2}{2\eta-1+\gamma^1}$, and after some algebra we get:

$$\gamma^1 + \gamma^2 = \frac{\alpha\left(2\eta - 1 + \gamma^1\right)}{\left(1-\gamma^1\right)^{1-\alpha}\left(x\right)^{1-\alpha}}$$

Hence, the values of $\gamma^j$ defined above satisfy the two equations that must hold at the steady state equilibrium for the system with $(\gamma_t^1, \gamma_t^2)$ . Since $R^j$ determines uniquely $\gamma^j$ and there is a unique equilibrium pair $(R^1, R^2)$ , then the pair $(\gamma^1, \gamma^2)$ which satisfies both equations is also unique.

The converse is also straightforward to show. Suppose there exists a unique pair $(\gamma^1, \gamma^2)$ , where $\gamma^1 > 1 - 2\eta$ and $\gamma^2 > 2\eta - 1$, satisfying

$$\gamma^1 + \gamma^2 = \frac{\alpha\left(2\eta - 1 + \gamma^1\right)}{\left(1-\gamma^1\right)^{1-\alpha}\left(x\right)^{1-\alpha}} = \frac{\left(2\eta - 1 + \gamma^1\right)\phi}{\left(1-\gamma^1\right)^{1-\alpha}}$$

and

$$\left(1-\gamma^2\right)^{1-\alpha}\left(2\eta - 1 + \gamma^1\right) = \left(1 - 2\eta + \gamma^2\right)\left(1-\gamma^1\right)^{1-\alpha}.$$

Define $R^j \equiv \frac{\phi}{(1-\gamma^j)^{1-\alpha}}$. We need to show that the pair $(R^1, R^2)$ satisfies both equations

$$1 = \frac{R^1 R^2}{R^1 + R^2},$$

and

$$2\left(1-\eta\right)R^2 - \phi^{\frac{1}{1-\alpha}}\left(R^2\right)^{\frac{-\alpha}{1-\alpha}} = 2\eta R^1 - \phi^{\frac{1}{1-\alpha}}\left(R^1\right)^{\frac{-\alpha}{1-\alpha}},$$

which is equivalent to

$$R^2\left[2\left(1-\eta\right) - \left(\frac{\phi}{R^2}\right)^{\frac{1}{1-\alpha}}\right] = R^1\left[2\eta - \left(\frac{\phi}{R^1}\right)^{\frac{1}{1-\alpha}}\right].$$

We basically work backwards relative to the first part of the proof. We know that

$$\gamma^1 + \gamma^2 = \frac{\left(2\eta - 1 + \gamma^1\right)\phi}{\left(1-\gamma^1\right)^{1-\alpha}} \Longrightarrow \frac{\gamma^1 + \gamma^2}{\left(2\eta - 1 + \gamma^1\right)} = \frac{\phi}{\left(1-\gamma^1\right)^{1-\alpha}},$$

and

$$\frac{\gamma^1 + \gamma^2}{2\eta - 1 + \gamma^1} = \frac{\gamma^1 + 2\eta - 1 + 1 - 2\eta + \gamma^2}{2\eta - 1 + \gamma^1} = 1 + \frac{1 - 2\eta + \gamma^2}{2\eta - 1 + \gamma^1}.$$

Hence, $\frac{\phi}{(1-\gamma^1)^{1-\alpha}} = 1 + \frac{1-2\eta+\gamma^2}{2\eta-1+\gamma^1}$. From the second equation, $(1-\gamma^2)^{1-\alpha}(2\eta - 1 + \gamma^1) = (1 - 2\eta + \gamma^2)$ $(1-\gamma^1)^{1-\alpha}$ , so

$$1 + \frac{1 - 2\eta + \gamma^2}{2\eta - 1 + \gamma^1} = 1 + \frac{\left(1-\gamma^2\right)^{1-\alpha}}{\left(1-\gamma^1\right)^{1-\alpha}}.$$

