The Dynamic Process of Economic Takeoff and Industrial Transformation

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January 2011

Abstract: This paper studies the patterns and key determinants of staged economic development. We construct a two-sector dynamic general equilibrium model populated with one-period lived non-overlapping generations, featuring endogenous enhancement in modern technology and endogenous accumulation of labor skills and capital funds. We consider preference biases toward the traditional sector of necessities, capital barriers to the modern sector, and imperfect substitution between skilled and unskilled workers. By calibrating the model to fit historic U.S. development, we find that modern technologies, saving incentives and capital fundings are most important determinants of the takeoff time. By evaluating the process of economic development, we identify that saving incentives is most crucial for the speed of modernization. We also study how labor and capital allocations toward the modern industry respond to various preference, technology and institutional changes. We further establish that labor, capital and output are most responsive to the initial state of modern technologies but least responsive to the initial state of skills, along the dynamic transition path. (*JEL Classification Numbers*: O330, O410)

Keywords: Economic takeoff and industrial transformation; endogenous skill and technological advancements; saving incentives, preference biases and capital barriers.

Acknowledgments: We are grateful for valuable comments and suggestions from Eric Bond, Juin-Jen Chang, Been-Lon Chen, Tain-Jy Chen, Hsueh-Fang Tsai, and Henry Wan, as well as seminar participants of Fudan University, Fu-Jen Catholic University, National Central University, National Chengchi University, Wuhan University, the Globalization Conference, the International Conference in Memory of John C.H. Fei and the Midwest Macroeconomic Conference. Financial support from Academia Sinica, the Granada Program of the Ministry of Education, the National Science Council (NSC 98-2911-H-001-001), the Research Grants Council of the Hong Kong SAR (RGC HKUST-6470/06H), and the Weidenbaum Center on the Economy, Government, and Public Policy to enable this international collaboration is gratefully acknowledged. The usual disclaimer applies.

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1 Introduction

The process of economic development has played a central role in the economics platform. Since the classic works by Lewis (1955), Rostow (1960), Rosenstein-Rodan (1961), Fei and Ranis (1964), and Tsiang (1964) more than four decades ago, there have been numerous studies devoting to understanding the patterns and key determinants of staged economic development. A sample of issues examined include:

(i) How important is the role of technological advancements played in industrialization?

(ii) Whether are saving incentives more important than skill accumulation in early development?

(iii) Why did some countries, such as the Newly Industrialized Countries (NICs), take off successfully but not others?

Despite the long tradition over the past five decades, not until recently have the aforementioned issues been addressed within a more rigorous general equilibrium framework. A representative set of such studies include Goodfriend and McDermott (1995), Fei and Ranis (1997), Laitner (2000), Gollin, Parente and Rogerson (2002), Hansen and Prescott (2002) and Wang and Xie (2004), to name but a few. In this paper, we extend the static general equilibrium model of Wang and Xie (2004) to a dynamic general equilibrium setting, thereby enabling us to examine the dynamic process of economic take-off explicitly. To fully characterize the dynamic process is by no means an easy task. The main challenge is: how to design a model to permit an analysis of the global dynamics, as local transitional dynamics techniques would not suit the purpose of characterizing the long process of transformation from traditional to modern economies.[1]

Specifically, we maintain several key features considered by Wang and Xie (2004), such as preference biases (away from the necessary agriculture good), heterogeneous capital costs (between the agricultural and modern sectors), and imperfect factor substitution (between skilled and unskilled workers). However, we generalize their static framework by allowing capital funds, labor skills, as well as sector-specific technologies to grow over time, depending on optimizing decisions by households and firms. While funds can be accumulated due to intergenerational savings, skills can evolve

[1] Gollin, Parente and Rogerson (2002), Lucas (2004) and Bond, Trask and Wang (2005) are three of the very few studies in which global dynamics can be analyzed. An alternative is to formulate models featuring unbalanced growth, as in Kongsamut, Rebelo, and Xie (2001), Bond, Trask and Wang (2003), Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008), which are also very difficult to construct.

due to educational effort. Moreover, the modern technology can be enhanced by inputs of skilled research labor.

The main message delivered by this research is as follows. The modern industry is more likely to be activated and the economy can take off at a faster speed when:

(i) the initial level of the modern technology or the speed of modern technology growth is high;

(ii) the initial supply of fund or the subjective intergenerational discounting factor is high, or the modern sector capital barriers is low;

(iii) the initial fraction of skilled workers or the speed of knowledge accumulation is high, or the disutility scaling factor is low;

(iv) the preference bias away from the traditional good is low.

The quantitative results suggest that the initial levels of the modern technology and fund supply as well as the subjective intergenerational discounting factor are most influential for determining the timing of economic takeoff – a 10% increase in each of these three parameters can reduce the benchmark activation time of 45 years by more than one third.

Moreover, we also find in our quantitative analysis that the subjective discounting factor that affects individual's saving incentives is most influential in generating a rapid transition toward a modern society. While the preference bias, the skill accumulation and the capital allocation barrier are crucial for labor shifts from the traditional to the modern sector, the initial fraction of the skilled labor, the initial level of fund supply and the initial level of the modern technology are essential for capital reallocation toward the modern industry.

Finally, by examining the dynamic transition in our model economy, we identify that labor, capital and output all shift rapidly in response to changes in the initial level of modern technology, their responses to the initial fraction of skilled labor are more moderate. As a consequence, the per capita real income growth is most responsive to the initial level of modern technology and least responsive to the initial fraction of skilled labor.

Literature Review

Goodfriend and McDermott (1995) illustrate that scale economies in production and learning are crucial for staged economic development, whereas Laitner (2000) argues savings are key elements to enable an economy to be advanced. While Hansen and Prescott (2002) points out that faster modern technology growth can speed up economic take-off, Ngai (2004) adds to their basic setup barriers to technology adoption and to capital accumulation to explain the delay of modernization. Departing from the conventional overlapping-generations framework commonly used in this literature, Gollin, Parente and Rogerson (2002) formulate an infinite-horizon model that permits an analysis of global dynamics and illustrate that an initially higher agricultural technology can release resources more effectively from the agriculture to the modern sector to advance an economy.

In their second book, Fei and Ranis (1997) summarize that the main determinants of economic development include not only the technologies of the agricultural and the modern sectors but also the speed of capital accumulation and the ability to reallocate labor from the agriculture to the modern sector. Wang and Xie (2004) adds to this list that the degree of luxuries of modern products and the availability of the skilled labor are also crucial for economic take-off. In Bond, Jones and Wang (2005), it is emphasized that learning from exporting can lower unit production costs to enable activation of more advanced industries. More recently, Tung and Wan (2008) consider informational problems in a game with firms interacting with each other sequentially where the barrier to modernization is due to firms' lack of incentives to serve as a pioneer in bringing a new technology to the society, because the sunk cost incurred for doing so may not be recovered as the future profits will quickly disappear with new firms entering the industry.

Our paper contributes to this growing literature in several significant aspects. In contrast with all other papers except Gollin, Parente and Rogerson (2002) and Bond, Jones and Wang (2005), our paper permits analytic characterization of the global dynamics of the model to enable a thorough study of both the dynamic process of economic take-off. In contrast with Gollin, Parente and Rogerson (2002) and Bond, Jones and Wang (2005), we allow modern technology, skills and capital funds to be endogenously evolved over time and examine the roles of other important development-driving forces beyond the initial level of agricultural technology.

2 The General Setup of the Economy

We construct a closed economy dynamic general equilibrium model to study the process of economic take-off. There are two different industries in the model, a traditional industry (industry 1) and a modern industry (industry 2), respectively. In this general model economy, we assume that the accumulation of funds and skills are both endogenous. Thus, our general setup may be viewed as an extension of the static model developed by Wang and Xie (2004) by allowing funds and skilled labor supply to increase at endogenous rates over time.

