Sorting by foot: ‘travel-for’ local public goods and equilibrium stratification

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Abstract. We re-examine Tiebout’s hypothesis of endogenous sorting in a competitive spatial equilibrium framework, by considering both income and preference heterogeneity and by allowing agents to decide endogenously the number of visits to a ‘travel-for’ local public good. The equilibrium configuration may be completely segregated, incompletely segregated, or completely integrated, depending on relative market rents and income/preference/local tax parameters. A segregated equilibrium may feature endogenous sorting purely by income or by both income and preferences. While the rich need not be closer to the local public facility site, multiple equilibria may arise when the equilibrium configuration is incompletely segregated. JEL classification: D50, H41

Classification par déplacement : biens publics locaux accessibles par déplacement et stratification d’équilibre. Les auteurs réexaminent l’hypothèse de Tiebout de classification endogène dans un cadre d’équilibre spatial concurrentiel en tenant compte à la fois de l’hétérogénéité des revenus et des préférences, ainsi qu’en permettant aux agents de décider de manière endogène du nombre de visites pour se procurer un bien public local par déplacement. Il peut s’ensuivre des équilibres de ségrégation complète, de ségrégation incomplète, ou d’intégration complète, selon les rentes relatives et les paramètres de

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revenus, de préférences ou de fiscalité locale. Un équilibre de ségrégation peut entraîner une classification endogène sur la base seulement des revenus ou sur la base des revenus et des préférences. Alors que les riches n'ont pas besoin d'être plus proches du site du bien public local, des équilibres multiples peuvent surgir quand la configuration d'équilibre en est une de ségrégation incomplète.

1. Introduction

In the post-war period, residential stratification as a result of racial and economic forces have received considerable attention in both the economics and sociology professions. Based on a standard measure of segregation, the dissimilarity index, most of the 30 largest Metropolitan Statistical Areas (MSAs) in the United States are highly stratified and have become more so since 1980 (see table 1 for the year 2000).[1] Jargowsky (1996) uses Census tract data over the period 1970 to 1990 to show a nation-wide, sharp increase in economic segregation in the 1980s accompanied by a small decline in racial segregation. In the light of the increase in economic stratification, this paper provides an analysis of possible forces that explain this important phenomenon.

Stratification warrants attention for a variety of reasons. Not least, it leads to significant disparities in socioeconomic status (SES), particularly in earned income, education, housing, and social norms (cf. Weiss 1989; Jencks and Mayer 1990). Wilson (1987) hypothesizes that the ongoing increase in economic segregation plays a crucial role in the formation of urban ghettos, and Ihlanfeldt (1994) suggests that the rapid suburbanization of jobs and workers and the formation of suburban business districts (SBDs) have expedited both economic and racial segregation. As a result, the ratio of central city income per capita to suburban income per capita in the 85 largest MSAs has dropped from 105% in 1960 to 84% in 1989 (cf. Barnes and Ledebur 1998). The associated neighbourhood effect (cf. Bond and Coulson 1989) and urban labour-market networking effect (cf. O'Regan and Quigley 1993), in addition to changes in commuting technologies (cf. Sassen 1991), further lead to a spatial mismatch with high unemployment rates in central cities accompanied by high job vacancies in suburbs (see a discussion in Coulson, Laing, and Wang 2001 and papers cited therein). Urban unemployment and immobility across generations have generated the unfortunate pathologies of central cities. In particular, serious crimes per capita in central cities have far exceeded the comparable figures in suburbs (cf. Glaeser, Sacerdote, and Scheinkman 1996).

The existing literature emphasizes that differences in human/non-human wealth, accessibility to capital markets, and preferences for housing are crucial

1 The dissimilarity index, first constructed by Duncan and Duncan (1955), measures the degree to which two particular population characteristics of interest (such as white versus black or rich versus poor) are distributed differently within a population (such as residents in an MSA). For example, using the 2000 Census, DC-Baltimore had a shockingly high dissimilarity index of 0.78, Detroit had 0.70, eight MSAs were in the 0.6 range (including New York and Chicago), six in the 0.5 range (including Boston and Los Angeles), eleven in the 0.4 range (including Miami and San Francisco), and only three below 0.4 (Dallas, Seattle, and Portland).

TABLE 1
Segregation by the dissimilarity index in the 30 largest U.S. MSAs (2000)

Metropolitan statistical area (MSA)	Dissimilarity index
DC-Baltimore, Detroit	0.70 or higher
Milwaukee, Cleveland, St Louis, New York	0.60–0.69
Philadelphia, Cincinnati, Chicago, Indianapolis	
Pittsburgh, Atlanta, Kansas City	0.50–0.59
Houston, Boston, Los Angeles	
Tampa, San Antonio, Phoenix, Minneapolis	0.40–0.49
San Diego, Norfolk, San Francisco	
Miami, Denver, Sacramento, Orlando	
Dallas, Seattle, Portland	0.39 or lower

NOTE: Let H_i and L_i denote, respectively, the numbers of the rich (non-white) and the poor (black) in census tract $i = 1, 2, \ldots, m$, and define the cumulated population of the rich and the poor up to the k^{th} track as $a_k^H \equiv \Sigma_{i=1}^k H_i / \Sigma_{i=1}^m H_i$ and $a_k^L \equiv \Sigma_{i=1}^k L_i / \Sigma_{i=1}^m L_i$. The dissimilarity index is then measured by $D \equiv (1/2)\,\Sigma_{i=1}^m \left|a_k^L - a_k^H\right|$.

economic forces driving segregation. For instance, Benabou (1996a) studies the effect of wealth and human capital investment on spatial segregation and economic growth, whereas a separate paper (Benabou 1996b) considers the roles that capital market imperfections and local spillovers play in economic stratification. Much of the recent theoretical literature follows this vein of research. In a recent empirical investigation, however, Bayer, McMillan, and Rueben (2002) document that heterogeneous preferences for housing and neighbourhood characteristics play an important role in driving endogenous stratification in urban housing markets where social interactions are present. In this paper, we argue that heterogeneous preferences for the local public good (LPG) service can be a source of economic segregation. This is important because the analysis will shed light on possible equilibrium outcomes that correspond to some of the outcomes observed in MSAs. Moreover, the LPG preference and the conventional income/wealth factors may reinforce each other, strengthening the process of stratification.

In this paper, we examine endogenous sorting on the basis of heterogeneous incomes as well as heterogeneous preferences for a ‘travel-for’ local public good.[2] We highlight the nature of the travel-for LPG: agents decide endogenously the number of visits needed in order to consume the service (patronization), while bearing the commuting costs, where the quality of service is reduced as congestion (measured by the total number of users) increases; the existing literature fixes the number of visits to the public goods facility to be one. By developing a multi-class competitive spatial equilibrium

2 The concept of travel-for public goods is discussed in Fujita (1986) and formally modelled in Thisse and Wildasin (1992): it is a special type of Starrett’s (1991) congestable local public good that is available to consumers only in specific locations to which they must travel in order to enjoy the service.

model of 'sorting by foot,' we determine endogenously the pattern of economic stratification.[3] The idea that the provision of LPGs may induce voting by feet can be traced back to Tiebout (1956) and some important follow-up studies by Buchanan (1965), Pauly (1970), Arnott and Stiglitz (1979), Ellickson (1979), Rose-Ackerman (1979), Wooders (1980), and Bewley (1981).[4] We apply this insight to the study of economic segregation, establishing conditions for the local economy to be completely segregated, incompletely segregated, or fully integrated. We also examine in a segregated economy whether the rich are necessarily closer to the local public facility site.

For any set of income, preference, and local tax parameters as well as a given site of the local public facility, we establish the range of relative market land rents within which agents may choose to be segregated or mixed in competitive spatial equilibrium. Extending the techniques developed in the multi-class locational equilibrium framework by Hartwick et al. (1976), we use the Negishi (1960) approach to show that a multi-class competitive spatial equilibrium exists. The equilibrium configuration depends crucially on relative market rents, preference heterogeneity, income heterogeneity, local tax progressiveness, and the unit commuting cost.