But $1 + \frac{\left(1-\gamma^2\right)^{1-\alpha}}{(1-\gamma^1)^{1-\alpha}}$ is equal to $1 + \frac{\left(\frac{\phi}{(1-\gamma^1)^{1-\alpha}}\right)}{\left(\frac{\phi}{(1-\gamma^2)^{1-\alpha}}\right)}$ , which in equilibrium is equal to $1 + \frac{R^1}{R^2}$. Therefore the equality $\frac{\phi}{(1-\gamma^1)^{1-\alpha}} = 1 + \frac{1-2\eta+\gamma^2}{2\eta-1+\gamma^1}$ is equivalent to

$$R^1 = \frac{\phi}{(1-\gamma^1)^{1-\alpha}} = 1 + \frac{1-2\eta+\gamma^2}{2\eta-1+\gamma^1} = 1 + \frac{R^1}{R^2} = \frac{R^2+R^1}{R^2}$$

so $R^1R^2 = R^1 + R^2$ and the first equation is obtained.

From the second condition in the $(\gamma^1, \gamma^2)$ system, $(1-\gamma^2)^{1-\alpha}(2\eta - 1 + \gamma^1) = (1-2\eta+\gamma^2)$ $(1-\gamma^1)^{1-\alpha}$ implies after, some algebra, that $R^1 \left(2\eta - 1 + \gamma^1\right) = R^2 \left(1 - 2\eta + \gamma^2\right)$, and then $R^1$ $(2\eta - (1-\gamma^1))$ is equal to $R^2 \left(2\left(1-\eta\right) - \left(1-\gamma^2\right)\right)$. From the definition of $R^j$ we have $1-\gamma^j$ $= \frac{\phi^{\frac{1}{1-\alpha}}}{R^{j\frac{1}{1-\alpha}}}$. Hence, the equality $R^1 \left(2\eta - (1-\gamma^1)\right) = R^2 \left(2\left(1-\eta\right) - \left(1-\gamma^2\right)\right)$ is equivalent to

$$R^1 \left(2\eta - \frac{\phi^{\frac{1}{1-\alpha}}}{(R^1)^{\frac{1}{1-\alpha}}}\right) = R^2 \left(2\left(1-\eta\right) - \frac{\phi^{\frac{1}{1-\alpha}}}{(R^2)^{\frac{1}{1-\alpha}}}\right),$$

which is the second equation we wanted to get. This completes the proof of uniqueness of steady state.

We now undertake the proof of uniqueness of equilibrium in two steps, one corresponding to the case $\eta < \frac{1}{2}$ and the other to the case $\eta > \frac{1}{2}$ (the case $\eta = \frac{1}{2}$ is trivial). Recall that $\gamma^1 > 1 - 2\eta$, and $\gamma^2 > 2\eta - 1$.

- **Case 1**: $\eta > \frac{1}{2}$.

This implies that $2\eta - 1 > 0$ and so $\gamma^2 > 0$. Hence it remains to show in this case that $\gamma^1 > 0$ $> 1 - 2\eta$. To do this, recall that the dynamic system in $(\gamma_t^1, \gamma_t^2)$ can be written as:

$$\gamma_{t+1}^1 + \gamma_{t+1}^2 = \frac{\phi\left(\gamma_t^1 + 2\eta - 1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}},$$

$$\left(1-\gamma_t^1\right)^{1-\alpha}\left(\gamma_t^2 + 1 - 2\eta\right) = \left(1-\gamma_t^2\right)^{1-\alpha}\left(\gamma_t^1 + 2\eta - 1\right).$$

The second equation can also be expressed one period forward as

$$\left(1-\gamma_{t+1}^1\right)^{1-\alpha}\left(\gamma_{t+1}^2 + 1 - 2\eta\right) = \left(1-\gamma_{t+1}^2\right)^{1-\alpha}\left(\gamma_{t+1}^1 + 2\eta - 1\right)$$

and from the first equation we get both $\gamma_{t+1}^2 = \frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}} - \gamma_{t+1}^1$ and $1 - \gamma_{t+1}^2 = 1 + \gamma_{t+1}^1 -$

$\frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}$. Replacing these two expressions in the last equation we get:

$$\begin{aligned} & \left(1-\gamma_{t+1}^1\right)^{1-\alpha}\left(\frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}-\gamma_{t+1}^1+1-2\eta\right) \\ = & \left(1+\gamma_{t+1}^1-\frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}\right)\left(\gamma_{t+1}^1+2\eta-1\right) \end{aligned}$$

which is a one-dimensional dynamic system. We already know that this system has two steady states. One at $\gamma_{t+1}^1=1-2\eta$ (which is not an equilibrium) and the other one where $\gamma_{t+1}^1>1-2\eta$. The system defines implicitly a curve on the $\left(\gamma_t^1,\gamma_{t+1}^1\right)$ plane. We first show that this curve is strictly increasing on $\Re_{++}^2$. We will then show that this curve goes through a point $(\bar{\gamma},1)$, where $\bar{\gamma}<1$. These facts will ensure that the equilibrium steady state is in fact the unique possible equilibrium in this economy.