More specifically, we assume that the traditional industry is using a conventional Cobb-Douglas technology, but the modern industry exhibits increasing returns to scale at the aggregate level. We therefore model this feature by resorting to the literature of Marshallian externality in economic growth theory (Romer 1986, Benhabib and Farmer 1994, Wang and Xie 2004). We have:

$$Y_{1t} = A_{1t} K_{1t}^{\alpha_1} L_{1t}^{1-\alpha_1}, \quad (1)$$

$$Y_{2t} = A_{2t} K_{2t}^{\alpha_2} L_{2t}^{1-\alpha_2} \bar{K}_2^{1-\alpha_2}, \quad (2)$$

where $\alpha_2 > \alpha_1$. K_{2t} denotes the capital input in a typical firm and $\bar{K}_2$ is the industry average and treated by individual firms as exogenous given. Both K_{1t} and K_{2t} are fully depreciated in one period. A_{1t} and A_{2t} represent the productivity of a specific technology used by an unskilled and skilled labor respectively to produce final goods, Y_{1t} and Y_{2t}. If A_{10} and A_{2t-1} denote the innovations in the initial level in industry 1 and the leading-edge productivity in period t-1 in industry 2, respectively, and L_{At} denotes the labor work in the research and development (R&D) sector at date t, and then the innovations of the two industries are as follow:[2]

$$A_{1t} = A_{10}(1+\gamma_1)^t, \quad (3)$$

$$A_{2t} = A_{2t-1}\left[1+\gamma_2\left(1-\zeta e^{-\phi L_{At}}\right)\right], \quad (4)$$

where $\gamma_1, \gamma_2 \in (0,1)$, $\phi > 0$ and $\zeta \geq 0$. We will restrict our attention to the case where the speed of modern technology growth $\widehat{\gamma}_2 = \gamma_2\left(1-\zeta e^{-\phi L_{At}}\right)$ exceeds that of the traditional sector γ_1, with $\gamma_2(1-\zeta) < \gamma_1$.

The population of the economy at time t is denoted by N_t, which grows at a constant rate, $n > 0$: $N_t = N_0(1+n)^t$, where $N_0 > 0$. There are two types of workers: skilled (whose supply is N_{st}) and unskilled (whose supply is N_{ut}). Under full employment, we have the following population identity: $N_{ut} + N_{st} = N_t$. At time 0, the initial fraction of skilled workers is given by $s = N_{s0}/N_0 > 0$. Denoting χ_t as the (endogenous) effort to acquire skills at time t, we can then specify the supply of skilled workers as follows:

$$N_{st} = N_t\left[1-(1-s)e^{-\psi\sum_{\tau=1}^{t}\chi_\tau}\right], \quad 0 < s < 1, \quad (5)$$

where $\psi > 0$ measures the speed of knowledge accumulation. Without the consideration of the endogenous effort of knowledge accumulation, the knowledge accumulation process reduces to that in Aghion (2002). Our setup of endogenous knowledge accumulation is significantly simpler than

[2] The industry 2's innovation is based on the setup of Aghion (2002).

the evolution of human capital in the human capital-based endogenous growth model developed by Lucas (1988) because we avoid modeling skills as a stock variable.

Assume that only the skilled workers can handle work in R&D sector and industry 2, but all workers can produce the traditional good in industry 1. Thus, the allocation of skilled and unskilled labor must satisfy:

$$\begin{aligned} L_{1t} + L_{2t} + L_{At} &= N_{ut} + N_{st} = N_t, \\ L_{2t} + L_{At} &\leq N_{st}. \end{aligned} \tag{6}$$

The first expression equates labor demand with labor supply in the absence of unemployment. While the second inequality indicates the demand for labor in the traditional sector cannot exceed total labor supply, the third restricts the aggregate demand for skilled labor by the modern industry and R&D.

To endogenize the fund accumulation process, one may follow the conventional optimal growth framework considering individual's consumption and saving decisions over time. Alternatively, one may construct a one-period nonoverlapping-generations setup (Saint-Paul and Verdier, 1993) with income being divided between consumption and bequests periodically (see Aghion and Bolton, 1997). While the latter simplifies the analysis greatly, individuals are short-lived and intergenerational skill accumulation is difficult to be constructed in a convincing manner. In our paper, we follow in the spirit of the latter setup to simplify the consumption tradeoff but allow individuals to be long-lived. This is done by assuming that a representative agent's lifetime utility function takes the following form:

$$U_t = \ln C_{1t} + \ln (C_{2t} + \theta N_t) + \delta \ln(S_t + \theta N_t) - z \frac{\chi_t^{1+\sigma}}{1+\sigma}, \tag{7}$$

where the representative agent chooses whether to enjoy consumption of the traditional and the modern goods ($\frac{C_{1t}}{N_t}$ and $\frac{C_{2t}}{N_t}$) now or to save for the future ($\frac{S_t}{N_t}$).[3] While the traditional good is assumed a necessity, the modern product is a luxury good so that $\theta > 0$ captures the preference bias away from the modern good; that is, the higher θ is, the less the modern good is necessary to consumers (see Wang and Xie, 2004). Also, $\delta \in (0, 1)$ is a subjective intergenerational discounting factor. Additionally, the utility cost knowledge accumulation effort takes a simple constant elasticity

[3] Without loss of generality, the original utility function in per capita form is assumed to be:

$$\ln \frac{C_{1t}}{N_t} + \ln \left(\frac{C_{2t}}{N_t} + \theta \right) + \delta \ln \left(\frac{S_t}{N_t} + \theta \right) - z \frac{\chi_t^{1+\sigma}}{1+\sigma},$$

which, aside from an exogenous term $-(2+\delta) \ln N_t$, is equivalent to (7).

of intertemporal substitution form with a convex disutility cost of effort, where $z > 0$ is a disutility scaling factor and $\sigma > 0$ is the elasticity parameter of effort disutility.[4]

Denoting ρ_t as the (endogenous) rate of savings in proportion to Y_{2t} (Y_{1t} is assumed to be perishable and can not be saved):

$$S_t = \rho_t Y_{2t}. \tag{8}$$

Intergenerational savings, S_t, will be added to the funding supply across generations, that is,

$$F_t = F_{t-1} + S_t, \tag{9}$$

where F_{t-1} measures the beginning-of-lifetime funds supply in the economy. The available funds can be used to purchase capital. Assuming that installing capital in the modern sector is more costly, we can then specify the funds (capital) allocation constraint as:

$$K_{1t} + qK_{2t} \leq F_{t-1}, \tag{10}$$

where $q > 1$ captures the barrier to the allocation of capital to the modern sector.[5] As it can be seen, our model, despite its simplicity, can capture the emphasis by Laitner (2000) that savings play an important role in the dynamic process of modernization.

In the absence of international trade, goods market must clear sector-by-sector, as given by:

$$C_{1t} = Y_{1t}(A_{1t}, K_{1t}, L_{1t}), \tag{11}$$

$$C_{2t} = (1 - \rho_t)Y_{2t}(A_{2t}, K_{2t}, L_{2t}). \tag{12}$$

We can thus rewrite the optimization problem facing the integrated consumer-producer as follows:

$$V_t = \max\{\ln C_{1t} + \ln (C_{2t} + \theta N_t) + \delta \ln (S_t + \theta N_t) - z\frac{\chi_t^{1+\sigma}}{1+\sigma}\},$$

such that

$$C_{1t} = Y_{1t}(A_{1t}, K_{1t}, L_{1t}),$$

$$C_{2t} = (1 - \rho_t)Y_{2t}(A_{2t}, K_{2t}, L_{2t}),$$

$$S_t = \rho_t Y_{2t}(A_{2t}, K_{2t}, L_{2t}),$$

[4] Let the market wage be w. Individual optimization then implies that the marginal disutility of effort is equal to the shadow wage: $z\chi^\sigma = w$. Taking log and differentiating, one obtains: $\frac{d \ln \chi}{d \ln w} = \sigma^{-1}$.

[5] In contrast with the conventional wisdom, this modern sector capital barrier is not in forms of fixed costs or adjustment costs but variable costs in proportion to the capital stock, as proposed by Wang and Xie (2004). As the reader will see, such a setup simplifies the modern industry activation condition greatly. It should also be noted that an alternative simplifying setup is to follow Ngai (2004) by introducing a barrier factor directly into the production function.

$$K_{1t} + qK_{2t} = F_{t-1},$$

$$L_{1t} + L_{2t} + L_{At} = N_t,$$

$$L_{2t} + L_{At} = N_{st}(\chi_t),$$

$$A_{2t} = A_{2t}(L_{At}),$$

based on which we shall proceed with solving the dynamic competitive equilibrium.

3 Optimization and Equilibrium

We now turn to solving the optimization problem. We begin by deriving conditions under which the returns on capital and labor are equalized across the two sectors, which are referred to as no-arbitrage conditions. We then characterize intertemporal tradeoffs, which pin down the consumption-saving choice and the knowledge accumulation decision. Finally, we examine when the modern sector will be activated as an intertemporal equilibrium outcome and when the periodic profit becomes positive.