Our paper contributes to the literature on economic segregation by highlighting that heterogeneity in preferences towards a travel-for congestable LPG can generate stratification. When their preference differentials are sufficiently strong, the rich and the poor are always segregated. In a segregated equilibrium where the rich are closer to the public facility site (frequently observed in Asian and European cities), endogenous sorting is both by income and preference towards LPG consumption. In a segregated equilibrium where the poor are closer to the public facility site (frequently observed in American cities, but a result that cannot be supported theoretically by previous studies on stratification), endogenous sorting is purely on the basis of income. The former (latter) configuration is likely to arise when the relative market rent augmented by the property tax of the community with the local public facility is sufficiently high (low). In contrast to the existing literature, multiple equilibria may occur when the equilibrium configuration is incompletely segregated, as when either the rich or the poor may reside closer to the public facility site.

3 Thus, the equilibrium concept used in our paper differs from that in the club or coalition formation literature (cf. Berglas and Pines 1981; Epple and Romano 1996; and papers cited in Wildasin 1987). We will discuss briefly the implications of this alternative in the concluding section.

4 See a comprehansive survey by Wildasin (1987) of early research in this area. Recently, there has been a growing amount of literature to formalize this Tiebout hypothesis in various dimensions. To name but a few, this includes Konishi, Le Breton, and Weber (1997), using non-cooperative equilibrium concepts, as well as Oates and Schwab (1991), Brueckner (1994), Scotchmer (1994), Konishi (1996), Nechyba (1997), and Conley and Wooders (2001), under the conventional competitive equilibrium or core set-up.

2. The model

We consider a simple model with two types of agents ($i = H, L$) with total mass $N > 0$ and two communities ($j = 1, 2$), where each community has a size conveniently normalized to $N/2$. Type-H agents (of mass H) have higher income and stronger preferences towards public good consumption than type-L agents (of mass L), where $H + L = N$. Denoting a type-i agent's income as Y^i and letting his/her preference towards public good consumption be captured by γ^i, we have: $Y^H > Y^L > 0$ and $\gamma^H > \gamma^L > 0$. Throughout the paper, we will refer to type-H agents as 'the rich' and type-L agents as 'the poor.' Denote the (endogenously determined) population of type-i agents in community j as i_j ($i = H, L$; $j = 1, 2$). Upon acquiring one unit of land, each agent can enjoy the consumption of the composite good, in addition to the service of the LPG.[5] In the absence of vacant land, each community j accepts residents of mass $N/2$ and population balance conditions require ($j = 1, 2$):

$$H_j + L_j = N/2. \tag{1}$$

The population identities can be conveniently summarized by

Population	1	2	Total
Type-H	H_1	H_2	H
Type-L	L_1	L_2	L
Total	$N/2$	$N/2$	N

A consumable local public good of size K is provided *only* in community 1, which may be thought of as the central city. The service of the LPG, G, depends on the degree of congestion from patronization by users as measured by M. Such an LPG may include a community park, library, museum, theater hall, swimming pool, exercise field, and other recreational facilities open to the public. For simplicity, the service of the LPG *inclusive of the congestion factor* is assumed to take the following form:

$$G = \frac{K}{M^\alpha}, \tag{2}$$

where $\alpha \in [0, 1]$ reflects the degree of congestion – the local public facility is a non-rival pure public good if $\alpha = 0$ and a completely rival private good if $\alpha = 1$. The facility of this variable usage LPG may be visited more than once by the same agent. We use n_j^i to measure the number of trips for a type-i agent

5 Since market rents are positive, no one will aquire more than one unit of land. This simplifying assumption on the inelastic use of land follows the spatial economics literature with local public goods (cf. Thisse and Wildasin 1992; Berliant, Peng, and Wang 2005; and papers cited therein).

residing in community *j* to patronize the local public facility. Letting $x_j \equiv H_j/N_j$ denote the fraction of the rich in community *j*, we thus have

$$\sum_{j=1}^{2} \left[n_j^H x_j + n_j^L (1 - x_j) \right] N_j = M, \tag{3}$$

which summarizes the total number of visits both by the rich and the poor.

Upon acquiring one unit of land, each agent can enjoy the consumption of the composite good consumption (denoted *C*), in addition to the service of the LPG. Agents of type-*i* are assumed to have an identical preference of the following form: $U = C + \gamma^i \cdot n^\beta \cdot \log G$, where $\beta \in (0, 1)$. This utility functional form extends those separable quasi-linear functions proposed by Bergstrom and Cornes (1983) and Berliant, Peng, and Wang (2005) by considering *endogenous number of trips* to the local public facility – an increase in the number of visits is assumed to lead to more enjoyment, but at a diminishing rate. An increase in γ^i represents a shift in preferences away from the composite good towards the LPG. In the case where $\beta = 0$ and $\gamma^i = \gamma$, our utility specification reduces to that in Berliant, Peng, and Wang (2005).

Denote R_j as the market rent (on land and housing property) in community *j* and $t(x_j)$ as the associated property tax rate that depends positively on the fraction of the rich (i.e., $t' > 0$).[6] Let *T* measure the unit commuting cost to the local public facility's site. Since the local public facility is in community 1, only community 2 residents bear the commuting cost. We thus define an indicator function I_j taking values in the set $\{0, 1\}$ with $I_1 = 0$ and $I_2 = 1$ and use $T \cdot I_j$ to measure the commuting cost per visit facing each agent residing in community *j*.[7] The local government charges a uniform user fee (per visit) at $\phi > 0$ for patronizing the local public facility.[8] Moreover, it sets the local income tax rate at τ_j^i. We assume $\tau_j^H > \tau_j^L$ to capture the progressiveness of income taxes and $\tau_1^i > \tau_2^i$ to capture the tax surcharge imposed at the site of the local public facility.[9]

A representative agent of type-*i* will solve a two-stage optimization problem. In stage 1, he/she chooses the residential location; in stage 2, he/she chooses composite good consumption and the number of visits to the local public facility in order to maximize the utility subject to the budget constraint. To ensure subgame perfection, this optimization problem is solved backward. In step 1, the stage-2 problem is solved: for a given residential location *j*, a representative agent of type-*i* takes G, R_j, x_j, and tax/user fee parameters as given to solve

6 Indeed, all we need to assume is that the market rents augmented by the property tax are increasing in the proportion of the rich. The assumption of $t' > 0$ is imposed for analytical convenience.

7 Although *T* is a flat unit commuting cost, the total commuting costs incurred by each agent depend on his/her endogenous number of visits to the LPG site.

8 User fees are frequently charged for variable usage public goods (as discussed in Wildasin 1987). A toll imposed on congestable goods is one such fee (e.g., see Brueckner 2002).

9 For example, in many states in the United States the local tax rates in cities with LPG amenities are higher than those in rural counties and townships.

$$V_j^i = \max_{C,n} C + \gamma^i n^\beta \log G, \tag{4}$$

$$\text{s.t.} \quad C + R_j(1 + t(x_j)) + n(T \cdot I_j + \phi) = \left(1 - \tau_j^i\right) Y^i. \tag{5}$$

In step 2, a representative agent of type-i solves the stage-1 problem by comparing the values (indirect utilities) obtained in step 1 to choose residential location. That is, he/she will reside in community $h \in \{1, 2\}$ if and only if $V_h^i \geq V_j^i$ for $j \neq h$ and $j \in \{1, 2\}$.

To close the model, we assume that the local public facility is financed entirely by revenues from local property taxes, user fees, and local income taxes:

$$K = \sum_{j=1}^{2} t(x_j) R_j + M\phi + \sum_{i=H,L} \sum_{j=1}^{2} \tau_j^i Y^i. \tag{6}$$

The local government in our economy is thus passive; its role is *to provide the LPG by maintaining a balanced budget*.