**Lemma 1** *Let*

$$\begin{aligned} F\left(\gamma_{t+1}^1,\gamma_t^1\right) \equiv & \left(\frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}-\left(\gamma_{t+1}^1-1+2\eta\right)\right)\left(1-\gamma_{t+1}^1\right)^{1-\alpha} \\ & -\left(1+\gamma_{t+1}^1-\frac{\phi\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}\right)^{1-\alpha}\left(\gamma_{t+1}^1+2\eta-1\right) \end{aligned}$$

*Then the equation* $F\left(\gamma_{t+1}^1,\gamma_t^1\right)=0$ *defines implicitly a curve* $\gamma_{t+1}^1$ *as a function of* $\gamma_t^1$, *and* $\frac{d\gamma_{t+1}^1}{d\gamma_t^1}>0$ *for* $\gamma_t^1$ *and* $\gamma_{t+1}^1$ *greater than or equal to* $1-2\eta$ *and strictly less than one.*

**Proof.** To show that there is an implicit function, the Implicit Function Theorem ensures that it is enough to show that $F_{\gamma_{t+1}^1}\left(\gamma_t^1,\gamma_{t+1}^1\right)\neq 0$ (we will in fact show that this derivative is strictly negative). By direct computation:

$$\begin{aligned} & F_{\gamma_{t+1}^1}\left(\gamma_{t+1}^1,\gamma_t^1\right) \\ = & -\left(1-\gamma_{t+1}^1\right)^{1-\alpha}-\left(\frac{\alpha\left(\gamma_t^1+2\eta-1\right)}{x^{1-\alpha}\left(1-\gamma_t^1\right)^{1-\alpha}}-\left(\gamma_{t+1}^1-1+2\eta\right)\right)(1-\alpha)\left(1-\gamma_{t+1}^1\right)^{-\alpha} \\ & -(1-\alpha)\left(1+\gamma_{t+1}^1-\frac{\alpha\left(\gamma_t^1+2\eta-1\right)}{x^{1-\alpha}\left(1-\gamma_t^1\right)^{1-\alpha}}\right)^{-\alpha}\left(\gamma_{t+1}^1+2\eta-1\right) \\ & -\left(1+\gamma_{t+1}^1-\frac{\alpha\left(\gamma_t^1+2\eta-1\right)}{x^{1-\alpha}\left(1-\gamma_t^1\right)^{1-\alpha}}\right)^{1-\alpha} \end{aligned}$$

From this expression it is straightforward to see that for any $\gamma_t^1\geq 1-2\eta$, $\gamma_{t+1}^1\geq 1-2\eta$, and less than one, then $F_{\gamma_{t+1}^1}\left(\gamma_{t+1}^1,\gamma_t^1\right)<0$. This shows that the implicit function is well defined. On the other hand, to sign the implicit derivative $\frac{d\gamma_{t+1}^1}{d\gamma_t^1}$ we need to get $F_{\gamma_t^1}\left(\gamma_{t+1}^1,\gamma_t^1\right)$. This is equal to:

$$\begin{aligned}F_{\gamma_t^1}\left(\gamma_{t+1}^1,\gamma_t^1\right) &= \phi\left(\frac{\left(1-\gamma_t^1\right)^{(1-\alpha)}+\left(\gamma_t^1+2\eta-1\right)(1-\alpha)\left(1-\gamma_t^1\right)^{-\alpha}}{\left(1-\gamma_t^1\right)^{2(1-\alpha)}}\right)\cdot \\ &\left[\left(1-\gamma_{t+1}^1\right)^{1-\alpha}+\left(\gamma_{t+1}^1+2\eta-1\right)(1-\alpha)\left(1+\gamma_{t+1}^1-\phi\frac{\left(\gamma_t^1+2\eta-1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}\right)^{-\alpha}\right]\end{aligned}$$