A *dynamic competitive equilibrium* is a tuple $\{C_{1t}, C_{2t}, \rho_t, \chi_t, S_t, N_{st}, N_t, Y_{1t}, Y_{2t}, L_{1t}, L_{2t}, L_{At}, K_{1t}, K_{2t}, F_{t-1}, p_t, A_{1t}, A_{2t}\}_{t=1}^{\infty}$ such that:

(i) each agent chooses consumption of the traditional and the modern goods as well as intergenerational savings and knowledge accumulation effort to maximize the lifetime utility (a total of four optimization conditions);

(ii) both labor and capital are allocated optimally between different sectors (a total of three no-arbitrage conditions);

(iii) sectoral outputs are given by (1) and (2);

(iv) production technologies evolve according to (3) and (4);

(v) intergenerational savings is determined by (8);

(vi) skills and funds are accumulated according to (5) and (9), respectively;

(vii) labor, funds, capital and both goods markets are all clear, i.e., (6), (9), (10), (11) and (12) hold;.

There are 18 equations for each t and 17 sequences of endogenous variables in our dynamical system. One can easily show that one of the equations at each point in time is redundant and the Walras law is met (specifically, when both labor and goods markets are clear, the funds market must also be clear).

3.1 Skilled Labor No-arbitrage Condition

Let us focus on the case where both sectors are active and the labor market is completely segmented in the sense that no skill labor works in the traditional sector (i.e., $L_{2t} + L_{At} = N_{st}$) – we will derive later the conditions for this case to arise. Under this circumstance, the marginal product of skilled labor (MPL) must exceed that of unskilled labor.

For any given productivity level A_{2t}, the skilled labors at date t must be indifferent between working at industry 2 or at R&D sector. Therefore, the following *research production no-arbitrage equation* must hold in equilibrium:

$$MPL_{2t} = MPL_{At},$$

where, from equations (2) and (4),

$$\begin{aligned} MPL_{2t} &= (1-\alpha_2) A_{2t} K_{2t} L_{2t}^{-\alpha_2}, \\ MPL_{At} &= \phi\gamma_2\zeta A_{2t-1} e^{-\phi L_{At}} K_{2t} L_{2t}^{1-\alpha_2}. \end{aligned}$$

The no-arbitrage condition can thus be written as:

$$(1-\alpha_2)\left[1+\gamma_2(1-\zeta e^{-\phi L_{At}})\right] = \phi\gamma_2\zeta e^{-\phi L_{At}} L_{2t},$$

$$L_{2t} = \frac{(1-\alpha_2)}{\phi\gamma_2\zeta}\left[(1+\gamma_2)e^{\phi L_{At}} - \gamma_2\zeta\right], \tag{13}$$

which together with equation (5) yields the fraction of research labor:

$$\frac{(1-\alpha_2)}{\phi\gamma_2\zeta}\left[(1+\gamma_2)\, e^{\phi L_{At}} - \gamma_2\zeta\right] + L_{At} = N_t\left[1-(1-s)e^{-\psi\sum_{\tau=1}^{t}\chi_\tau}\right], \tag{14}$$

from which one can solve for $L_{At} = Q_t(N_t, \{\chi_\tau\}_{\tau=1}^{t}; s, \psi, \alpha_2, \phi, \zeta, \gamma_2)$, which depends positively on the endogenous knowledge accumulation effort. From (13),

$$L_{2t} = \frac{(1-\alpha_2)}{\phi\gamma_2\zeta}\left[(1+\gamma_2)e^{\phi Q_t} - \gamma_2\zeta\right]. \tag{15}$$

Hence,

$$\begin{aligned} \frac{\partial L_{2t}}{\partial Q_t} &= \frac{(1-\alpha_2)(1+\gamma_2)}{\gamma_2\zeta} e^{\phi Q_t}, \\ &= \phi L_{2t} + (1-\alpha_2). \end{aligned}$$

Also, equation (14) implies:

$$\begin{aligned}\frac{\partial Q_t}{\partial \chi_t} &= \frac{\psi(1-s)N_t e^{-\psi \sum\limits_{\tau=1}^{t} \chi_\tau}}{\frac{(1-\alpha_2)}{\gamma_2 \zeta}\left(1+\gamma_2\right) e^{\phi Q_t}+1} \\ &\equiv \Psi_t(\{\chi_\tau\}_{\tau=1}^{t}; s, \psi, \alpha_2, \phi, \zeta, \gamma_2). \end{aligned} \tag{16}$$

Intuitively, an increase in the fraction of research labor generates two opposing effect on the fraction of skilled labor in modern good production: (i) by labor substitution, less skilled labor is allocated to modern good production and (ii) by skill accumulation, more skilled workers become available. As a result, the relationship between the fraction of skilled labor in modern good production and the fraction of research labor need not be monotone. It is clear that the fraction of research labor depends positively on endogenous knowledge accumulation effort (χ). Until knowledge accumulation effort is pinned down, one cannot easily characterize the fraction of research labor.

3.2 Capital No-arbitrage Condition

Substituting the funds allocation constraint (with equality) into the first-order conditions with respect to K_{1t} and K_{2t} implies:

$$\rho_t = \frac{\delta}{1+\delta} - \frac{(1-\delta)\theta N_t}{(1+\delta)A_{2t}K_{2t}L_{2t}^{1-\alpha_2}}, \tag{17}$$

$$K_{1t} = \frac{q\alpha_1}{\left[\frac{\alpha_2(1-\rho_t)+\alpha_2\rho_t}{(1-\rho_t)A_{2t}K_{2t}L_{2t}^{1-\alpha_2}+\theta N_t}\right] A_{2t}L_{2t}^{1-\alpha_2}}. \tag{18}$$

The first equation illustrates consumption-intergenerational saving tradeoff, which can be best elaborated by Figure 1. Specifically, the marginal benefit of intergenerational saving as a result of altruistic preferences must be equal to its marginal cost measured by foregone consumption. We can then obtain the expression for funds allocated to the modern sector:

$$K_{2t} = \frac{\frac{F_{t-1}}{q} - \frac{2\alpha_1\theta N_t}{\alpha_2(1+\delta)A_{2t}L_{2t}^{1-\alpha_2}}}{1+\frac{\alpha_1}{(1+\delta)\alpha_2}}. \tag{19}$$

The condition for $K_{2t} > 0$ requires:

$$\frac{F_{t-1}}{q} > \frac{2\alpha_1\theta N_t}{\alpha_2(1+\delta)A_{2t}L_{2t}^{1-\alpha_2}}, \tag{20}$$

together with the implicit restriction that $\rho_t > 0$, namely,

$$\frac{\delta}{(1-\delta)} > \frac{\theta N_t}{A_{2t} K_{2t} L_{2t}^{1-\alpha_2}}, \tag{21}$$

which both hold if $\theta = 0$ (i.e. the sector 2 will be activated immediately if $\theta = 0$). When θ is positive, then activation can take place if A_{2t} and L_{2t} eventually become large to outweigh N_t.

3.3 Intertemporal Tradeoff Conditions

Under our setup, the consumption-saving tradeoff is parsimonious as there is no need for deriving the Benveniste-Scheinkman condition associated with states variables in dynamic programming. Moreover, the knowledge accumulation decision is also simple because it is in terms of effort without any feasibility condition accompanied.

$$p_t = \frac{MU_{2t}}{MU_{1t}} = \frac{A_{1t} K_{1t}^{\alpha_1} L_{1t}^{1-\alpha_1}}{(1-\rho_t) A_{2t} K_{2t} L_{2t}^{1-\alpha_2} + \theta N_t} = \frac{Y_{1t}}{(1-\rho_t) Y_{2t} + \theta N_t}. \tag{22}$$

3.3.1 Effort toward Knowledge Accumulation

The first-order condition for knowledge accumulation effort χ_t is:

$$\begin{aligned} & MB_\chi \\ = \; & \Psi_t \left(\{\chi_\tau\}_{\tau=1}^{t}; s, \psi, \alpha_2, \phi, \zeta, \gamma_2 \right) \cdot \Bigg\{ -\frac{1-\alpha_1}{L_{1t}} + \\ & [\phi L_{2t} + (1\text{-}\alpha_2)] \left[\frac{1\text{-}\alpha_2}{L_{2t}} \frac{(1\text{-}\rho_t) Y_{2t}}{(1\text{-}\rho_t) Y_{2t} + \theta N_t} + \frac{\delta\,(1\text{-}\alpha_2)}{L_{2t}} \frac{\rho_t Y_{2t}}{\rho_t Y_{2t} + \theta N_t} - \frac{1\text{-}\alpha_1}{L_{1t}} \right] \Bigg\} \\ = \; & z \chi_t^{\sigma} = MC_\chi. \end{aligned} \tag{23}$$

This condition equates the marginal benefit of knowledge accumulation effort with its marginal cost. Figure 2 illustrates the determination of knowledge accumulation intuitively, where the marginal benefit is decreasing in knowledge accumulation effort and the marginal cost is increasing in it. In response to a reduction in Y_{1t} or s, the marginal benefit of accumulating knowledge rises, thereby increasing the speed of knowledge accumulation and the supply of skilled labor. An increase in Y_{2t} or a decrease in θ creates a direct positive effect on the marginal benefit as well as an indirect positive effect via its negative effect on intergenerational saving. As a consequence, the speed of knowledge accumulation and the supply of skilled labor rise unambiguously. Moreover, a greater disutility of effort devoted to knowledge accumulation increases the marginal cost and hence slows down the rate of knowledge accumulation and skilled labor expansion.

