Locational Stratification by Environment

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Abstract: This paper develops an equilibrium sorting model wherein perfectly mobile agents decide their optimal residential locations. Our local economy features two cities, a clean city and a dirty city in which production factories are located. In contrast with previous studies on economic stratification, equilibrium configurations in our framework depend crucially on environmental considerations. While residents in a dirty city suffer a higher level of pollution, they incur less commuting cost when going to work. When workers of different abilities choose different locations in which to reside, a segregated spatial configuration emerges. In this stratified equilibrium, those residing in a clean city have higher working time, human capital and income than those in the dirty city. The higher the degree of pollution generated or the better the commuting technology, the larger the size of a clean city is. A location-biased pollution factor results in not only more aggregate pollution but also less aggregate output of the local economy. In addition, our numerical exercises suggest that the welfare-maximizing local tax scheme requires a positive surcharge on residents in the clean city.

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1 Introduction

Economic stratification has been a central issue over the past two decades. Disparities in socioeconomic status, earned income, education, and housing concern policy-makers (Weiss 1989; Mayer and Jencks 1990), because such disparities are also widely believed to be a primary driving force in the creation of urban ghettos (Wilson 1987). About a decade ago, Jargowsky (1996) documented a nation-wide, sharp increase in economic segregation in the 1980's, accompanied by a small decline in racial segregation. This compels us to ponder what the underlying economic factors are that cause economic segregation.

The leading economic factors believed to be responsible for the widening economic stratification outcome are: (i) heterogeneity in ability, skills, or human capital (cf. Benabou 1996a; Nechyba, 2003; Chen, *et al.* 2009; Kempf and Moizeau 2009; Berliant and Kung 2010), (ii) heterogeneity in non-human wealth (cf. Benabou 1996a; Nechyba, 1997), (iii) heterogeneity in an individual's accessibility to capital markets (cf. Benabou 1996b), and (iv) heterogeneity in individual preferences for housing, local public goods, or location-specific peers (cf. Peng and Wang 2005). In this paper, we propose a fifth and potentially important factor, namely, the environment surrounding residential communities populated with ability-heterogeneous agents. This consideration captures real world observations, as we have seen in the development of many local economies, some featuring a large clean city and a small dirty city (e.g., Seattle vs. Tacoma, or San Francisco vs. Oakland), some featuring a small clean city and a large dirty city (e.g., Ann Arbor vs. Detroit, or Aurora vs. Denver), and some featuring comparably sized clean and dirty cities (e.g., Washington, D.C. vs. Baltimore, or Champaign vs. Urbana). Just how do people make their residential choices within these local economies when pollution is a concern in addition to other economic factors? Can a local economy improve its residents' welfare by means of local taxes?

The observation that agents' optimization behavior depends on their concerns regarding environmental quality is hardly new. In recognizing the trade-off between environmental quality and final goods through individual preferences, Cremer and Thisse (1999) use a vertical differentiation model to address the effects of tax. They show that taxation may change consumers' willingness-to-pay in regard to products of different qualities, and then change environmental outcomes and utility levels. With similar preference specifications, Stokey (1998) and Jones and Manuelli (2001) lend theoretical support to the environmental Kuznets curve in a dynamic equilibrium framework, showing that an inverted-U relationship exists between pollution and income growth. We complement this strand of the research by examining how environmental considerations may interact with ability heterogeneity to influence the decisions of residential choice and the configurations of stratification arising in an equilibrium.

More specifically, we consider a local economy with two cities: (i) a clean city and (ii) a dirty city in which production factories are located. Agents differ in their ability to work and their human capital evolves based on their parents' human capital as well as their home city's aggregate human capital. While residents in the dirty city suffer a higher level of pollution, they incur less commuting cost when going work. Agents have perfect foresight and are altruistic, valuing their own consumption, their children's human capital, as well as the environment of the city where they reside. The local government collects local taxes to provide pollution subsidies to those residing in a dirty city.