For any $\gamma_t^1\geq 1-2\eta$, $\gamma_{t+1}^1\geq 1-2\eta$, and less than one, then $F_{\gamma_t^1}\left(\gamma_{t+1}^1,\gamma_t^1\right)>0$. Therefore, by the Implicit Function Theorem it is obvious that

$$\frac{d\gamma_{t+1}^1}{d\gamma_t^1}=-\frac{F_{\gamma_t^1}\left(\gamma_{t+1}^1,\gamma_t^1\right)}{F_{\gamma_{t+1}^1}\left(\gamma_{t+1}^1,\gamma_t^1\right)}>0,$$

which concludes the proof of this lemma. ■

The second part shows that this map goes through $(\bar{\gamma},1)$ with $\bar{\gamma}<1$.

**Lemma 2** *When* $\gamma_{t+1}^1\to 1$ *then* $\gamma_t^1\to\bar{\gamma}<1$.

**Proof.** From the equation defining the dynamic system in $\gamma_t^1$ take limits on both sides with $\gamma_{t+1}^1\to 1$ and $\gamma_t^1\to\bar{\gamma}$. We must have that

$$\begin{aligned}&(1-1)^{1-\alpha}\left(\frac{\phi\left(\bar{\gamma}+2\eta-1\right)}{\left(1-\bar{\gamma}\right)^{1-\alpha}}-1+1-2\eta\right)\\ =&\left(1+1-\frac{\phi\left(\bar{\gamma}+2\eta-1\right)}{\left(1-\bar{\gamma}\right)^{1-\alpha}}\right)(1+2\eta-1).\end{aligned}$$

Given that the left hand side is zero, this is equivalent to $0=2\eta\left(2-\frac{\phi(\bar{\gamma}+2\eta-1)}{(1-\bar{\gamma})^{1-\alpha}}\right)$, and given that $\eta>0$, what this implies is

$$\frac{\phi\left(\bar{\gamma}+2\eta-1\right)}{\left(1-\bar{\gamma}\right)^{1-\alpha}}=2$$

To get this equality, it is necessary that $\bar{\gamma}>1-2\eta$ (which is true since the map is strictly increasing) and that $(1-\bar{\gamma})^{1-\alpha}>0$, which implies $\bar{\gamma}<1$ as desired. ■

Given that the curve $\gamma_{t+1}^1\left(\gamma_t^1\right)$ goes through the $45^o$ line only through two points, one at $(1-2\eta,1-2\eta)$ and another one above this, the two lemmas imply that the curve must cut the $45^o$ line at the second steady state (the stationary equilibrium) from below. This shows that this steady state is unstable and so, if $\gamma^1>0$, then it is the unique equilibrium for this economy, since any other combination of

$(\gamma_t^1, \gamma_{t+1}^1)$ outside the stationary equilibrium leads to either the point $(1-2\eta, 1-2\eta)$ or to some value greater than one. Neither of the two situations can be an equilibrium.

It remains to be shown that $\gamma^1 > 0$. Given the last lemma, it is sufficient to show that when $\gamma_{t+1}^1 = 0$ then $\gamma_t^1 > 0$. To prove this, recall that the system can be reduced to:

$$\begin{aligned} F\left(\gamma_{t+1}^1, \gamma_t^1\right) \equiv & \left(\frac{\phi\left(\gamma_t^1 + 2\eta - 1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}} - \left(\gamma_{t+1}^1 - 1 + 2\eta\right)\right)\left(1-\gamma_{t+1}^1\right)^{1-\alpha} \\ & - \left(1 + \gamma_{t+1}^1 - \frac{\phi\left(\gamma_t^1 + 2\eta - 1\right)}{\left(1-\gamma_t^1\right)^{1-\alpha}}\right)^{1-\alpha}\left(\gamma_{t+1}^1 + 2\eta - 1\right) = 0. \end{aligned}$$