This can be written in an implicit function as: $\chi_t = X(\{\chi_\tau\}_{\tau=1}^{t-1}, Q_t, Y_{1t}, Y_{2t}; \psi, N_0, s, n, \alpha_1, \alpha_2, \theta, \delta, z, \sigma)$. While an increase in the endogenous output of the modern sector raises knowledge accumulation effort, an increase in the endogenous output of the traditional sector reduces it. Since the fraction of research labor (Q_t) has a positive effect on the fraction of skilled labor allocated to modern good production, it reduces individuals' effort to acquire skills for given past effort and current outputs. Finally, the direct effect of the relative price of the modern product is to increase knowledge accumulation effort.

3.4 Activation of Modern Industry

Combining results in previous subsections, we can solve the nondegenerate dynamic equilibrium in which both sectors are operative. Our main task is to examine when the modern industry is activated.

Consider a case where intergenerational saving is strictly positive ($\rho_t > 0$), i.e., (21) holds true. Under $s > 0$, skilled labor is available since time 0. Yet, funds may not be sufficient for production of the necessity (the traditional good). Therefore, industry 2 will be operative only if the supply of funds is more than sufficient to cover the capital demand in industry 1 that is used to produce the necessities, i.e., the sufficient funding condition (20) is met. Since A_{2t}, L_{2t} and F_{t-1} are growing over time, this above inequality is more likely to be met as time goes by. The earliest time at which (20) holds is denoted by $T_F \equiv \min\{t \mid F_{t-1} \geq K_{1t}\}$.

To ensure that the skilled labor is willing to work in the modern industry, the shadow wage ratio of industry 1 relative to 2 must exceed one:

$$\begin{aligned}\Omega(L_{2t}) &= p_t \frac{MPL_{2t}}{MPL_{1t}} \\ &= \frac{(1-\alpha_2)}{(1-\alpha_1)} \frac{L_{1t}}{L_{2t}} \frac{(1+\delta)\left[(1+\delta)\alpha_2 A_{2t} L_{2t}^{1-\alpha_2} F_{t-1} - 2\alpha_1 \theta q N_t\right]}{(1+\delta)\alpha_2 A_{2t} L_{2t}^{1-\alpha_2} F_{t-1} + 2\left[(1+\delta)\alpha_2 q - \alpha_1(1-q)\right]\theta N_t} > 1. \quad (24)\end{aligned}$$

The earliest time at which (24) holds is denoted by $T_W \equiv \min\{t \mid \Omega(L_{2t}) \geq 1\}$.

Intuitively, when the marginal product of labor in industry 2 turns positive, the sufficient funding condition (20) holds, which can be seen by comparing the numerator of (24) with (20). However, not until the marginal product of labor in industry 2 become sufficiently large such that the shadow wage condition (24) is met, the skilled labor that is required for producing the modern good is unwilling to work in industry 2. Thus, one may expect throughout the dynamic process of development, $T_F < T_W$ and only after T_W the modern industry is activated. We can therefore define T_W as *the* *time for the modern industry to be activated* at which we shall say that the economy takes off. After the takeoff ($t > T_W$), the modern sector fully absorbs the entirety of the skilled labor and hence complete labor specialization occurs,

$$\begin{aligned} L_{1t} &= N_{ut}, \\ L_{2t} + L_{At} &= N_{st}. \end{aligned}$$

From (20), and applying (4), (14), (15), the modern industry is more likely to be activated when (i) the initial supply of fund (F_0) is high, (ii) the initial level of modern technology (A_{20}) is high, (iii) the preference bias toward the traditional good (θ) is low, or (iv) the modern sector capital barriers (q) is low. In addition, we can provide further insights toward understanding the *dynamic process* of take-off. As time goes by, the supply of funds increases at rate $\frac{S_t}{F_t}$, the modern technology increases at rate $\widehat{\gamma}_2 = \gamma_2 \left(1 - \zeta e^{-\phi L_{At}}\right)$, and the skilled labor increases at a gross growth factor approximately:

$$(1+n)\left(1 + (1-s)e^{-\psi \sum_{\tau=1}^{t-1} \chi_\tau}(1 - e^{\psi \chi_t})\right).$$

An increase in any of these rates will raise the levels of funds, modern technology and skilled labor, thus speeding up the modernization and take-off process. It may be noted that an increase in the skilled labor growth rate not only enhances the supply of skilled workers to produce modern goods but also improves the modern technology which in turn raises marginal products of capital and labor in the modern industry.

4 Calibrating the Dynamic Process of Economic Development

We focus on examining the dynamic process of activating the modern industry, which requires that the capital funds are sufficient and that the generation-discounted cumulative shadow wage ratio exceed one. Due to analytic complexity, we will conduct numerical exercises to compute the takeoff time and plotting the dynamic paths of some key variables and comparative dynamics throughout the entire development process.

4.1 A Benchmark Case

We begin by providing a benchmark case that generally captures the development of the United States over the past 100 years. To begin, we normalize both the initial level of per capita fund supply and the ratio of sectoral technologies to one (i.e., $\frac{F_1}{N_0} = 1$ and $\frac{A_{10}}{A_{20}} = 1$). We set the initial fraction of skilled workers about half of the current level, $s = 0.2$. Given that we did not have good data on production factor shares for the actual industries, we choose to set capital's share equal to 0.25 in industry 1 and 0.35 in industry 2. In the absence of a prior for the effort elasticity, we choose it as 2 such that $\sigma = 0.5$. Similarly, there is no direct measure of modern sector capital barriers; we simply pick a reasonable value $q = 1.2$, which implies a moderate degree of barriers at 20%.

We calibrated, based on data from Maddison (1995), the population grew at about 1.4% per year on average over the past century, which pins down the value of n. In the absence of a longer historic series in U.S. sectoral outputs, we use the corresponding U.K. data from Maddison to approximate the U.S. economy. By following the same computation as in Hansen and Prescott (2002), the growth rates of per capita GNP for agricultural-based Malthusian and capital-intensive Solovian technologies are 1.032 and 1.518, respectively. These give the respective annual rates $\gamma_1 = 0.000975$ and $\widehat{\gamma}_2 = 0.0088$, where the latter leads to $\gamma_2 = 0.013$.

We further calibrate the remaining parameters as follows: (i) the speed of knowledge accumulation parameter $\psi = 0.2205$ and the modern technology growth parameter $\zeta = 0.9722$ are calibrated such that, at the time of takeoff, the fraction of labor allocated to the modern sector is approximately 40% (40.24%) and the fraction of labor allocated to the research sector is small, below 0.5% (0.28%), respectively; (ii) modern industry's R&D productivity parameter $\phi = 347.8$ is chosen such that the rate of economic growth at the takeoff time is about 2% (1.938%); (iii) the subjective intergenerational discounting factor $\delta = 0.5415$ and the initial supply of fund $F_1 = 0.6$ are such that the saving ratio ($\frac{S}{Y}$) is about 6% (6.33%) and the fraction of capital funds allocated to the modern sector is about 50% (52.67%), at the time of take-off; (iv) the initial level of modern technology is chosen as $A_{20} = 0.7$ such that the modern technology at takeoff time is about 30% (29.04%) higher than the traditional technology; (v) the disutility scaling factor $z = 0.4443$ is such that, at the time of takeoff, the disutility of effort is measured (in consumption equivalence) about 10% (9.543%); and, (vi) the preference bias parameter θ is set to 0.1 to produce the targeted timing of economic takeoff at $T_W = 45$. We summarize these figures in Table 1.