3. Optimization and equilibrium

We are now prepared to solve the optimization problem in step 1 facing the representative agent of each type $i = H, L$. Define the Lagrangian function as

$$L^i(C,n,\lambda^i) \equiv C + \gamma^i n^\beta \log G + \lambda^i \left[\left(1 - \tau_j^i\right) Y^i - C - R_j(1 + t(x_j) - n(T \cdot I_j + \phi)\right].$$

Straightforward differentiation yields

$$\frac{\partial L^i}{\partial C} = 1 - \lambda^i = 0 \tag{7}$$

$$\frac{\partial L^i}{\partial n} = \beta \gamma^i n^{\beta - 1} \log G - \lambda^i (T \cdot I_j + \phi) = 0 \tag{8}$$

While (7) is standard under the quasi-linear utility function specification, (8) equates the marginal benefit from patronizing the local public facility with the associated commuting cost and user fee. In deriving (8), we note that the representative agent of each type takes the service of the LPG as given when he/she solves the optimization problem, where each individual is atomistic without accounting for the effect of his/her decision of patronization on the congestion factor via M.

Using (2), (3), and (6), we can express the logged level of the LPG service log G as a function of (n_j^i; x_j, R_j; Y^i). Utilizing this result, we can then solve from (7) and (8) the number of visits:

$$n_j^i = \left[\frac{\beta\gamma^i \log G}{T \cdot I_j + \phi}\right]^{\frac{1}{1-\beta}} \equiv q\left(n_j^i; x_j, R_j; Y^i, \gamma^i\right), \tag{9}$$

which constitutes a fixed point mapping in n_j^i. Since $\gamma^H > \gamma^L$, $I_1 = 0$, and $I_2 = 1$, we can rank: $n_1^H > \max\{n_2^H, n_1^L\} > \min\{n_2^H, n_1^L\} > n_2^L$. Moreover, straightforward differentiation yields

$$\frac{\partial q}{\partial n_j^i} = -\frac{1}{1-\beta}\Gamma_j^i, \tag{10}$$

where $\Gamma_j^i \equiv (n_j^i / \log G) x_j N_j (\phi/K - \alpha/M)$ captures the positive user fee revenue effect and the negative congestion effect resulting from an additional patronization. Without loss of generality, we impose:[10]

ASSUMPTION 1. $\Gamma_j^i = 0$.

This assumption can be justified by imagining that the local government sets the user fee ϕ to offset the congestion cost incurred by an additional visitor to the local public facility. Thus, other than this simple role, the local government is essentially passive, maintaining its budget balance (6) and providing the local public good given the income and property tax schedules, $\{\tau_j^i, t(x_j)\}$. Since the main purpose of our paper is to characterize various endogenous sorting equilibria, the consideration of optimal fiscal policy and social optimum is beyond the focus of this research.[11]

We can then establish:

LEMMA 1. *Under assumption 1, the balanced-budget local public good service, log* $G = g(x_1, x_2, R_j; Y^i)$, *is increasing in the fraction of the rich in both communities* (x_1 *and* x_2), *the market rents* (R_j), *and the level of income* (Y^i).

Proof. All proofs are relegated to the appendix. ■

A higher market rent (income) yields greater property (income) tax revenue, thus enabling a higher provision of the LPG. While an increase in the fraction of the rich also raises the balanced-budget LPG service, it generates two additional effects via an increased number of users measured by M. On the one hand, the user fee revenue increases; on the other, the resulting congestion deteriorates the service of the local public facility. Under assumption 1, these two additional effects offset each other, thereby giving an unambiguously positive effect of the fraction of the rich on the LPG service.[12]

10 The assumption is innocuous, since our results are robust as long as the direct effect dominates these two opposing indirect effects; that is, $1 - \partial q/\partial n_j^i > 0$.

11 We will discuss briefly the possible political mechanism determining these tax schedules in the concluding section.

12 Again, this assumption is imposed for convenience – all we need is to have these conflicting indirect effects dominated by the direct effect.

Utilizing (9) and lemma 1, we can conclude:

LEMMA 2. *Under assumption 1, the optimal number of visits to the local public facility* (n_j^i) *is increasing in the fraction of the rich* (x_j)*, the market rents* (R_j)*, the level of income* (Y^i)*, and the preference towards the local public good* (γ^i).

From (9), a higher level of the LPG induces more visits. Thus, any exogenous shift leading to a higher LPG service must raise the number of visits. Additionally, when the preference is biased towards the LPG, the local public facility offers greater enjoyment and is hence patronized more frequently.

Next, by substituting (9) into (5), we have

$$C_j^i = \left(1 - \tau_j^i\right) Y^i - R_j(1 + t(x_j)) - (\beta\gamma^i \log G)^{-\frac{1}{\beta-1}}(T \cdot I_j + \phi)^{-\frac{\beta}{1-\beta}}, \qquad (11)$$

which can be substituted into (4) to yield the indirect utility function:

$$V_j^i(x_j, R_j; I_j) = \left(1 - \tau_j^i\right) Y^i - R_j(1 + t(x_j)) + (1 - \beta)\gamma^i \left(n_j^i\right)^\beta \log G. \qquad (12)$$

Lemma 1 shows that an increase in the fraction of the rich induces a greater LPG service. However, since the property tax schedule is increasing in the fraction of the rich, there is a negative wealth effect. It is natural to impose a regularity condition such that the benefit from the additional LPG service is sufficient to compensate the loss of net wealth from the property tax payment. As a consequence, indirect utility is strictly increasing in the fraction of the rich, ensuring that richness is never harmful to the community. Even with this regularity condition, the effect of market rent on indirect utility remains ambiguous. In particular, an increase in market rents has a direct effect: it reduces an individual's wealth and hence his/her indirect utility. It, however, stipulates a higher property tax revenue, a greater provision of the LPG, as well as more visits to the local public facility – these indirect effects lead to higher indirect utility.

We can now define the *bid rent* Ψ_j as the slope of the indifference curves in (x_j, R_j) space:

$$\Psi_j = \Psi(x_j, R_j;\ \gamma^i, Y^i) \equiv \frac{dR_j}{dx_j}\Big|_{V_j^i = \overline{V}} = -\frac{dV_j^i/dx_j}{dV_j^i/dR_j}.$$

Denote $S_j^i \equiv [\gamma^i (n_j^i)^\beta / K - 1]$, which captures all the effects of γ^i and Y^i on the bid rent. To ensure positive bid rents, we need: $S_j^i < 1/t(x_j)$ for all i and j. Recall that n_j^i is strictly increasing in γ^i and $n_1^i > n_2^i$. We thus have $S_1^H = \max_{i,j} \{S_j^i\}$. Moreover, $t(x_j)$ is strictly increasing in x_j, implying $1/t(1) = \min_{x_j} \{1/t(x_j)\}$. Furthermore, from (6) the balanced-budget provision of the LPG must exceed the total income tax revenue, provided that the market rents are strictly positive (i.e., $K > \Sigma_{i=H,L} \Sigma_{j=1}^{2} \tau_j^i Y^i$). Denote

$$A \equiv \frac{1}{(\gamma^H)^{\frac{1}{\beta}}} \left(\frac{\phi}{\beta}\right) \left[\left(1 + \frac{1}{t(1)}\right) \sum_{i=H,L} \sum_{j=1}^{2} \tau_j^i Y^i \right]^{\frac{1-\beta}{\beta}}.$$

Using (9) and the definition of S_j^i, we can establish a sufficient condition:

ASSUMPTION 2. $g(x_1, x_2, R_j; Y^i) < A$.

The values of $\{x_1, x_2\}$ will be pinned later when we analyse each equilibrium configuration, where the range of the supporting market rents are also specified. At that time, we will express assumption 2 only in terms of exogenous parameters. It is useful to note that assumption 2 guarantees that indirect utility is decreasing in market rents and, by definition, the bid rents must be positive. More specifically, assumption 2 states that income tax revenues finance a significant portion of the LPG expenditure. As a result, the positive indirect effects of market rents on indirect utility via the property tax revenue and the endogenous number of trips are dominated by the direct negative net wealth effect. We can then obtain

LEMMA 3. *Under assumptions 1 and 2, the bid rents are positive, satisfying*

$$\Psi(R_j,\ x_j,\ \gamma^i, Y^i) = \frac{t'(x_j)R_j}{\left(S_j^i\right)^{-1} - t(x_j)} > 0. \tag{13}$$

Applying lemmas 2 and 3, we can derive

$$\frac{d\Psi_j}{d\gamma^i} = \frac{1}{K(1-\beta)} \left(n_j^i\right)^{\beta} \frac{t'(x_j)R_j}{\left\{1 - t(x_j)S_j^i\right\}^2} > 0, \tag{14}$$

$$\frac{d\Psi_j}{dY^i} = \frac{\gamma^i \left(n_j^i\right)^{\beta} \tau_j^i t'(x_j) R_j}{K^2 \log G \left[1 - t(x_j)S_j^i\right]^2} \left(\frac{\beta}{1-\beta} - \log G\right)$$

$$\gtrless 0 \text{ if } \log G \gtrless \frac{\beta}{1-\beta}. \tag{15}$$

Summarizing:

PROPOSITION 1. (bid rent functions). *Under assumptions 1 and 2, the bid rents are increasing in the preference bias towards the local public good; they are increasing (decreasing) in income if the level of local public good service is sufficiently small (large) that* $g(x_1, x_2, R_j; Y^i) < (>)\ \beta/(1-\beta)$.