The main findings are as follows. First, we show that there is a unique critical ability above which agents choose rationally to reside in a clean city. Thus, economic stratification emerges in equilibrium as a result of ability, heterogeneity and environmental concerns. Second, the stratified equilibrium exhibits higher working time, human capital, effective labor and income for those residing in a clean city relative to those in a dirty city. Third, the more severe pollution that engulfs the dirty city or the better the human capital productivity and the commuting technology is, the larger the size of the clean city will be. Fourth, a location-biased pollution factor not only leads to more aggregate pollution but is more harmful to the aggregate output of the local economy. Fifth, in response to the human capital evolution and tax surcharge parameters, aggregate pollution and aggregate net income are positively related; in response to changes in the pollution and commuting cost parameters, aggregate pollution and aggregate net income are negatively related. Finally, our numerical exercises suggest that the welfare-maximizing local tax scheme requires a positive surcharge on residents in the clean city.

The remainder of this paper is organized as follows. Section 2 sets up the basic structure of the model. Section 3 solves the optimization problem and the spatial equilibrium, whereas Section 4 characterizes the equilibrium analytically. In Section 5, we perform a numerical analysis to provide some quantitative results on both the economic stratification and welfare implications. Finally, some concluding remarks are offered in Section 6.

2 The Model

Consider a local economy with two cities, labelled A and B, where the manufacturing factory is located in city A. Following the conventional wisdom whereby we assume that pollution is a by-product of production (cf. Jones and Manuelli 2001), we may thus call A a dirty city and B a clean city. The corresponding population masses are denoted by x^A and x^B, respectively. For simplicity, we will detrend all ability, human capital, output and consumption variables by the national human capital stock that is assumed to grow at an exogenous rate. Upon detrending, all such variables cease to grow asymptotically.

Letting Z denote the distance between A and B, we can specify the levels of pollution in cities A and B as $\frac{\theta Y}{D}$ and $\frac{\theta Y}{D+Z}$, respectively, where Y denotes the aggregate income of the local economy and $\theta > 0$ and $D > 0$ are pollution parameters. Thus, the levels of pollution are endogenous, depending on the levels of output, which captures the convention that regards pollution as a by-product of production. More precisely, $\frac{\theta}{D}$ measures the degree of pollution generated by manufacturing activity in the local factory city, whereas $\frac{\theta}{D+Z}$ measures the degree of pollution spillovers from the factory city to the clean city. An increase in the "general pollution scaling factor" θ raises the level of pollution proportionately for both cities. A decrease in the "location-biased pollution parameter" D raises the level of pollution in the dirty city more than proportionately compared to the clean city (i.e., $\frac{\theta Y}{D}/\frac{\theta Y}{D+Z}$ is decreasing in D). The aggregate level of pollution P of the local economy is thus given by,

$$P = \frac{\theta Y}{D} + \frac{\theta Y}{D+Z} = \frac{\theta\,(2D+Z)}{D\,(D+Z)}Y \tag{1}$$

While residents in the dirty city suffer a higher level of pollution (noting $\frac{\theta Y}{D} > \frac{\theta Y}{D+Z}$), they incur less commuting cost when going to work. For simplicity, we normalize the intraregional commuting cost to zero and measure the unit interregional commuting cost (per distance) by $\widetilde{q} > 0$.

To simplify the dynamic optimization problem, we consider a one-period lived non-overlapping generations framework with altruistic agents. Each generation, indexed by $t = 0, 1, 2, \cdots$, consists of a continuum of agents of mass one. To ease the notational burden, we will not label generation/time whenever it does not cause confusion. Altruistic agents care about their children's human capital. During her one-period lifetime, each agent is endowed with one unit of time, which can be allocated to work (v^i) and to a child's education ($1 - v^i$), where $i = A, B$.