The computed activation time is 45 years. Figure 3 illustrates the determination of the critical time for the takeoff based on the sufficient funding and shadow wage conditions. In the calibrated economy, the sufficient funding condition is met at the initial period (year 1). The shadow wage condition is met at year 45 after which the modern industry is activated and the economy takes off. The dynamic paths of these endogenous indicators are plotted in Figure 4. At the time of takeoff, we see discrete jumps rather than a smooth transition. The chart on real GDP, although appears smooth, in fact indicates a jump in growth rate (the chart is in logarithmic scale). These jumps arise because prior to the takeoff when the only operative sector is industry 1, the skilled labor is treated equally as unskilled After the takeoff, the traditional labor share continues to fall whereas the modern labor share continues to rise. At year 84 (39 years after the takeoff), the former share is already below the latter.[6] Similar trends can be observed in the allocation of funds – the fraction of modern industry capital rises from 1/2 to almost 2/3 in 39 years. Upon activating the modern industry, the economy also experiences a much faster rate of growth with per capita real GDP rising sharply by about 3 times in 39 years. In addition to the reallocation of labor and capital, a key driving force of rapid growth is endogenous technical progress: the relative TFP rises from slightly over one at the time of takeoff to about 1.8. The widened technology gap together with production factor input reallocation leads to a significant increase in the relative production (of modern to traditional industries) from about 0.8 to over 1.5. As a result of the increased supply, the relative price of modern to traditional goods falls sharply.

4.2 Comparative-static Analysis

We conduct numerical comparative-static analysis with respect to the following nine parameters of particular interest. Our results suggest that the activation time is most responsive to changes in the initial level of the modern technology (A_{20}), the initial level of fund supply (F_1) and the subjective discounting factor (δ) – a 10% increase in each of these parameters can shorten the activation time from the benchmark 45 years to 26, 28 and 24 years, respectively, reducing the takeoff time by more than one third. By contrast, changes in the modern industry's R&D productivity parameter (ϕ) or the education effort disutility parameter (z) generate relatively small changes in activation timing.

So over the transition to a modern society, what happens to the labor shifts and capital reallocation away from the traditional sector and what happens to the economy-wide aggregate output? Our numerical results suggest that the subjective discounting factor is most influential in generating a rapid transition. In this respect, we echo Laitner (2000) who highlight saving incentives as the key driving force for long-term economic development. Moreover, we find that while the preference bias (θ), the skill accumulation (ψ) and the capital barrier (q) parameters are crucial for labor shifts,

[6] Notice that as a result of skill diminishing returns in the modern industry, the fraction of traditional sector labor does not fall significantly. Should we allow for additional modern sectors to emerge to prevent from rapid diminishing returns on modern labor usage, the traditional labor share will reduce to a more realistic number (10% or less).

their effects on capital reallocation or aggregate output advancement are not nearly as important. On the contrary, while the initial fraction of the skilled labor (s) and the initial level of fund supply have little impact on labor shifts, they are essential for capital reallocation and aggregate output advancement. Furthermore, concerning the initial level of the modern technology, our results indicate that it is most important for capital reallocation and least influential for aggregate output advancement. Finally, as always, changes in the modern industry's speed of growth parameter or the education effort disutility parameter have relatively little impact on factor shifts or aggregate outputs.

One may then wonder under which circumstances the modern industry can never be activated. In Table 2, we illustrate that a growth trap with the modern industry remaining nonoperative throughout can arise when the initial level of the modern industry production technology (A_{20}) is sufficiently low (as low as 0.5), which is consistent with the arguments by Hansen and Prescott (2002) who emphasize the role of modern technology played in economic development. We also find that activation of the modern industry may become impossible if (i) the initial funding (F_1) decreases from 0.6 to 0.4, (ii) the altruistic factor capturing saving incentives (δ) drops from 0.5415 to 0.45, (iii) the preference bias toward the traditional good (θ) increases from 0.1 to 0.2, or (iv) the shadow cost of capital allocated to the modern industry (q) rises from 1.2 to 1.5, (v) the initial size of the skill labor (s) falls from 0.2 to 0.1, (vi) the speed of knowledge accumulation parameter ψ decreases from 0.2205 to 0.15, (vii) industry 2's R&D productivity parameter ϕ also falls from 347.8 to 100. However, the modern industry can always be activated even though the education effort disutility parameter (z) approach infinity. The result regarding preference bias and capital allocation barrier is consistent with the conclusion obtained by Wang and Xie (2004) in a static framework.

In the interest of conciseness, we illustrate selectively the most representative comparative dynamics from the time of takeoff in year 45 to year 100 (55 years after the activation of the modern industry). The three cases highlighted are the dynamic transition in response to the initial level of modern technology, the initial fraction of the skill labor and the capital barrier measure. The results are depicted in in Figures 5a-5c, where the paths marked with "+" ("−") indicates those responding to a 10% increase (decrease) in one of the three exogenous parameters. While labor, capital and production all shift rapidly in response to such an increase in the initial level of modern technology, the resultant shifts in response to the initial fraction of skilled labor are more moderate. In response to a 10% decrease in the capital barrier parameter, both labor and capital shift rapidly from traditional to modern sectors, though changes in the relative output are more moderate over the transition. As a result of the aforementioned transition processes, the per capita real income grow at the highest rate in response to the initial level of modern technology and the lowest in response to the initial fraction of skilled labor.

4.3 Policy Implications

Our findings yield several useful policy implications. Specifically, the results suggest that there are many ways for the government to help activating a modern industry and enabling an economy to take off. Such public policies include at least (i) government subsidies to create sufficient incentives for industrial transformation, (ii) establishment of public enterprises in early development when modern industries are not profitable, and (iii) direct technology transfer or imitation to jump-start the modern industry. For example, should the government fully internalize capital externalities originated in the modern sector by ways of investment subsidy or public enterprising, the scale barrier can be completely removed. In this case, our numerical results suggest that the activation time is reduced all the way to zero and the economy can take off immediately.

To the end, it is useful to discuss plausible sets of parameters that may replicate the speed of take-off experienced by the UK, Canada, Korea and Taiwan. As documented by Gollin, Parente and Rogerson (2002), it took about 55 years for the UK to double its per capita real income from $2,000$ (1990 US$) to $4,000$, while it only took about 32, 15 and 10 years, respectively, for Canada, Korea and Taiwan to do so. For illustrative purposes, let us use these figures to capture the take-off time in our model. We can obtain the take-off time of 55 years as in the UK with lower initial levels of the modern technology and funding, a lower subjective discounting factor and a higher shadow cost associated with modern capital ($A_{20} = 0.69$, $F_1 = 0.59$, $\delta = 0.53$ and $q = 1.23$). On the contrary, the take-off time of 32 for Canada can be captured with higher initial levels of the modern technology and funding, a higher subjective discounting factor and a lower shadow cost associated with modern capital ($A_{20} = 0.715$, $F_1 = 0.61$, $\delta = 0.55$ and $q = 1.19$). With a slightly better initial condition ($A_{20} = 0.75$ and $F_1 = 0.65$) while maintaining $\delta = 0.55$ and $q = 1.19$, the take-off time becomes 15 years, thereby mimicking the cases of Korea. Similarly, with a much better initial condition ($A_{20} = 0.77$ and $F_1 = 0.66$) while maintaining $\delta = 0.55$ and $q = 1.19$, the take-off time becomes 10 years, thereby mimicking the cases of Taiwan. In the case of Taiwan, the government has undertaken a series of education reforms and established public programs by subsidizing investment in modern industries (partly via the operation of some strategic public enterprises) as well as by providing funding to the private sector using foreign aid and monopoly revenues.

5 Concluding Remarks

By constructing a dynamic general equilibrium model with endogenous activation of the modern industry, we have identified an array of preference, technology, funds and labor skill forces to enable the take-off of a closed economy. By calibrating the model to fit historic U.S. development, our quantitative results suggest that the timing of economic takeoff depends most crucially on the initial levels of the modern technology and fund supply as well as the subjective intergenerational discounting factor. While individual's saving incentives is most important for the speed of modernization, the preference bias, the skill accumulation and the capital allocation barrier are influential for labor reallocation, and the initial states of skills, funds and modern technologies are crucial for capital reallocation. Along the dynamic transition path, labor, capital and output are most responsive to the initial state of modern technologies but least responsive to the initial state of skills.