We evaluate $F$ at $(0,0)$, which gives

$$F(0,0) = (2\eta - 1)\left[\phi - 1 - (1 - \phi(2\eta-1))^{1-\alpha}\right].$$

We know that $\eta > \frac{1}{2}$, so $(2\eta - 1) > 0$ holds. Also, we know that $F$ is strictly decreasing in $\gamma_{t+1}^1$ and strictly increasing in $\gamma_t^1$. Therefore, it is sufficient to show that $F(0,0) < 0$. If this is true, then when $\gamma_t^1 = 0$ the corresponding value of $\gamma_{t+1}^1$ must be strictly negative. But $F(0,0) < 0$ if and only if $\phi - 1 < (1 - \phi(2\eta - 1))^{1-\alpha}$. To show that this inequality must hold, we proceed by contradiction. Suppose then that $\phi - 1 \geq (1 - \phi(2\eta - 1))^{1-\alpha} = (\phi + 1 - 2\phi\eta)^{1-\alpha}$. However, since $\phi < \frac{1}{\eta}$ (recalling that $\min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\} = \frac{1}{\eta}$ for $\eta > \frac{1}{2}$) then $\phi\eta < 1$ and so $-2\phi\eta > -2$, and so $1 - 2\phi\eta > -1$. Since $1 - \alpha > 0$ then $(\phi + 1 - 2\phi\eta)^{1-\alpha} > (\phi - 1)^{1-\alpha}$. Putting things together we get that $\phi - 1 \geq (1 - \phi(2\eta - 1))^{1-\alpha} = (\phi + 1 - 2\phi\eta)^{1-\alpha} > (\phi - 1)^{1-\alpha}$. Since $1 - \alpha < 1$, this implies that $\phi - 1 > 1$ or $\phi > 2$. But $\phi < \min\left\{\frac{1}{\eta}, \frac{1}{1-\eta}\right\} \leqq 2$, a contradiction. Thus, $\phi - 1 < (1 - \phi(2\eta - 1))^{1-\alpha}$ as desired. Hence $F(0,0) < 0$ and so, when $\gamma_t^1 = 0$ then $\gamma_{t+1}^1 < 0$.

- **Case 2:** $\eta < \frac{1}{2}$.

The proof follows similar arguments, so we just sketch part of it. First note that $\gamma^1 > 1 - 2\eta > 0$. Then it remains to show that $\gamma^2 > 0 > 2\eta - 1$. The difference in the procedure is that we will work with $\gamma_t^2$ as the variable instead of $\gamma_t^1$. Recall that the equilibrium conditions are:

$$\frac{p_t}{p_{t+1}} = \frac{\left(\gamma_{t+1}^1 + \gamma_{t+1}^2\right)}{\left(\gamma_t^1 + \gamma_t^2\right)} = \frac{q_t R_t^2}{1 + q_t},$$

$$q_t = \frac{\gamma_t^2 + 1 - 2\eta}{\gamma_t^1 + 2\eta - 1},$$

and

$$\left(1-\gamma_t^1\right)^{1-\alpha}\left(\gamma_t^2+1-2\eta\right)=\left(1-\gamma_t^2\right)^{1-\alpha}\left(\gamma_t^1+2\eta-1\right).$$

From the first three equations we get

$$\frac{\left(\gamma_{t+1}^1+\gamma_{t+1}^2\right)}{\left(\gamma_t^1+\gamma_t^2\right)}=\frac{q_t}{1+q_t}\left(\frac{\phi}{\left(1-\gamma_t^2\right)^{1-\alpha}}\right)=\left(\frac{\gamma_t^2+1-2\eta}{\gamma_t^1+\gamma_t^2}\right)\left(\frac{\phi}{\left(1-\gamma_t^2\right)^{1-\alpha}}\right)$$

Therefore, the dynamic system describing the equilibrium (which is equivalent to the one presented at the beginning of this proof) is

$$\left(\gamma_{t+1}^1+\gamma_{t+1}^2\right)=\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}$$

$$\left(1-\gamma_t^1\right)^{1-\alpha}\left(\gamma_t^2+1-2\eta\right)=\left(1-\gamma_t^2\right)^{1-\alpha}\left(\gamma_t^1+2\eta-1\right).$$

We proceed as before, reducing this system to a one-dimensional system in $\gamma_t^2$. From the first equation

$$\gamma_{t+1}^1=\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}-\gamma_{t+1}^2;\quad 1-\gamma_{t+1}^1=1+\gamma_{t+1}^2-\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}.$$

and replacing in the second equation forwarded one period gives:

$$\begin{aligned}&\left(1+\gamma_{t+1}^2-\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}\right)^{1-\alpha}\left(\gamma_{t+1}^2+1-2\eta\right)\\ =\ &\left(1-\gamma_{t+1}^2\right)^{1-\alpha}\left(\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}-\gamma_{t+1}^2+2\eta-1\right).\end{aligned}$$