Agents differ in their ability to work (which is detrended by the national human capital stock). The ability distribution is uniform over $[\underline{a}, \bar{a}]$. By denoting the range of ability distribution as $\Delta = \bar{a} - \underline{a}$, we can compute the constant density as $\frac{1}{\Delta}$. The human capital stock embodied in an agent of ability a in city i is simply measured by $h^i(a) = \psi a$, where $\psi > 0$ is assumed to be location-independent. In the presence of only one-dimensional heterogeneity, we can show (see Section 3) that there is a critical value a_c such that agents with ability $a \in [\underline{a}, a_c]$ reside in the dirty city (A), whereas agents with ability $a \in [a_c, \bar{a}]$ reside in the clean city (B). The population masses are hence given by:

$$x^A = \frac{a_c - \underline{a}}{\Delta}, \quad x^B = \frac{\bar{a} - a_c}{\Delta} \tag{2}$$

Thus, the aggregate human capital stocks in A and B can be derived as:

$$H^A = \psi \int_{\underline{a}}^{a_c} \frac{a}{\bar{a} - \underline{a}} da = \frac{\psi}{2} \frac{a_c^2 - \underline{a}^2}{\bar{a} - \underline{a}} \tag{3}$$

$$H^B = \psi \int_{a_c}^{\bar{a}} \frac{a}{\bar{a} - \underline{a}} da = \frac{\psi}{2} \frac{\bar{a}^2 - a_c^2}{\bar{a} - \underline{a}} \tag{4}$$

It is obvious that while H^A is increasing in a_c, H^B is decreasing.

We next define the aggregate effective labor in the local economy as:

$$L = L^A + L^B \tag{5}$$

where

$$L^A = \frac{1}{\Delta} \int_{\underline{a}}^{a_c} v^A(a) h^A(a) da \tag{6}$$

$$L^B = \frac{1}{\Delta} \int_{a_c}^{\bar{a}} v^B(a) h^B(a) da \tag{7}$$

The production function is then specified as a linear function of aggregate effective labor in the local economy:

$$Y = QL = Q(L^A + L^B) \tag{8}$$

where $Q > 0$ is a constant scaling factor (the inverse of unit labor requirements).

We assume that the clean and the dirty goods are perfect substitutes and that goods are shipped freely between the two cities. Thus, we can choose the clean good as the numeraire. Constructing an indicator function,

$$I = \begin{cases} 0 & if \quad a > a_c \quad (\text{city } B) \\ 1 & if \quad a \leq a_c \quad (\text{city } A) \end{cases}$$

Let $q = \widetilde{q}/w$ measure the interregional commuting cost per distance per unit of labor cost, which is paid in the form of the numeraire good.[1] Let us further denote τ^i as the local income tax rate prevailing in city $i = A, B$ and S as the total pollution subsidy to those residing in the dirty city. Each agent residing in city i takes the effective market wage w and the per capita pollution subsidy $\frac{S}{x^i}$ as given, and faces the following budget constraint:

$$c^i(a) = (1 - \tau^i)wv^i h^i(a) + \frac{S}{x^i}I - qwZ(1 - I) \tag{9}$$

That is, in addition to after-tax wage income, those in the dirty city receive a government subsidy while those in the clean city incur a commuting cost. In the absence of bequests or the resource cost of a child's education, individual net income (the right-hand side of (9)) is fully consumed by one-period lived agents.

The dynamics of the model economy is mainly driven by the evolution of the cohort-specific human capital stock and inter-generational altruistic behavior. The human capital evolves according to,

$$h^{i'}(a) = \phi[(1 - v^i)h^i(a)]^\alpha \left(H^i\right)^{1-\alpha} \tag{10}$$

where $\phi > 0$ and $\alpha \in (0, 1)$. That is, the human capital stock of the next generation residing in city i' depends on parental human capital ($h^i(a)$), the parents' time devoted to a child's education ($1 - v^i$), and the local aggregate human capital in city i where parents reside (H^i). Of particular note, the important role played by local human capital externalities in intergenerational human capital transmissions has been emphasized by Benabou (1996a) and Chen, Peng and Wang (2009).