The bid rent is always increasing in individuals' preferences towards the LPG. It is increasing in individuals' incomes only when the scale of LPG services is small. Intuitively, income generates two opposing effects. On the one hand, there is a *patronization effect*: higher income encourages the use of the local public facility (higher n_j^i), thereby raising the bid rent. On the other, it creates a *tax revenue effect*: higher income leads to a larger tax revenue and a greater provision of the LPG (K), which by diminishing marginal utility, results in a lower bid rent. When the scale of LPG services is small, the tax revenue effect is dominated by the patronization effect. In this case, the bid rent is increasing in income.

A type-i person will choose a location of residence $h \in \{1, 2\}$ if

$$V_h^i(I_h, R_h, t(x_h)) \geq V_j^i(I_j, R_j, t(x_j)) \qquad \forall \quad j \neq h, \tag{16}$$

where the relevant indirect utilities are given by (12). This may be referred to as the *optimal locational choice* condition. Following Negishi (1960), we will find *equilibrium price support* (market rents) for any equilibrium configuration to be established. We can now define the concept of equilibrium with endogenous sorting:

DEFINITION 1. *A* multi-class competitive spatial equilibrium *(MCSE) is a tuple* $\{n_j^i, C_j^i, I_j, \Psi_j, G, M, K, H_j, L_j\}$ *together with the market land rents* $\{R_j\}$ *such that the following conditions are satisfied*:

i) given the residential location, each agent maximizes his/her utility subject to the budget constraint, i.e., (5), (9), and (11) are met;

ii) the market rents are in an appropriate range such that the optimal locational choice conditions (16) are met for all agents and each agent only resides in one location $(I_1 + I_2 = 1)$;

iii) the bid rents satisfy (13);

iv) the service of the local public good is captured by (2), the number of users patronizing the local public good is captured by (3), and the government budget is balanced as given by (6);

v) the population balance conditions (1) are met for both communities.

In the next two sections we will characterize the MCSE. There are two types of MCSE: *segregated* and *integrated*. In an integrated equilibrium, both communities must be populated by residents of both types. In a segregated equilibrium, at least one of the two communities must have residents of a homogeneous type. Depending on the relative size of the population of the rich to the poor, however, a segregated equilibrium may have both types residing in one of the two communities. When each community only contains one type of resident, the equilibrium is called *completely segregated*; otherwise, it is called *incompletely segregated*. In figure 1 we depict six possible types of segregated equilibrium configurations: there are two completely segregated and

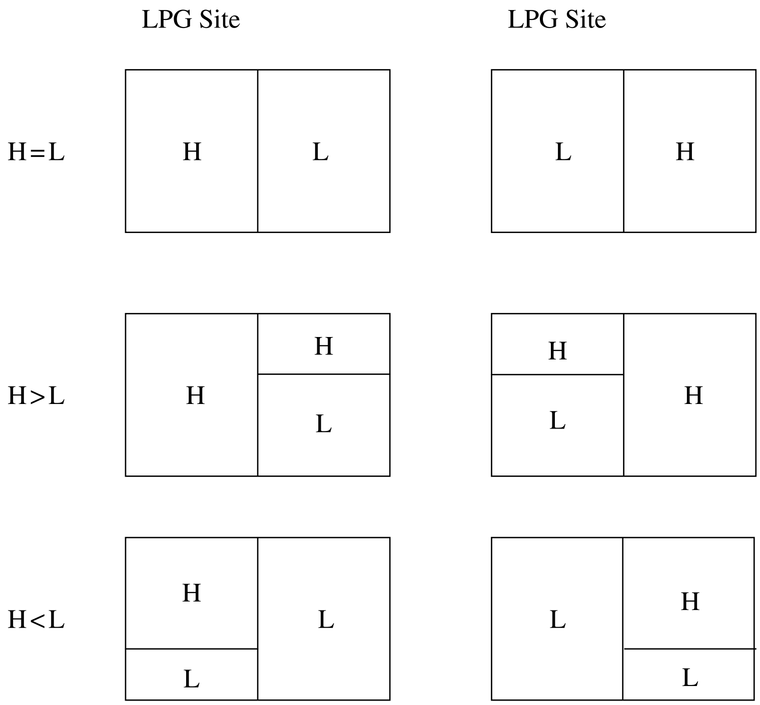

FIGURE 1 Configurations of segregated equilibria

four incompletely segregated cases. Formally, these equilibrium configurations can be defined as

DEFINITION 2. *A multi-class competitive spatial equilibrium* $\{n_j^i, C_j^i, I_j, \Psi_j, G, M, K, H_j, L_j; R_j\}$ *is called*

i) completely segregated *if* $H_j \cdot L_j = 0$ *for both* $j = 1$ *and* $j = 2$;

ii) incompletely segregated *if* $H_j \cdot L_j = 0$ *for either* $j = 1$ *or* $j = 2$ *(but not both)*;

iii) integrated *if* $H_j + L_j = N/2$ *and* $H_j \cdot L_j > 0$ *for both* $j = 1$ *and* $j = 2$.

We will establish conditions on the underlying parameters and ranges of market rents under which each type of equilibrium configuration is supported. Obviously, from the population balance condition, a completely segregated equilibrium is possible only when $H = L = N/2$.

4. Segregated equilibrium

We begin by examining the case of segregated equilibrium. From the bid rent function (13), this type of equilibrium requires that the bid rents be increasing in Y^i. That is, the *willingness to pay* for a higher market land/property rent must be matched by the *ability to pay*. If we utilize (15) and proposition 1, this condition is equivalent to

CONDITION S $g(x_1, x_2, R_j; Y^i) < \beta/(1-\beta)$.

Again, we will express this condition in primitives when we analyse each configuration.

There are two cases to consider: (i) equal population of the rich and the poor and (ii) unequal population of the rich and the poor. While the former case may result in complete segregation, the stratification between the rich and the poor in the latter case must be incomplete. Depending on the parameters, it is possible that the rich reside in community 1 where the local public facility is provided or in community 2 where they need to travel in order to consume the LPG service. If we regard community 1 as the city centre with publicly provided museums and activity facilities and community 2 as suburbs, the former case captures many Asian and European cities (e.g., London, Paris, Rome, Taipei, and Tokyo), whereas the latter is consistent with the configurations of typical American cities (e.g., Atlanta, Chicago, Detroit, Los Angeles, and Washington, DC).

4.1. Complete segregation

Consider the case where the rich and the poor are of equal size: $H = L = N/2$. We will show that when the market rents are within an appropriate range, the equilibrium features a complete segregation in the sense that all the rich reside in one community and the poor in another. Since all the rich may reside in community 1 or in community 2, depending on the underlying set of parameters, each case must be analysed accordingly:

a) The rich are closer to the site of the public facility: in this case, the rich reside in community 1 and the poor travel for public good services. That is, we have $x_1 = 1$ and $x_2 = 0$. To ensure this is an optimal locational choice to every individual, it requires $V_1^H > V_2^H$ and $V_1^L < V_2^L$.
b) The poor are closer to the site of the public facility: now, all the poor reside in community 1 and the rich travel for public good services. That is, we have $x_1 = 0$ and $x_2 = 1$. Optimal locational choice requires $V_1^H < V_2^H$ and $V_1^L > V_2^L$.