We then define

$$\begin{aligned}&G\left(\gamma_t^2,\gamma_{t+1}^2\right)\\ \equiv\ &\left(1-\gamma_{t+1}^2\right)^{1-\alpha}\left(\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}-\gamma_{t+1}^2+2\eta-1\right)\\ &-\left(1+\gamma_{t+1}^2-\frac{\phi\left(\gamma_t^2+1-2\eta\right)}{\left(1-\gamma_t^2\right)^{1-\alpha}}\right)^{1-\alpha}\left(\gamma_{t+1}^2+1-2\eta\right).\end{aligned}$$

An equilibrium path is characterized by $G\left(\gamma_t^2,\gamma_{t+1}^2\right)=0$, which implicitly defines a function $\gamma_{t+1}^2\left(\gamma_t^2\right)$ provided that the conditions for the Implicit Function Theorem hold. Following identical arguments as in the lemma before, it can be shown that $G_{\gamma_{t+1}^2}\left(\gamma_t^2,\gamma_{t+1}^2\right)<0$ and $G_{\gamma_t^2}\left(\gamma_t^2,\gamma_{t+1}^2\right)>$

0 for all $\gamma_t^2$ and $\gamma_{t+1}^2$ less than one and strictly greater than $2\eta - 1$. Hence $\gamma_{t+1}^2\left(\gamma_t^2\right)$ is well defined and $\frac{d\gamma_{t+1}^2}{d\gamma_t^2} > 0$. In a similar fashion as in case 1, it is easy to prove that if $\gamma_{t+1}^2 \rightarrow 1$ then $\gamma_t^2 \rightarrow \widehat{\gamma} < 1$, which implies that the steady state $\gamma^2 > 2\eta - 1$ is locally unstable. This concludes the proof of the proposition.■

## A.5 Pareto optimality of the steady state equilibrium in the economy with banks, markets, and a central bank

The central planner chooses a steady-state equilibrium allocation of the resources available in the economy, subject to the constraints imposed by nature on the total amount of output, and the by technology used to transform inputs into output. The central planner, however, is not subject to the environmental constraints imposed by limited diversification in each island, and spatial separation and limited communication. We solve the problem of a central planner who maximizes the expected utility of lenders, young and old, at time $t$, subject to the constraint of providing a fixed level of utility to borrowers.

Let $d$ represent the (steady-state) amount of consumption good available to each *young* agent of region $j$. The central planner must allocate $d$ between investment (recall that agents consume only when *old*) and consumption for the current old generation. Let $\gamma^j$ represent the fraction of available resources destined to finance old agents consumption. The remaining fraction $(1 - \gamma^j)$ is invested in the random technology. The amount of resources available to the planner at a given time includes the return from the investment of the previous period in the amount of $(1 - \gamma^s) R^s$, where $s$ indicates the region where the investment outcome is successful. The total amount of resources available to the planner in a given period is then $(\gamma^1 + \gamma^2) + (1 - \gamma^s) R^s$.Finally, assume that the planner transfers a lump-sum amount to entrepreneurs. Let this amount be denoted by $\hat{T}_j$. For notational convenience, recall that the population of lenders in each region has unit measure, let $T_j \equiv \frac{\widehat{T}_j}{2}$.

The planner maximizes the expected utility of lenders subject to providing a level $\widehat{T}_j$ of utility to borrowers. We can rewrite this problem as[22]

$$\max_{\gamma^1,\gamma^2} \quad \eta \ln\left[\left(\gamma^1+\gamma^2\right)d+\left[\left(1-\gamma^1\right)d\right]^\alpha-2T_1\right]$$
$$+\left(1-\eta\right)\ln\left[\left(\gamma^1+\gamma^2\right)d+\left[\left(1-\gamma^2\right)d\right]^\alpha-2T_2\right].$$