The lifetime utility of a representative agent with ability a residing in city i takes the following form:

$$U^i(a) = \left[\ln c^i(a) - \frac{\theta Y}{D + Z(1 - I)}\right] + \beta \ln[h^{i'}(a)] \tag{11}$$

where $\beta > 0$ measures intergenerational discounting (the more altruistic an agent is, the higher the value of β will be). That is, an agent values her own consumption, the child's human capital and the cleanliness of the city where she resides.

[1] Should the commuting cost be paid by time, there would be three possible time allocations: work time, time devoted to the child's education and commuting. This would lead to a great complication particularly because it is then difficult to integrate the tax rate to obtain the effective labor in each city (which is a crucial step for establishing the unique determination of the equilibrium cutoff ability).

To avoid algebraic complexity, we set the tax rate in the dirty city to zero, which we can integrate in a recursive manner to obtain the effective labor in each city as displayed in (6) and (7). This simplifying assumption is innocuous particularly because our attention is mainly focused on the local tax differential that is crucial for locational choice by mobile agents. In the absence of other taxes or spending, the government budget constraint is:

$$S = \tau^A w L^A + \tau^B w L^B = \eta w L^B \tag{12}$$

Finally, we can compute the aggregate net income Y_N of the local economy, which is defined as aggregate income net of the social cost incurred due to commuting by residents in the clean city:

$$Y_N = Y - qwx^B \tag{13}$$

3 Optimization

We can divide an agent's optimization problem into two steps. In step 1, the agent solves the second-stage optimization problem given her residential location in city $i = A, B$. In step 2, the agent solves the first-stage discrete-choice problem by comparing the values (indirect utility) obtained in step 1 for the two cities.

By backward solving, we begin with the second-stage optimization problem given location i. Substituting (9) and (10) into (11), we can write the optimization problem as an unconstrained maximization problem:

$$\begin{aligned} V^i(a) &= \max_{v^i} \ln\left[(1-\tau^i) w v^i h^i(a) + \frac{S}{x^i} I - qwZ(1-I)\right] - \frac{\theta Y}{D + Z(1-I)} \\ &\quad + \beta \ln\left\{\phi[(1-v^i)h^i(a)]^\alpha \left(H^i\right)^{1-\alpha}\right\} \end{aligned}$$

Using the first-order condition, we can solve the agents' working time ($i = A, B$):

$$v^A(a) = \frac{1}{1+\alpha\beta} - \left(\frac{\alpha\beta}{1+\alpha\beta}\right) \frac{S/x^A}{wh^A(a)} \tag{14}$$

$$v^B(a) = \frac{1}{1+\alpha\beta} + \left(\frac{\alpha\beta}{1+\alpha\beta}\right) \frac{qZ}{(1-\eta)h^B(a)} \tag{15}$$

When net non-wage income increases (as a result of an increase in the subsidy for those residing in city A or a reduction in the commuting cost for those in city B), consumption is higher and the marginal utility of consumption is lower, thus reducing the marginal benefit of working. As a consequence, agents reallocate their time from work to a child's education in response to an increase in net non-wage income; moreover, those residing in the clean city work more than those in the dirty city ($v^B(a) > v^A(a)$ as can be seen from (14) and (15)). The substitution of (14) and (15) into (6) and (7) yields aggregate effective labor in cities A and B and aggregate effective labor for the local economy:

$$L^A(a_c) = \frac{1}{1+\alpha\beta}\{\frac{\psi}{\Delta}(\frac{a_c^2 - \underline{a}^2}{2}) - \frac{\bar{a}-a_c}{\Delta}\frac{\alpha\beta}{1+\alpha\beta}\eta[\frac{\psi(\bar{a}+a_c)}{2} + \frac{\alpha\beta qZ}{1-\eta}]\} \quad (16)$$

$$L^B(a_c) = (\frac{\bar{a}-a_c}{\Delta})(\frac{1}{1+\alpha\beta})[\frac{\psi(\bar{a}+a_c)}{2} + \frac{\alpha\beta qZ}{1-\eta}] \quad (17)$$

$$L(a_c) = \frac{1}{\Delta(1+\alpha\beta)}\{\frac{\psi}{2}[\bar{a}^2(1-\frac{\alpha\beta\eta}{1+\alpha\beta})-\underline{a}^2+\frac{\alpha\beta\eta}{1+\alpha\beta}a_c^2]+(1-\frac{\alpha\beta\eta}{1+\alpha\beta})(\frac{\alpha\beta qZ}{1-\eta})(\bar{a}-a_c)\} \quad (18)$$