Denote $\Delta\tau^i \equiv \tau_1^i - \tau_2^i$ and $B \equiv (\beta/\phi)^\beta(1-\beta)^{1-\beta}\{1-[\phi/(T+\phi)]^{\beta/(1-\beta)}\}^{1-\beta}$. Further define

$$\Omega \equiv \frac{\Delta\tau^H Y^H - \Delta\tau^L Y^L}{(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{1-\beta}}} \text{ and}$$

$$\Lambda \equiv \frac{\Delta\tau^H Y^H - \left(\frac{\gamma^H}{\gamma^L}\right)^{\frac{1}{1-\beta}} \Delta\tau^L Y^L}{\left(\frac{\gamma^H}{\gamma^L}\right)^{\frac{1}{1-\beta}} - 1} = (\gamma^i)^{\frac{1}{1-\beta}}\Omega - \Delta\tau^i Y^i,$$

where Ω is positive, because $\gamma^H > \gamma^L$, $Y^H > Y^L$ and $\Delta\tau^H > \Delta\tau^L$. While B is increasing in the unit commuting cost, Ω and Λ are increasing in the income/income tax differential and decreasing in the preference differential.

We will show that the two different types of completely segregated equilibria can arise only if the following conditions are met: (i) $g(1, 0, R_j; Y^i) < \min\{A, \beta/(1-\beta)\}$ for the case where the rich all reside in community 1 and (ii) $g(0, 1, R_j; Y^i) < \min\{A, \beta/(1-\beta)\}$ for the case where the rich all reside in community 2. Notably, the inequality, $g(x_1, x_2, R_j; Y^i) < \min\{A, \beta/(1-\beta)\}$, corresponds to assumption 2 and condition S under the respective configurations. These conditions are necessary, which will become sufficient if the market rents fall into certain ranges. More precisely, we consider

ASSUMPTION 3. $[B \; min \; \{A, \beta/(1-\beta)\}]^{1/(1-\beta)} > \Omega.$

This inequality is more likely to hold true if the preference differential is large relative to the income/income tax differential (Ω small) or if the unit commuting cost is high (B large). We can then establish

PROPOSITION 2. (completely segregated MCSE). *Under assumption 1 with* $H = L$,

a) there exists a multi-class competitive spatial equilibrium with complete segregation where the rich are closer to the site of the public facility if assumption 3 is met and if

$$\frac{\Lambda + R_2(1+t(0))}{1+t(1)} < R_1 < \frac{\left[B\gamma^L \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L + R_2(1+t(0))}{1+t(1)}; \quad (17)$$

b) there exists a multi-class competitive spatial equilibrium with complete segregation where the poor are closer to the site of the public facility if

$$R_1 < \frac{\min\left\{\Lambda, \left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H\right\} + R_2(1+t(0))}{1+t(1)}. \quad (18)$$

Assumption 3 ensures that the inequalities in (17) are valid. Inequalities (17) and (18) are mutually exclusive,[13] thereby ruling out the possibility of multiple equilibrium configurations.

Notice that in equilibrium one can only pin down the relative market rent between the two communities. Should we fix R_2 at the agricultural land rent or as an exogenous multiple of the agricultural land rent, the range of R_1 is fully determined by (17) and (18) in the respective cases.[14] When the relative market rent (adjusted by the property tax) of the LPG site (community 1) is high, it discourages the poor from residing there, since they do not care as much about the service of the LPG. On the contrary, when the relative market rent of community 1 is sufficiently low, the poor are closer to the LPG site. The effect of property taxes works in an analogous manner.

As can be seen from (17) and the definition of Λ, to have the rich closer to the LPG site in a completely segregated equilibrium requires the preference differential be sufficiently large to overcome the differences in income and income tax progressiveness. In this case, endogenous sorting is *by both income and preferences towards local public good consumption*. On the contrary, (18) is met if the preference differential is small and the level of the LPG provision is low. Thus, in the completely segregated equilibrium with the poor closer to the LPG site, endogenous sorting is *entirely driven by income heterogeneity*, since those with stronger preferences towards the LPG (the type-H) now reside *away from* the site of the public facility. The main results concerning the completely segregated equilibrium are summarized in table 2(A).

4.2. Incomplete segregation

When the rich and the poor have different population masses, one of the two communities must have mixed residents of both types. There are two cases to be studied: (i) more rich than poor ($H > L$, or, $H/N > 1/2$) and (ii) more poor than rich ($H/N < 1/2$). In the first case, community 2 (1) is mixed with both types if the rich (poor) are closer to the public facility site, whereas in the second, community 1 (2) is mixed if the rich (poor) are closer to the public facility site.

4.2.1. More rich than poor

Since the population of the rich exceeds that of the poor, some of the rich must reside with the poor by construction. Depending on the underlying parameters, the poor may reside in community 1 or community 2:

a) The rich are closer to the site of the public facility: in this case, only the rich reside in community 1 and all the poor travel for public good services. That

13 Specifically, (17) requires $R_1 > [\Lambda + R_2(1 + t(0))]/(1 + t(1))$, whereas (18) requires $R_1 < [\Lambda + R_2(1 + t(0))]/(1 + t(1))$.

14 In a closed-city model with a finite number of communities, the land rent at the boundary need not equal the agricultural land rent.

TABLE 2
Summary of segregated equilibrium outcomes

(A) H = L

Large preference heterogeneity with intermediate R_1	Small preference heterogeneity with intermediate R_1
Complete segregation The poor closer to the LPG site Sorting purely by income	Complete segregation The rich closer to the LPG site Sorting by income/preference

(B) H > L or H < L

Small R_1	Intermediate R_1	Large R_1
Incomplete segregation	Incomplete segregation	Incomplete segregation
The poor closer to the LPG site	Multiple equilibria in locational choice	The rich closer to the LPG site
Sorting purely by income	Sorting sources ambiguous	Sorting by income/preference

is, we have $x_1 = 1$ and $x_2 = 2H/N - 1$. Locational equilibrium requires $V_1^H = V_2^H$ and $V_1^L < V_2^L$.

b) The poor are closer to the site of the public facility: now, all the poor reside in community 1 and some of the rich travel for public good services. That is, we have $x_1 = 2H/N - 1$ and $x_2 = 1$. Locational equilibrium requires $V_1^H = V_2^H$ and $V_1^L > V_2^L$.

By similar arguments to those in the completely segregated case, the validity of incompletely segregated equilibria requires (i) $g(1,\ 2H/N - 1,\ R_j;\ Y^i) < \min\ \{A,\ \beta/(1-\beta)\}$ for the case where the poor all reside in community 2, and (ii) $g(2H/N - 1,\ 1,\ R_j;\ Y^i) < \min\ \{A,\ \beta/(1-\beta)\}$ for the case where the poor all reside in community 1. We now establish

PROPOSITION 3. (incompletely segregated MCSE). *Under assumption 1 with $H > L$,*
a) there exists a multi-class competitive spatial equilibrium with incomplete segregation where the rich are closer to the site of the public facility if assumption 3 is met and if

$$\frac{\Lambda + R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right)}{1 + t(1)} < R_1 < \frac{\left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H + R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right)}{1 + t(1)}; \tag{19}$$

b) there exists a multi-class competitive spatial equilibrium with incomplete segregation where the poor are closer to the site of the public facility if

$$R_1 < \frac{\min\left\{\Lambda, \left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H\right\} + R_2(1+t(1))}{1+t\left(\frac{2H}{N}-1\right)}; \quad (20)$$

c) The equilibrium is indeterminate in which either the rich or the poor may be closer to the site of the public facility if assumption 3 is met and if

$$R_1 < \frac{\left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H + R_2\left(1+t\left(\frac{2H}{N}-1\right)\right)}{1+t(1)} \quad (21)$$

and

$$\frac{\Lambda + R_2\left(1+t\left(\frac{2H}{N}-1\right)\right)}{1+t(1)} < R_1 < \frac{\Lambda + R_2(1+t(1))}{1+t\left(\frac{2H}{N}-1\right)}. \quad (22)$$

The incompletely segregated equilibrium configuration with the rich being closer to the public facility site is supported by a relatively high market rent/property tax in the community with the public facility (community 1), whereas that with the poor being closer to the public facility site is supported by a relatively low market rent/property tax in community 1. Parallel to the case of completely segregated equilibrium, endogenous sorting is both by income and preferences towards LPG consumption when the rich are closer to the LPG site, while endogenous sorting is entirely driven by income heterogeneity when the poor are closer.