The first-order conditions to this problem are given by

$$\frac{\eta}{c\left(s_1\right)}\left[d-\alpha\left(1-\gamma^1\right)^{\alpha-1}d^\alpha\right]+\frac{1-\eta}{c\left(s_2\right)}d=0,$$

$$\frac{\eta}{c\left(s_1\right)}d+\frac{1-\eta}{c\left(s_2\right)}\left[d-\alpha\left(1-\gamma^2\right)^{\alpha-1}d^\alpha\right]=0,$$

where $c\left(s\right)\equiv\left(\gamma^1+\gamma^2\right)d+\left[\left(1-\gamma^s\right)d\right]^\alpha-2T_s$. Therefore,

$$d\left[\frac{\eta}{c\left(s_1\right)}+\frac{1-\eta}{c\left(s_2\right)}\right]=\frac{\eta}{c\left(s_1\right)}\alpha\left(1-\gamma^1\right)^{\alpha-1}d^\alpha=\frac{1-\eta}{c\left(s_2\right)}\alpha\left(1-\gamma^2\right)^{\alpha-1}d^\alpha.$$

The last expression implies that

$$\frac{\eta}{c\left(s_1\right)}\frac{c\left(s_2\right)}{1-\eta}=\frac{\alpha\left(1-\gamma^2\right)^{\alpha-1}d^\alpha}{\alpha\left(1-\gamma^1\right)^{\alpha-1}d^\alpha}=\frac{\alpha\left(1-\gamma^2\right)^{\alpha-1}d^{\alpha-1}}{\alpha\left(1-\gamma^1\right)^{\alpha-1}d^{\alpha-1}}=\left(\frac{1-\gamma^1}{1-\gamma^2}\right)^{1-\alpha}. \tag{20}$$

This is one of the conditions that characterizes the solution to the planner's problem, stating that at optimum

$$\frac{1-\eta}{c\left(s_2\right)}=\frac{\eta}{c\left(s_1\right)}\left(\frac{1-\gamma^2}{1-\gamma^1}\right)^{1-\alpha}.$$

This expression implies that

$$\frac{\eta}{c\left(s_1\right)}+\frac{1-\eta}{c\left(s_2\right)}=\frac{\eta}{c\left(s_1\right)}\left[1+\left(\frac{1-\gamma^2}{1-\gamma^1}\right)^{1-\alpha}\right]$$

We also had

$$d\left[\frac{\eta}{c\left(s_1\right)}+\frac{1-\eta}{c\left(s_2\right)}\right]=\frac{\eta}{c\left(s_1\right)}\alpha\left(1-\gamma^1\right)^{\alpha-1}d^\alpha,$$

so

$$\frac{\eta d}{c\left(s_1\right)}\left[1+\left(\frac{1-\gamma^2}{1-\gamma^1}\right)^{1-\alpha}\right]=\frac{\eta}{c\left(s_1\right)}\alpha\left(1-\gamma^1\right)^{\alpha-1}d^\alpha.$$

[22] We maintain this notation to keep the notation close to that of the decentralized problem analyzed in Section 5. It would of course be equivalent to solve the problem of the planner in terms of island-wide allocations, rather than region-wise allocations.

Since $\eta > 0$ and $c(s_1) > 0$ we can rewrite the last expression as

$$1 + \left(\frac{1-\gamma^2}{1-\gamma^1}\right)^{1-\alpha} = \alpha\left(1-\gamma^1\right)^{\alpha-1} d^{\alpha-1} = \frac{\alpha}{\left[d\left(1-\gamma^1\right)\right]^{1-\alpha}}. \tag{21}$$

Equations (20) and (21) define the solution to the planner problem. We now relate equations (20) and (21) to the decentralized solution obtained in the main text, and show that it is exactly the solution to the planner problem. Recall that in the economy with banks, asset markets, and the central bank we had $R_t^1(s_1) = q_t R_t^2(s_2)$, which in steady state is equivalent to

$$\frac{1}{q} = \frac{R^2}{R^1}$$

We also showed that in this stationary equilibrium that:

$$1 + \frac{\left(1-\gamma^2\right)^{1-\alpha}}{\left(1-\gamma^1\right)^{1-\alpha}} = \frac{\alpha}{\left(1-\gamma^1\right)^{1-\alpha} x^{1-\alpha}} \tag{22}$$