As plotted in Figure 1, aggregate effective labor (as well as the population) in city A is increasing in critical ability (a_c), while aggregate effective labor (and the population) in city B is decreasing. Due to these offsetting forces, the effect of critical ability on aggregate effective labor (and aggregate output) in the local economy is ambiguous. When critical ability is sufficiently small ($a_c < \hat{a} = \frac{1+\alpha\beta}{\psi\eta}[1 - \frac{\alpha\beta}{1+\alpha\beta}\eta\Delta]\frac{qZ}{\Delta(1-\eta)}$), the dirty city is small and the clean city is large. In response to an increase in critical ability, those who relocate from a clean city to a dirty city enjoy higher net non-wage income and hence reduce their work time. As a result, aggregate effective labor in the local economy decreases. When critical ability is sufficiently large, the reverse result holds true. Those with very low ability ($a \in [\underline{a}, a_c]$) do not work, their sole income being derived from the government lump-sum subsidy ($\frac{S}{x^A}$).

We next turn to solving the first-stage locational choice problem. In particular, we have a unique critical ability a_c such that:

$$\begin{cases} V^A(a_c) > V^B(a_c) & \text{for } a < a_c \\ V^A(a_c) = V^B(a_c) & \text{for } a = a_c \\ V^A(a_c) < V^B(a_c) & \text{for } a > a_c \end{cases} \quad (19)$$

We impose the following condition under which pollution is bad even after a government subsidy:

Condition 1: $\Gamma = 1 - \eta(1+\alpha\beta) > 0$.

The indifference at $a = a_c$ implies a locational no-arbitrage condition as follows:

$$(1\text{-}\alpha)\beta\ln(\frac{H^A}{H^B}) - \frac{\theta ZY}{D(D+Z)} = \ln\{\frac{(1\text{-}\eta)\frac{\psi a_c}{1+\alpha\beta} - \frac{\Gamma qZ}{1+\alpha\beta}}{\frac{\psi a_c}{1+\alpha\beta} + \frac{\eta L^B}{x^A}(1\text{-}\frac{\alpha\beta}{1+\alpha\beta})}\} + \alpha\beta\ln\{\frac{\psi a_c - \frac{qZ}{1-\eta}}{\psi a_c + \frac{\eta L^B}{x^A}}\} \quad (20)$$

where the left-hand side may be thought of as the net marginal benefit for residing in city A as compared to city B (denoted by NMB) and the right-hand side as the net marginal cost (denoted by NMC). As to be shown in the proof of Theorem 1 below, NMB is always increasing in a_c and NMC is also increasing in a_c as long as η is not too large. When the intergenerational human capital externality $(1-\alpha)$ is not too strong, NMB responds to changes in a_c less sensitively than NMC. This ensures that the interior solution of a_c is optimal (i.e., the second-order condition is met).

4 Equilibrium Characterization

A *dynamic competitive spatial equilibrium* is a tuple of individual choice variables $\{v_t^i, h_t^{i'}, c_t^i\}$ and aggregate variables $\{x_t^i, H_t^i, L_t^i, S_t, Y_t, Y_{N_t}, P_t\}$ ($i = A, B$ and $t = 0, 1, 2, \cdots$), together with a locational stratification variable a_c, such that:

(i) all individuals optimize, i.e., (14), (15), and (9) are met;

(ii) human capital evolves according to (10);

(iii) locational no-arbitrage (20) holds;

(iv) a location-specific population distribution, aggregate human capital and effective labor satisfy (2), (3), (4), (6), and (7), respectively;

(v) aggregate pollution, aggregate income and aggregate net income satisfy (1), (8) and (13), respectively; and

(vi) the government budget is balanced, i.e., (12) is met.