When the relative market rent (adjusted by the property tax) of community 1 falls in the range specified in (22), it is not high enough to discourage the poor from residing in the community with the public facility and not low enough to always attract the poor to community 1. As a result, multiple equilibria arise, since the poor may reside either in community 1 or in community 2. The equilibrium selection depends purely on the self-fulfilling prophecies, where the corresponding configuration is supported by endogenous adjustments in the proportion of the rich. The main results concerning the incompletely segregated equilibrium are summarized in table 2(B).

4.2.2. More poor than rich

The population of the poor now exceeds that of the rich, thereby forcing some of the poor to reside with the rich. Again, depending on the underlying parameters and relative market rents, the rich may reside in community 1 or community 2:

a) The rich are closer to the site of the public facility: in this case, all the rich reside in community 1 and some of the poor travel for public good services.

That is, we have $x_1 = 2H/N$ and $x_2 = 0$. Locational equilibrium requires $V_1^H > V_2^H$ and $V_1^L = V_2^L$.

b) The poor are closer to the site of the public facility: now, only the poor reside in community 1 and all the rich travel for public good services. That is, we have $x_1 = 0$ and $x_2 = 2H/N$. Locational equilibrium requires $V_1^H < V_2^H$ and $V_1^L = V_2^L$.

By similar arguments to those in the completely segregated case, the validity of incompletely segregated equilibria requires: (i) $g(2H/N, 0, R_j; Y^i) < \min\{A, \beta/(1-\beta)\}$ for the case where the poor all reside in community 2 and (ii) $g(0, 2H/N, R_j; Y^i) < \min\{A, \beta/(1-\beta)\}$ for the case where the poor all reside in community 1. Parallel to the case with more rich than poor, we can obtain the following:

PROPOSITION 4. (incompletely segregated MCSE). *Under assumption 1 with $H < L$,*

a) there exists a multi-class competitive spatial equilibrium with incomplete segregation where the rich are closer to the site of the public facility if assumption 3 is met and if

$$\frac{\Lambda + R_2(1+t(0))}{1+t\left(\frac{2H}{N}\right)} < R_1 < \frac{\left[B\gamma^L \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L + R_2(1+t(0))}{1+t\left(\frac{2H}{N}\right)}; \qquad (23)$$

b) there exists a multi-class competitive spatial equilibrium with incomplete segregation where the poor are closer to the site of the public facility if

$$R_1 < \frac{\min\left\{\Lambda, \left[B\gamma^L \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L\right\} + R_2\left(1+t\left(\frac{2H}{N}\right)\right)}{1+t(0)}; \qquad (24)$$

c) The equilibrium is indeterminate in which either the rich or the poor may be closer to the site of the public facility if assumption 3 is met and if

$$R_1 < \frac{\left[B\gamma^L \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L + R_2(1+t(0))}{1+t\left(\frac{2H}{N}\right)} \qquad (25)$$

and

$$\frac{\Lambda + R_2(1+t(0))}{1+t\left(\frac{2H}{N}\right)} < R_1 < \frac{\Lambda + R_2\left(1+t\left(\frac{2H}{N}\right)\right)}{1+t(0)}. \qquad (26)$$

Since the intuition behind proposition 4 resembles that underlying proposition 3 (differing only in the relative population size, H and L), it is omitted for the sake of brevity. Again, when the relative market rent falls in the range given in (26), the economy features multiple equilibria in which the rich may reside in community 1 or 2.

5. Integrated equilibrium

We now study the case of integrated equilibrium under which both communities must be populated by residents of both types. From the bid rent functions (13), this type of equilibrium requires that the bid rents be decreasing in income, or using (15), the following condition must be met:

CONDITION I. $g(x_1, x_2, R_j; Y^i) > \beta/(1-\beta)$.

That is, the service of the LPG is sufficiently large that the tax revenue effect dominates the patronization effect. In an integrated equilibrium, it requires $V_1^i = V_2^i$ for $i = H, L$.

In the appendix we show that the level of the LPG service satisfies

$$g(x_1, x_2, R_j; Y^i) = \frac{1}{B}\Omega^{1-\beta}. \tag{27}$$

Thus, condition I is equivalent to $\Omega > [B\beta/(1-\beta)]^{1/(1-\beta)}$. Moreover, assumption 2 now becomes $\Omega < (BA)^{1/(1-\beta)}$. These inequalities can be combined as

ASSUMPTION 4. $[B\beta/(1-\beta)]^{1/(1-\beta)} < \Omega < (BA)^{1/(1-\beta)}$.

These inequalities are met only when the preference differential is not too large relative to the income/income tax differential, provided that income tax revenues are sufficiently large to ensure positive bid rents (A large).

We now establish

PROPOSITION 5. (integrated MCSE). *Under assumptions 1 and 4, a multi-class competitive spatial equilibrium with an integrated configuration exists if*

$$\frac{\left[1 + t\left(\frac{2H}{N} - 1\right)\right] R_2 + \Lambda}{1 + t(1)} < R_1 < \frac{R_2\left(1 + t\left(\frac{2H}{N}\right)\right) + \Lambda}{1 + t(0)}, \tag{28}$$

where the level of local public good services is determined by (27).

Condition (28) ensures that for each pair of market rents within a specific range, there is a corresponding value of x_1 (and $x_2 = 2H/N - x_1$) which falls within the unit interval, (0, 1). Importantly, condition I indicates that in order for an integrated equilibrium to exist, we need the preference differential to be not too large compared to the income/income tax differential. Notice that when $R_1 = R_2 + \Lambda/[1 + t(H/N)]$, we have $x_1 = x_2 = H/N$. This features a *symmetric* integrated equilibrium where both communities have an identical fraction of the rich. In general, the proportions of the rich need not be equalized across the two communities, depending crucially on the relative market rent.

6. Segregation versus integration

We now examine the underlying forces driving different equilibrium configurations, particularly segregated versus integrated equilibria. Among all such forces, we focus on three: the preference differential, the income/income tax differential, and the commuting cost.

When the preference differential is sufficiently large, Ω is so low that condition I is violated. This implies that endogenous sorting can never result in an integrated equilibrium. Intuitively, with a higher preference differential, the rich are more willing to outbid the poor, thus driving towards a segregated economy.

Now consider a sufficiently large income/income tax differential (relative to the preference differential) that a sufficiently high value of Ω causes condition S to be violated. In this case, a higher income creates a large tax revenue effect that dominates the patronization effect. Thus, the rich is less willing to outbid the poor and endogenous sorting results in an integrated equilibrium.

Additionally, an increase in the unit commuting cost raises B, which tends to cause condition I to be violated and hence rules out integration. Specifically, higher commuting costs discourage patronization of the local public facility and lower the LPG provision. Thus, the tax revenue effect is dominated by the patronization effect, implying that the rich are more willing to outbid the poor and a segregated equilibrium arises.

In summary, when the preference differential is large relative to the income differential, the economy is more likely to be segregated. This explains why many observed stratification cases feature clustering by ethnic groups. On the contrary, with extremely high tax progressiveness as observed in socialist countries or with a superior transportation infrastructure that lowers commuting costs, one would expect the society to be more integrated. Furthermore, with relatively low market rents (adjusted by property taxes) in the central city, the poor may reside there despite a lower preference towards the public good service, as we see in many U.S. cities.

7. Conclusions

We have characterized competitive spatial equilibrium with endogenous sorting where the equilibrium configuration may be completely segregated, incompletely segregated, or completely integrated, depending on the relative market rents augmented by property taxes and the underlying parameters (especially, preference heterogeneity, income heterogeneity, and local income tax progressiveness). The segregation outcome depends on relative market rents augmented by property taxes and parameters describing preference heterogeneity, income heterogeneity, and local income tax progressiveness. Our paper extends the literature on economic segregation by examining the role of preference heterogeneity with respect to a travel-for congestable local public good in driving economic stratification. With a sufficiently large differential in preferences, the rich and the poor are always segregated. However, when the relative market rent (adjusted by the property tax) of the community with the local public facility is sufficiently low, the rich need not be closer to the public facility site and, in this case, endogenous sorting is purely on the basis of income. When the relative market rent falls in an intermediate range, multiple incompletely segregated equilibria may arise, in which either the rich or the poor may reside closer to the public facility site. Stratification warrants attention, because it induces socioeconomic disparities and is therefore important for public policy considerations. Our analysis sheds light on possible equilibrium outcomes that correspond to some of the outcomes observed in various metropolitan areas.