In order to accomplish our analysis, we have imposed a number of simplifying assumptions that help the tractability of our model framework. It is therefore natural to relax some of these assumptions by further simplifying other parts of the model structure to check the robustness of our main conclusions. For example, in the aspects of the dynamic take-off theory, one may endogenize capital accumulation process based on intertemporal consumption-savings trade-off as in the standard Ramsey optimal growth framework, or endogenize the knowledge accumulation process based on learning-by-doing (as in Lucas 1993). Since trade is believed to play a major role in many newly industrialized economies, one may also extend the model to a small open economy case (as in Bond, Jones and Wang, 2005 or as in Trindade, 2005) to understand how globalization may help advance an economy and to whether tariff reduction, export learning or foreign direct investment may speed up the activation of a modern industry.

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Table 1: Parameters for numerical analysis

Par.	Benchmark case	Definition
α_1	0.25	traditional sector capital share
α_2	0.35	modern sector capital share
ϕ	347.78798	R&D productivity parameter
ζ	0.97215	modern technology growth
θ	0.1	preference bias
γ_1	0.000975	speed of traditional technology growth
γ_2	0.013	speed of modern technology growth
q	1.2	modern sector capital barriers
n	0.014	population growth
s	0.2	initial fraction of skilled workers
F_1	0.6	initial supply of fund
N_0	0.6	initial population
A_{10}	0.7	initial level of traditional technology
A_{20}	0.7	initial level of modern technology
δ	0.54149	subjective intergenerational discounting factor
σ	0.5	elasticity parameter of effort disutility
ψ	0.22052	speed of knowledge accumulation
z	0.4443	disutility scaling factor
$\widehat{\gamma}_2$	0.0088	speed of modern technology growth (total)

Notes: $\widehat{\gamma}_2 = \gamma_2 \left(1 - \zeta e^{-\phi L_{At}}\right)$.

Table 2: Activation time, growth traps and comparative static adjustments

			Comparative statics Size of responses to 10% increase in each parameter			
Par.	+(−)10%	Traps	Activation time	Labor	Capital	Output
A_{20}	26 (57)	0.5 [0.7]	Large	Median	Large	Small
s	40 (50)	0.1 [0.2]	Median	Small	Median	Median
ψ	40 (53)	0.15 [0.221]	Median	Large	Median	Median
ϕ	42 (48)	100 [347.8]	Small	Small	Small	Small
F_1	28 (55)	0.4 [0.6]	Large	Small	Median	Median
δ	24 (63)	0.45 [0.541]	Large	Large	Large	Large
z	46 (43)	N/A [0.444]	Small	Small	Small	Small
q	56 (25)	1.5 [1.2]	Median	Large	Small	Small
θ	54 (26)	0.2 [0.1]	Median	Large	Small	Small

Notes: The activation time, $T_W \equiv \min\{t \mid \Omega(L_{2t}) \geq 1\}$. T_W for benchmark case is 45. $T_W = 62$ as $z \to \infty$. 10% decreases in parameters place in parentheses. Benchmark parameters appear in square brackets.