In total, there are 17 equations, determining 17 endogenous variables. It should be noted that we do not include the aggregate goods market equilibrium condition, because it holds automatically once (6)-(9) and (12) are met. A *steady-state equilibrium* is a *dynamic competitive spatial equilibrium* where all variables are constant.

To begin with, we must verify the existence and interiority of the steady-state equilibrium. The key step is to determine critical ability (a_c) using the locational no-arbitrage condition (20).

Theorem 1: (Existence and Interiority of Steady-state Equilibrium) Under Condition 1, there exists a steady-state equilibrium. The steady-state equilibrium configuration may feature a unique interior critical ability, or multiple interior critical abilities, or no interior critical ability.

It may be best to illustrate the determination of steady-state equilibrium critical ability by plotting the $NMB - NMC$ locus against a_c (see Figure 2). In the first case, as depicted in Figure 2(a), there exists a unique interior steady-state equilibrium critical ability (see point B) under which those with ability $a \geq a_c$ self-sort into the clean city C, whereas those with ability $a < a_c$ self-sort into the dirty city D. In the second case, as depicted in Figure 2(b), there exist multiple interior steady-state equilibrium critical abilities, where we plot the simplest scenario with two of them (see points B and D). The equilibrium selection in this case is purely driven by self-fulfilling prophecies. Should agents expect critical ability to be low, they will elect to reside in the clean city and in so doing equilibrium critical ability will turn out to be low (i.e., self-fulfilling prophecies lead to point B as the equilibrium outcome). Conversely, should agents expect critical ability to be high, equilibrium critical ability will turn out to be high. In the last case as depicted in Figure 2(c), there exists no interior steady-state equilibrium critical ability. In this case, if every agent expects critical ability to be below his/her own ability, then critical ability is at the lower bound of the support; if every agent expects critical ability to be above his/her own ability, then critical ability is at the upper bound of the support.

One may then inquire under what circumstances a degenerate steady-state equilibrium with a corner solution for critical ability is likely to arise as the equilibrium configuration. We can simply examine (20) to conclude:

Corollary 1: (Degenerate Steady-state Equilibrium) Under Condition 1, the steady-state equilibrium configuration is degenerate under the following circumstances:

(i) if $\alpha \to 0$ and $\eta \to 1$, then $a_c \to \bar{a}$;

(ii) if $\alpha \to 1$ and $\eta \to 0$, then $a_c \to \underline{a}$.

That is, when the intergenerational externality is extremely high ($1-\alpha \to 1$) and the local tax differential is extremely large ($\eta \to 1$), no one would have the incentive to reside in the clean city. On the contrary, when the intergenerational externality is extremely low ($1 - \alpha \to 0$) and the local tax differential is extremely small ($\eta \to 0$), the dirty city is evacuated.

Before conducting a full characterization of the equilibrium, we would like to examine the equilibrium configuration of the local economy. In particular, we are interested in how critical ability (a_c) responds to changes in preference, technology, commuting, distribution and policy parameters. It can be shown below that the equilibrium relative size of the two cities plays a crucial role. When the equilibrium value of critical ability is low ($a_c < \frac{1}{2}(\bar{a}+\underline{a})$), the local economy features a large clean city and a small dirty city (e.g., Seattle vs. Tacoma). Alternatively, when the equilibrium value of critical ability is high ($a_c > \frac{1}{2}(\bar{a}+\underline{a})$), it features a small clean city and a large dirty city (e.g., Ann Arbor vs. Detroit).

4.1 Changes in Pollution, Technology and Policy

We begin by summarizing the comparative-static results with respect to pollution, technology and policy parameters in Table 1 and in the following two propositions. Specifically, we consider changes in the following pollution, technology and policy parameters:

(i) pollution parameters (θ and D),

(ii) the scaling factor for human capital formation (ψ),

(iii) the unit commuting cost (q), and

(iv) the tax surcharge on those residing in the clean city (η).