holds. In addition, the equilibrium conditions imply that

$$MRS_{1,2}^j = \frac{\eta}{r^j(s_1)}\frac{r^j(s_2)}{1-\eta} = \frac{\left(\frac{1}{q}+1\right)\gamma^j + \frac{1}{q}\left(1-\gamma^j\right)R^j(s_1) + \left(1-\gamma^j\right)R^j(s_2)}{(1+q)\gamma^j + \left(1-\gamma^j\right)R^j(s_1) + q\left(1-\gamma^j\right)R^j(s_2)},$$

and that

$$\frac{\left(\frac{1}{q}+1\right)\gamma^j + \frac{1}{q}\left(1-\gamma^j\right)R^j(s_1) + \left(1-\gamma^j\right)R^j(s_2)}{(1+q)\gamma^j + \left(1-\gamma^j\right)R^j(s_1) + q\left(1-\gamma^j\right)R^j(s_2)} = \frac{1}{q},$$

so $\frac{1}{q}$ is the marginal rate of substitution between consumption in state $s_1$ and state $s_2$. We also know that $\frac{R^2}{R^1}$ is equal to the ratio $\frac{f'(k_2)}{f'(k_1)} = \frac{f'\left(1-\gamma^2\right)}{f'\left(1-\gamma^1\right)} = \frac{\alpha\left(1-\gamma^2\right)^{\alpha-1}d^{\alpha-1}}{\alpha\left(1-\gamma^1\right)^{\alpha-1}d^{\alpha-1}}$. Hence, in equilibrium:

$$\frac{\eta}{r^j(s_1)}\frac{r^j(s_2)}{1-\eta} = \frac{\alpha\left(1-\gamma^2\right)^{\alpha-1}d^{\alpha-1}}{\alpha\left(1-\gamma^1\right)^{\alpha-1}d^{\alpha-1}} \tag{23}$$

holds. Now note that in equilibrium each lender in region $s$ receives $\eta(s)R^s d$, but it is easy to show that $\eta(s)R^s = \frac{\gamma^1+\gamma^2+R^s(1-\gamma^s)}{2}$, and therefore

$$\eta(s)R^s d = \left(\frac{\gamma^1+\gamma^2}{2}\right)d + \frac{\alpha\left(\left(1-\gamma^s\right)d\right)^{\alpha}}{2}.$$

With this expression it is not difficult to see that the First Welfare Theorem holds here.

Suppose that $\left(\gamma_{eq}^1, \gamma_{eq}^2\right)$ are the equilibrium allocations. Hence, they satisfy equations (22) and

(23). Define

$$\gamma_{op}^{j} \equiv \gamma_{eq}^{j}, \quad \widehat{T}_j \equiv \frac{\left[\left(1-\gamma_{eq}^{j}\right) d\right]^{\alpha}(1-\alpha)}{2}$$

Note that if $\gamma_{op}^{j}$ satisfies equation (23) then it immediately satisfies (20). The same holds true for equations (22) and (21). Finally, note that in equilibrium we have that $c_1(s) = \left(\frac{\gamma_{eq}^{1}+\gamma_{eq}^{2}}{2}\right) d + \frac{\alpha\left(\left(1-\gamma_{eq}^{s}\right)d\right)^{\alpha}}{2}$, and, given our definitions,

$$\frac{\alpha\left(\left(1-\gamma_{eq}^{s}\right) d\right)^{\alpha}}{2} = \frac{\alpha\left(\left(1-\gamma_{op}^{s}\right) d\right)^{\alpha}}{2} = \frac{\left(\left(1-\gamma_{op}^{s}\right) d\right)^{\alpha}}{2}(\alpha-1+1) =$$

$$= \frac{\left(\left(1-\gamma_{op}^{s}\right) d\right)^{\alpha}}{2} - (1-\alpha)\frac{\left(\left(1-\gamma_{op}^{s}\right) d\right)^{\alpha}}{2} = \frac{\left(\left(1-\gamma_{op}^{s}\right) d\right)^{\alpha}}{2} - \widehat{T}_j.$$

This shows that the constructed allocation $\left(\gamma_{op}^{j}\right)_{j=1}^{2}$, and the constructed transfers $\left(\widehat{T}_j\right)_{j=1}^{2}$ satisfy all the conditions for $\left(\gamma_{op}^{j}\right)_{j=1}^{2}$ to be the solution of the Pareto problem, which proves the result.

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