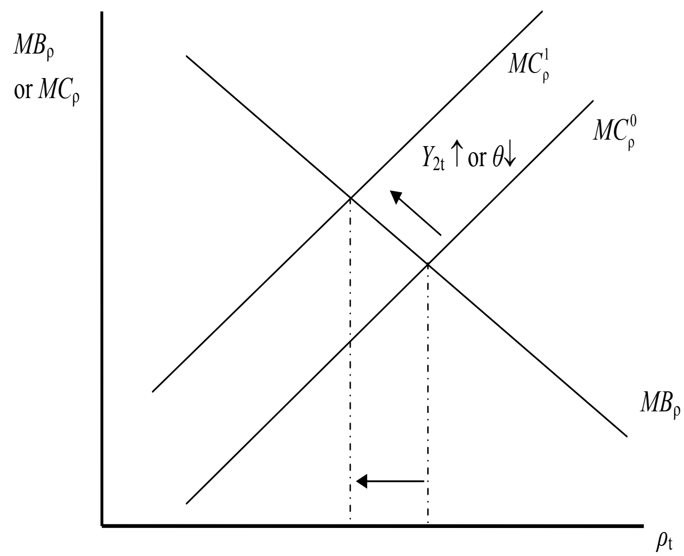

Figure 1: Consumption-saving choice condition

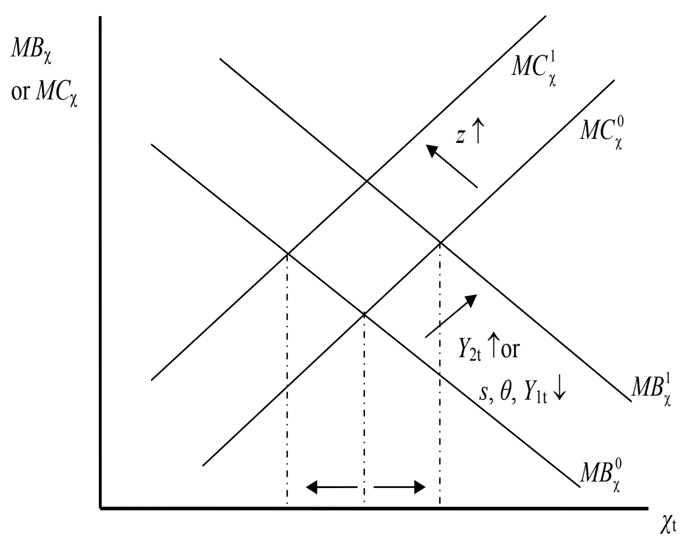

Figure 2: Knowledge accumulation decision condition

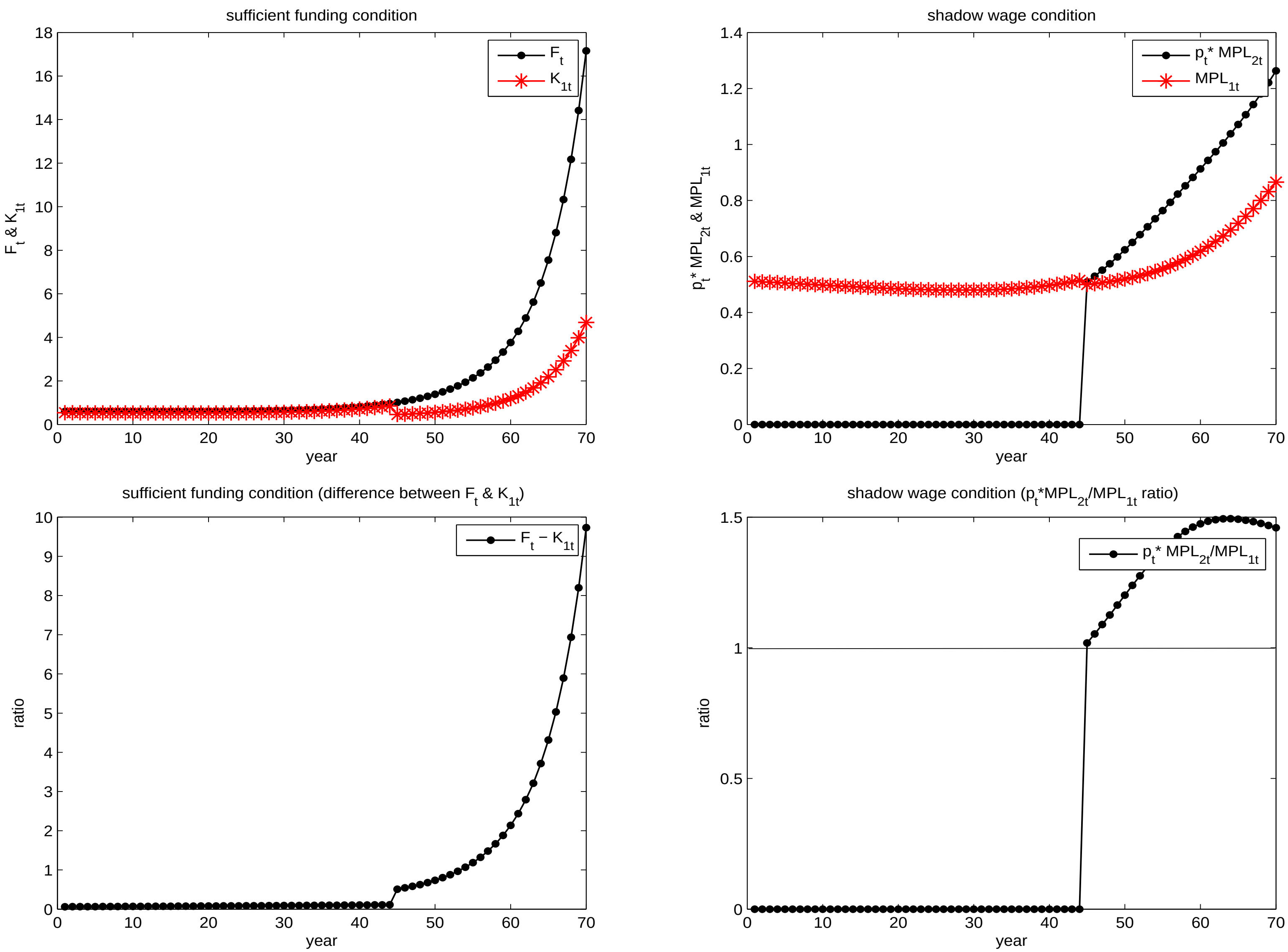

Figure 3: Benchmark model of economic take-off for the modern industry

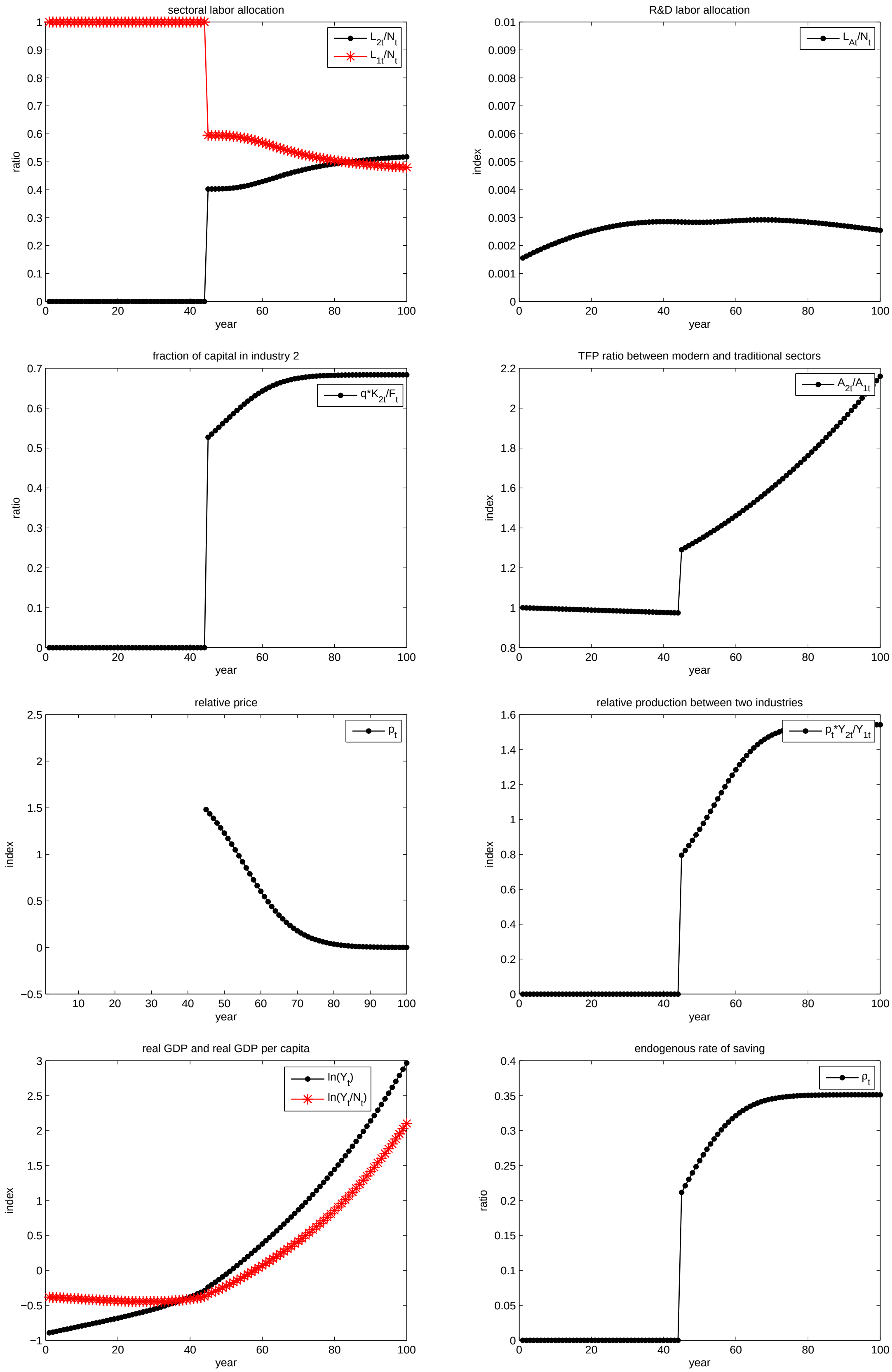