The results obtained in this paper should be taken as suggestive rather than definitive, because our model is simplified in several aspects for analytical convenience. There are at least five possible avenues for future research if some simplifying assumptions are relaxed. First, the assumption of a positive correlation between income and preferences for the local public good can be relaxed: agents may be divided into four categories: high income and high preferences for the local public good, low income and high preferences for the local public good, high income and low preferences for the local public good, and low income and low preferences for the local public good. One may then ask whether pure stratification by preferences towards local public good consumption may occur. Second, one may consider a more realistic urban structure (say, monocentric city) with elastic land/housing demand. One may then study the shape of the distribution of the rich and the poor as well as the role of immobile property demand played in equilibrium sorting. Third, it may be interesting to consider an alternative equilibrium concept based on club theory (especially the 'clubs and the market' framework developed by Ellickson et al. 1999, 2001). In particular, one can view different stratified classes as clubs and establish the formation of these clubs. Fourth, one may introduce political economy issues, establishing voting or other mechanisms to determine endogenously the provision of local public good or the schedules of the fiscal instruments (user fee and local income/property tax rates). Of course, one must bear in mind the such a political equilibrium need not exist in general. Finally, our paper exclusively conducts a positive analysis concerning equilibrium sorting by income and preferences for the travel-for local public good. It would also be interesting to undertake a normative analysis, examining, on the basis of efficiency and equity, which policy may achieve higher welfare for both communities as a whole.

Appendix

Proof of lemma 1. Straightforward differentiation leads to

$$\frac{d\log G}{dn_j^i} = \frac{d\log K}{dn_j^i} - \alpha\frac{d\log M}{dn_j^i} = x_j N_j\left(\frac{\phi}{K} - \frac{\alpha}{M}\right)$$
$$\frac{d\log G}{dx_j} = \frac{1}{K}t'(x_j)R_j + \left(\frac{\phi}{K} - \frac{\alpha}{M}\right)\frac{dM}{dx_j}$$
$$\frac{d\log G}{dR_j} = \frac{d\log K}{dR_j} = \frac{t(x_j)}{K}$$
$$\frac{d\log G}{dY^i} = \frac{d\log K}{dY^i} = \frac{\tau_j^i}{K}.$$

Under assumption 1, the balanced-budget local public good service is invariant to the number of visits on the margin, thus yielding the required properties. ■

Proof of lemma 2. Totally differentiating (9) leads to

$$\left(1 - \frac{\partial q}{\partial n_j^i}\right)dn_j^i = \frac{\partial q}{\partial R_j}dR_j + \frac{\partial q}{\partial x_j}dx_j + \frac{\partial q}{\partial Y^i}dY^i + \frac{\partial q}{\partial \gamma^i}d\gamma^i,$$

where, under assumption 1, $\Gamma_j^i = 0$, and hence $\partial q/\partial n_j^i = 0$. Using (6), it is trivial that $\partial K/\partial M = \phi$, $\partial K/\partial R_j = t(x_j)$, and $\partial K/\partial x_j = t'(x_j)R_j$. Thus, we have

$$\frac{\partial q}{\partial x_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t'(x_j)R_j$$
$$\frac{\partial q}{\partial R_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t(x_j)$$
$$\frac{\partial q}{\partial Y^i} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}\tau_j^i$$
$$\frac{\partial q}{\partial \gamma^i} = \frac{1}{1-\beta}\frac{n_j^i}{\gamma^i}.$$

These imply the following results:

$$\frac{dn_j^i}{dx_j} = \frac{1}{K}t'(x_j)R_j > 0 \tag{A1}$$

$$\frac{dn_j^i}{dR_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t(x_j) > 0 \tag{A2}$$

$$\frac{dn_j^i}{dY^i} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{\tau_j^i}{K} > 0 \tag{A3}$$

$$\frac{dn_j^i}{d\gamma^i} = \frac{1}{1-\beta}\frac{n_j^i}{\gamma^i} > 0, \tag{A4}$$

thereby completing the proof. ■

Proof of lemma 3. From (12) and (A2), we obtain

$$\frac{dV_j^i}{dx_j} = -R_j t'(x_j) + (1-\beta)\gamma^i \left(n_j^i\right)^{\beta} \frac{1}{K} t'(x_j) R_j + \beta(1-\beta)\gamma^i \left(n_j^i\right)^{\beta-1} (\log G)\left[\frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K} t'(x_j) R_j\right],$$

which can be simplified to

$$\frac{dV_j^i}{dx_j} = t'(x_j) R_j \left[\gamma^i \left(n_j^i\right)^{\beta} \frac{1}{K} - 1\right].$$

Similarly, from (12), (A1), and lemma 1, we have

$$\frac{dV_j^i}{dR_j} = -(1 + t(x_j)) + \gamma^i \left(n_j^i\right)^{\beta} \frac{1}{K} t(x_j).$$

Substituting these results into the bid rent functions gives

$$\Psi(R_j,\ x_j,\ \gamma^i, Y^i) = -\frac{t'(x_j) R_j \left[\gamma^i \left(n_j^i\right)^{\beta} \frac{1}{K} - 1\right]}{t(x_j)\left[\gamma^i \left(n_j^i\right)^{\beta} \frac{1}{K} - 1\right] - 1},$$

which can be manipulated to yield (13). ■

Proof of proposition 1. Straightforward differentiation of the bid rent function (13) yields

$$\frac{d\Psi}{dS_j^i} = \frac{t'(x_j) R_j}{\left\{1 - t(x_j) S_j^i\right\}^2} > 0$$

$$\frac{dS_j^i}{d\gamma^i} = \frac{1}{K}\left(n_j^i\right)^{\beta} \frac{1}{(1-\beta)} > 0$$

$$\frac{dS_j^i}{dY^i} = \frac{\gamma^i}{K^2}\left(n_j^i\right)^{\beta} \frac{\tau_j^i}{\log G}\left[\frac{\beta}{1-\beta} - \log G\right] \gtrless 0,$$

which can be combined to derive the required properties. ■

Proof of proposition 2. We first consider part (a). From (12), the indirect utility differential for a type-i agent between the two communities is

$$V_1^i - V_2^i = (1-\beta)(\gamma^i \log G)^{\frac{1}{1-\beta}} \left(\frac{\beta}{\phi}\right)^{\frac{\beta}{1-\beta}} \left[1 - \left(\frac{\phi}{T+\phi}\right)^{\frac{\beta}{1-\beta}}\right] + [R_2(1+t(0)) - R_1(1+t(1))] - \Delta\tau^i Y^i.$$

It is useful to define $W(x_1, x_2, R_j; Y^i) \equiv [B \cdot g(x_1, x_2, R_j; Y^i)]^{1/(1-\beta)}$, which is increasing in each argument by lemma 1. Thus, $V_1^H - V_2^H > 0$ requires

$$(\gamma^H)^{\frac{1}{1-\beta}} W(1, 0, R_j; Y^i) - \Delta\tau^H Y^H > R_1(1+t(1)) - R_2(1+t(0)). \tag{A5}$$

Similarly, $V_1^L - V_2^L < 0$ requires

$$(\gamma^L)^{\frac{1}{1-\beta}} W(1, 0, R_j; Y^i) - \Delta\tau^L Y^L < R_1(1+t(1)) - R_2(1+t(0)). \tag{A6}$$

Combining (A5) and (A6), we obtain

$$R_1 > \frac{\Lambda + R_2(1+t(0))}{1+t(1)}. \tag{A7}$$

Utilizing (A5) and (A6), we can verify that in this configuration, assumption 2 holds if

$$R_1 < \frac{(B\gamma^L A)^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L + R_2(1+t(0))}{1+t(1)}, \tag{A8}$$

whereas condition S is met if

$$R_1 < \frac{\left(B\gamma^L \frac{\beta}{1-\beta}\right)^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L + R_2(1+t(0))}{1+t(1)}. \tag{A9}$$

Combining (A7), (A8), and (A9) yields the required condition in proposition 2(a), which is valid under

$$\left[B\gamma^L \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^L Y^L > \Lambda.$$

By applying the definition of Λ, this condition reduces to the inequality in assumption 3.