Figure 4: The dynamic paths of endogenous indicators

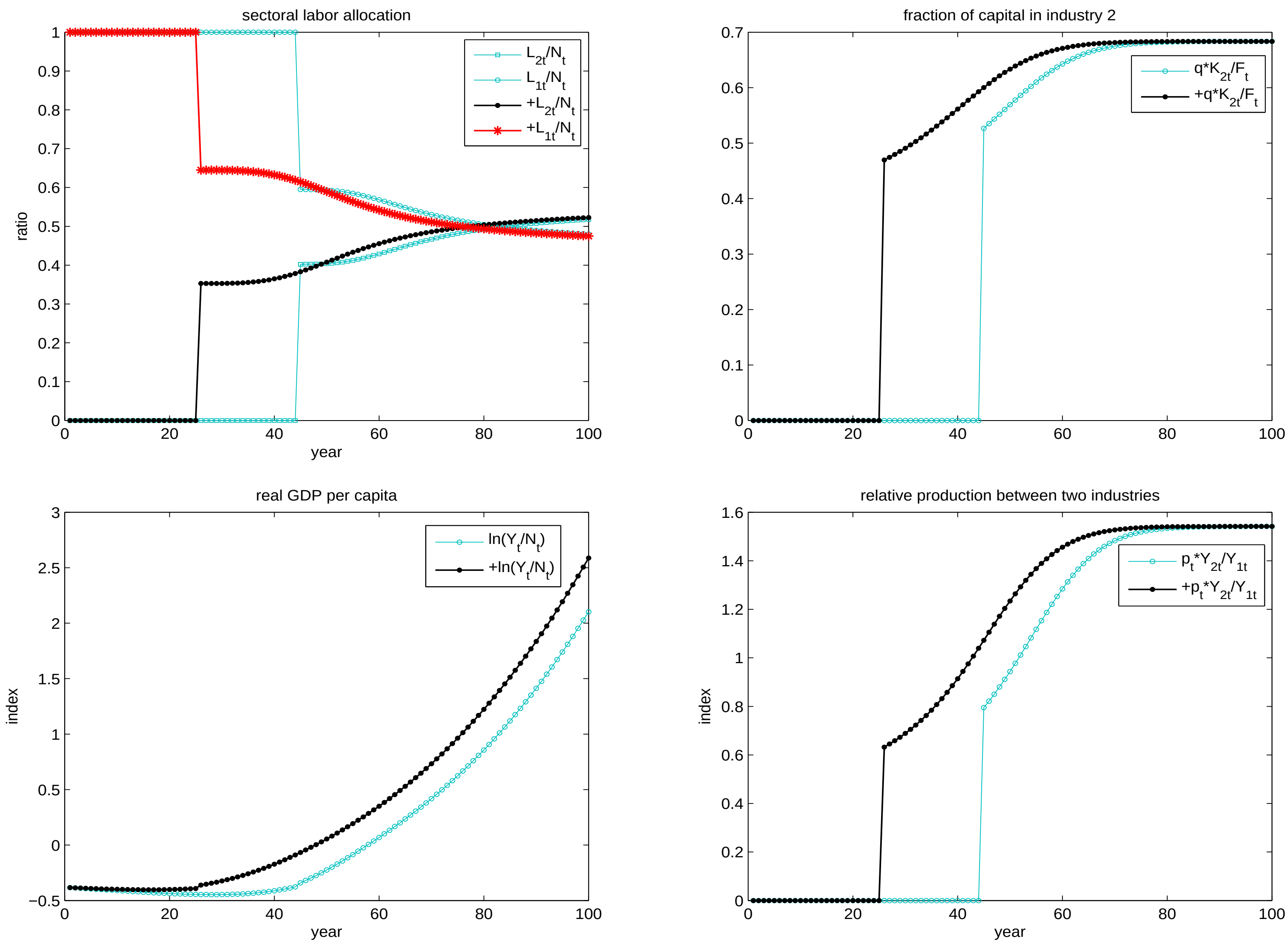

Figure 5a: Comparative dynamics: + denotes 10% increases in A_{20}

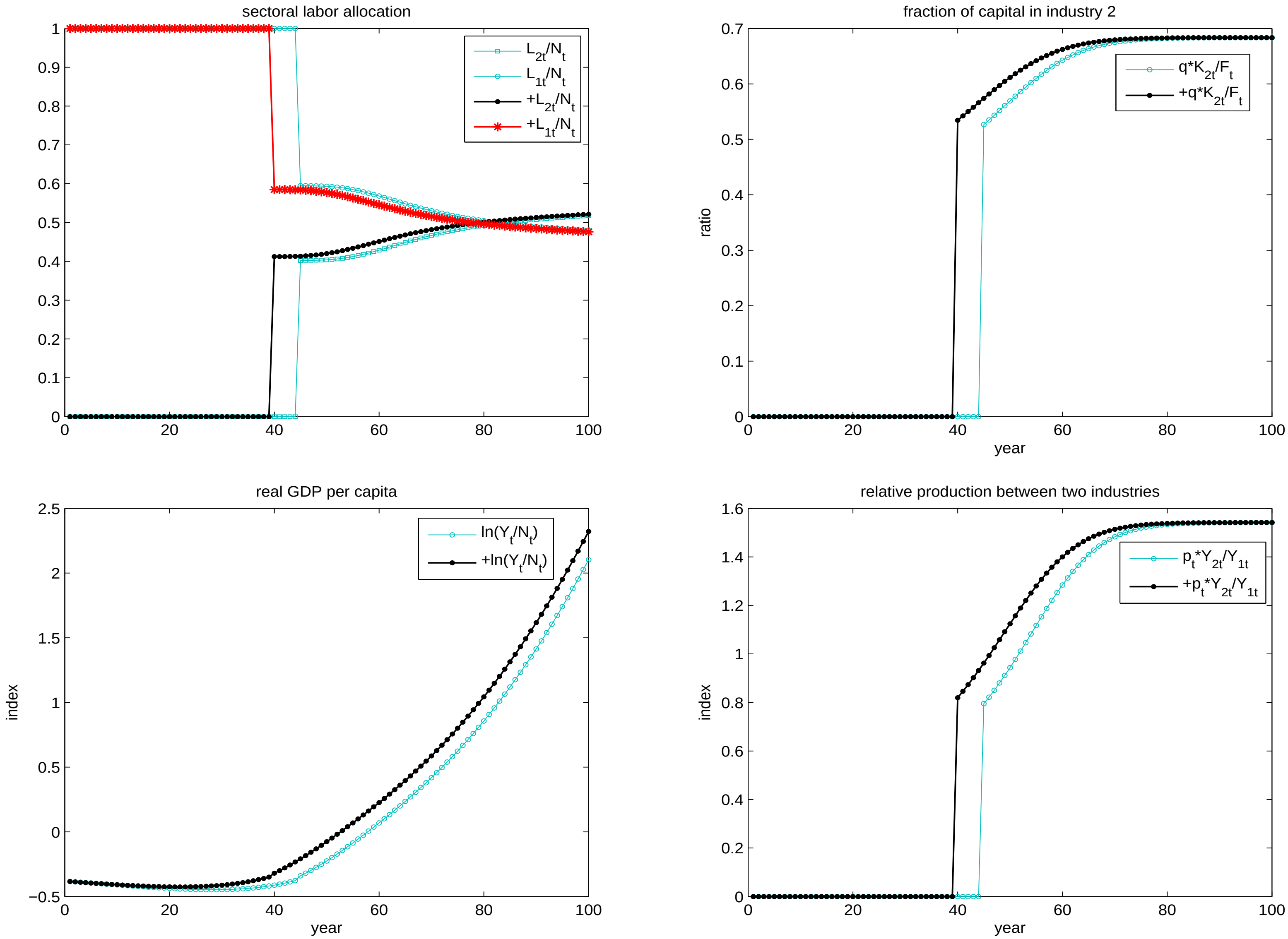

Figure 5b: Comparative dynamics: + denotes 10% increases in s

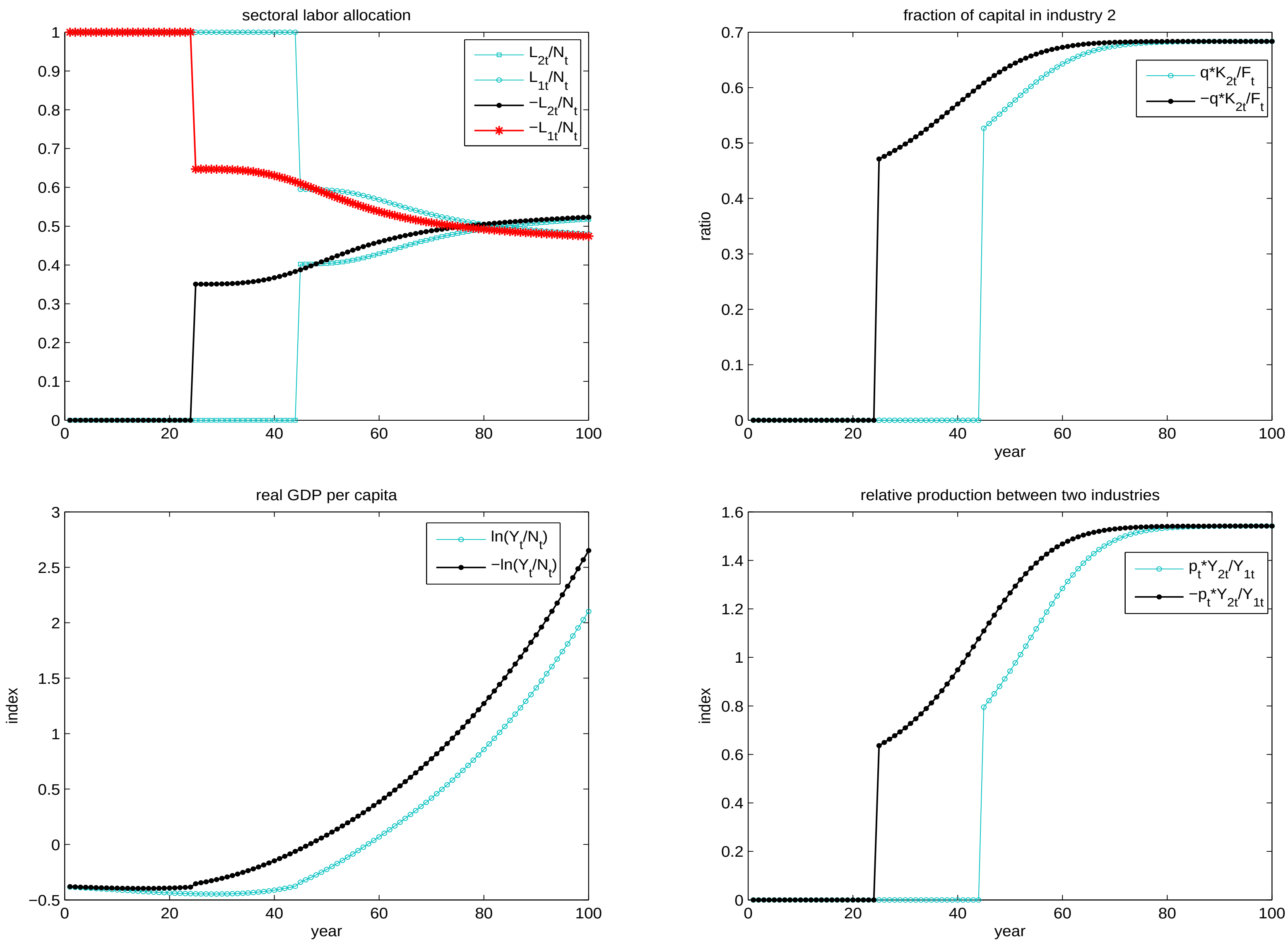

Figure 5c: Comparative dynamics: $-$ denotes 10% decreases in q