For part (b), we utilize (12) and repeat the same steps as before: $V_1^H - V_2^H < 0$ and $V_1^L - V_2^L > 0$ require, respectively, that

$$(\gamma^H)^{\frac{1}{1-\beta}} W(0, 1, R_j; Y^i) - \Delta\tau^H Y^H < R_1(1+t(0)) - R_2(1+t(1)) \tag{A10}$$

$$(\gamma^L)^{\frac{1}{1-\beta}} W(0, 1, R_j; Y^i) - \Delta\tau^L Y^L > R_1(1+t(0)) - R_2(1+t(1)). \tag{A11}$$

Combining (A10) and (A11) gives

$$R_1 < \frac{\Lambda - \Delta\tau^H Y^H + R_2(1 + t(0))}{1 + t(1)}. \tag{A12}$$

By utilizing (A10) and (A11), assumption 2 and condition S under this configuration hold true if

$$R_1 < \frac{\left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H + R_2(1 + t(0))}{1 + t(1)}. \tag{A13}$$

Combining (A12) and (A13), we obtain the required condition in proposition 2(b). ■

Proof of proposition 3. Consider part (a). Following similar steps to those in the proof of proposition 2, we can express $V_1^H - V_2^H = 0$ as

$$(\gamma^H)^{\frac{1}{1-\beta}} W\left(1, \frac{2H}{N} - 1, R_j; Y^i\right) - \Delta\tau^H Y^H = R_1(1 + t(1)) - R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right), \tag{A14}$$

and $V_1^L - V_2^L < 0$ as

$$(\gamma^L)^{\frac{1}{1-\beta}} W\left(1, \frac{2H}{N} - 1, R_j; Y^i\right) - \Delta\tau^L Y^L < R_1(1 + t(1)) - R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right). \tag{A15}$$

We then use (A14) to eliminate $W(1, 2H/N - 1, R_j; Y^i)$ in (A15) to obtain

$$R_1 > \frac{\Lambda + R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right)}{1 + t(1)}.$$

Moreover, assumption 2 and condition S are met if

$$R_1 < \frac{\left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H + R_2\left(1 + t\left(\frac{2H}{N} - 1\right)\right)}{1 + t(1)}. \tag{A16}$$

These inequalities can be combined to yield the required condition which is valid under assumption 3.

Consider, next, part (b). By straightforward manipulation, $V_1^H - V_2^H = 0$ implies

$$(\gamma^H)^{\frac{1}{1-\beta}} W\left(\frac{2H}{N} - 1, 1, R_j; Y^i\right) - \Delta\tau^H Y^H = R_1\left(1 + t\left(\frac{2H}{N} - 1\right)\right) - R_2(1 + t(1)), \tag{A17}$$

whereas $V_1^L - V_2^L > 0$ requires

$$(\gamma^L)^{\frac{1}{1-\beta}} W\left(\frac{2H}{N} - 1, 1, R_j; Y^i\right) - \Delta\tau^L Y^L > R_1\left(1 + t\left(\frac{2H}{N} - 1\right)\right) - R_2(1 + t(1)). \quad \text{(A18)}$$

Both (A17) and (A18) together yield

$$R_1 < \frac{\Lambda - \Delta\tau^H Y^H + R_2(1 + t(1))}{1 + t\left(\frac{2H}{N} - 1\right)},$$

whereas assumption 2 and condition S hold if

$$R_1 < \frac{\left[B\gamma^H \min\left\{A, \frac{\beta}{1-\beta}\right\}\right]^{\frac{1}{1-\beta}} - \Delta\tau^H Y^H + R_2(1 + t(1))}{1 + t\left(\frac{2H}{N} - 1\right)}. \quad \text{(A19)}$$

Together these give the required condition.

Part (c) is a direct consequence of parts (a) and (b) by comparing the equilibrium ranges of market rents and by noting that (A16) implies (A19). ■

Proof of proposition 4. To prove part (a), repeat the same exercises used in the proof of proposition 3: $V_1^H > V_2^H$ and $V_1^L = V_2^L$ require, respectively, that

$$(\gamma^H)^{\frac{1}{1-\beta}} W\left(\frac{2H}{N}, 0, R_j; Y^i\right) - \Delta\tau^H Y^H > R_1\left(1 + t\left(\frac{2H}{N}\right)\right) - R_2(1 + t(0)) \quad \text{(A20)}$$

$$(\gamma^L)^{\frac{1}{1-\beta}} W\left(\frac{2H}{N}, 0, R_j; Y^i\right) - \Delta\tau^L Y^L = R_1\left(1 + t\left(\frac{2H}{N}\right)\right) - R_2(1 + t(0)). \quad \text{(A21)}$$

By eliminating $W(2H/N, 0, R_j; Y^i)$ and expressing assumption 2 and condition S in primitives under this configuration, we then derive (23), which is valid under assumption 3.

For part (b), the requirements that $V_1^H < V_2^H$ and $V_1^L = V_2^L$ imply, respectively, that

$$(\gamma^H)^{\frac{1}{1-\beta}} W\left(0, \frac{2H}{N}, R_j; Y^i\right) - \Delta\tau^H Y^H < R_1(1 + t(0)) - R_2\left(1 + t\left(\frac{2H}{N}\right)\right) \quad \text{(A22)}$$

$$(\gamma^L)^{\frac{1}{1-\beta}} W\left(0, \frac{2H}{N}, R_j; Y^i\right) - \Delta\tau^L Y^L = R_1(1 + t(0)) - R_2\left(1 + t\left(\frac{2H}{N}\right)\right). \quad \text{(A23)}$$

Repeating the same exercises to eliminate $W(0, 2H/N, R_j; Y^i)$ and to express assumption 2 and condition S in the primitives gives (24).

Part (c) is shown by comparing the equilibrium ranges of market rents in (23) and (24). ■

Proof of proposition 5. Equalizing the indirect utilities specified in (12) for each type across the two communities yields

$$(\gamma^H)^{\frac{1}{1-\beta}} W(x_1, x_2, R_j; Y^i) - \Delta\tau^H Y^H = R_1(1 + t(x_1)) - R_2(1 + t(x_2)) \quad \text{(A24)}$$

$$(\gamma^L)^{\frac{1}{1-\beta}} W(x_1, x_2, R_j; Y^i) - \Delta\tau^L Y^L = R_1(1 + t(x_1)) - R_2(1 + t(x_2)). \quad \text{(A25)}$$

Combining (A24) and (A25) to eliminate market rents, we obtain

$$W(x_1, x_2, R_j; Y^i) = \Omega, \quad \text{(A26)}$$

which together with the definition of W gives (27). Using (27), we establish that assumption 2 and condition I are met under assumption 4.

It is easily seen from the definition of Ω that for any set of income and local tax parameters, Ω is decreasing in the degree of heterogeneity in preferences towards the local public good. Substituting $x_2 = 2H/N - x_1$ and (A26) into (A25) leads to

$$\begin{aligned} \chi(x_1) &\equiv R_1(1 + t(x_1)) - R_2\left(1 + t\left(\frac{2H}{N} - x_1\right)\right) \\ &= (\gamma^L)^{\frac{1}{1-\beta}}\Omega - \Delta\tau^L Y^L \\ &= \Lambda. \end{aligned} \quad \text{(A27)}$$

Note that $\chi(x_1)$ is strictly increasing in x_1. By the mean value theorem, the inequalities, $\chi(0) < \Lambda$ and $\chi(1) > \Lambda$, are sufficient to ensure a unique fixed point $x_1 \in (0, 1)$, satisfying (29). These inequalities can be combined to obtain (28). ■

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