Proposition 1: (Effects of Pollution, Technology and Policy Parameter Shifts on Equilibrium Value of Critical Ability) *The equilibrium value of critical ability decreases if*

(i) *the general pollution scaling factor or the human capital scaling factor rises,*

(ii) *the location-biased pollution parameter, the unit commuting cost, or the tax surcharge on those residing in the clean city falls.*

An increase in the general pollution scaling factor (θ) raises the level of pollution proportionately for both cities, but widens the pollution differential between the dirty and the clean cities ($\frac{\theta Y}{D} - \frac{\theta Y}{D+Z}$), thus encouraging residents in a dirty city to move out and lowering the equilibrium value of critical ability. Since a decrease in the location-biased pollution parameter (D) raises the level of pollution in a dirty city proportionately more compared to a clean city, it generates a larger reduction in the equilibrium value of critical ability than the general pollution scaling factor.

Since human capital in a clean city is higher, an increase in the scaling factor of human capital formation (ψ) creates greater benefits for residents in the clean city compared to those in a dirty city. As a consequence, people relocate from a dirty to a clean city, causing the equilibrium value of critical ability to fall. Conversely, an increase in the unit commuting cost (q) or the tax surcharge on those residing in a clean city (η) discourages people from residing there and hence raises the equilibrium value of critical ability.

Proposition 2: (Effects of Pollution, Technology and Policy Parameter Shifts on Equilibrium Population, Human Capital and Effective Labor)

(i) *An increase in the general pollution scaling factor or a decrease in the location-biased pollution parameter relocates the population from a dirty to a clean city, lowers human capital and effective labor in the dirty city, raises human capital and effective labor in the clean city, and reduces non-movers' working time in the dirty city without affecting non-movers' working time in the clean city.*

(ii) *An increase in the human capital scaling factor relocates the population from the dirty to a clean city, raises human capital and effective labor and lowers non-movers' working time in a clean city; its effects on human capital, effective labor and non-movers' working time in a dirty city are all ambiguous.*

(iii) *An increase in the unit commuting cost or the tax surcharge on those residing in a clean city relocates the population from a clean to a dirty city, raises human capital in a dirty city and non-movers' working time in a clean city, and reduces human capital in a clean city; its effects on effective labor and non-movers' working time in a dirty city are ambiguous.*

From Proposition 1, an increase in the general pollution scaling factor or a decrease in the location-biased pollution parameter reduces critical ability, thereby causing the population to move from a dirty to a clean city. Since these parameter shifts do not affect individual human capital, the reduction in critical ability leads to lower aggregate human capital in a dirty city but to higher aggregate human capital in a clean city. Since the reduction in a dirty city's population raises the per capita pollution subsidy, those residing in a dirty city have less incentive to work. Since clean-city residents do not receive a pollution subsidy, their working time remains unchanged (referring to non-movers only throughout the paper). While both a smaller population and lower working time in the dirty city cause its aggregate effective labor to fall, a larger population in the clean city results in higher aggregate effective labor.

An increase in the human capital scaling factor has a positive direct effect on both individual and aggregate human capital. By reducing critical ability, it causes a dirty city to shrink and a clean city to expand. While the population effect reinforces the direct effect in a clean city to result in higher aggregate human capital, the opposing effects in a dirty city lead to an ambiguous outcome. In the clean city, the increase in individual human capital encourages time reallocation from work to children's education. (Recall that the parents' human capital and their time devoted to the children's education are Pareto complements in accumulating human capital.) Under our human capital formation specification, this latter negative effect on working time is always dominated by the positive direct effect; as a consequence, aggregate effective labor rises.

In response to an increase in the unit commuting cost or the tax surcharge on those residing in a clean city, critical ability is higher, a dirty city rises, and a clean city falls. In the absence of a direct effect, aggregate human capital in the dirty city increases whereas that in the clean city decreases. Moreover, the reduction in net income lowers consumption and raises the marginal utility of consumption, thereby increasing the marginal benefit from work. In a dirty city, however, the effect on a per capita pollution subsidy is ambiguous. Thus, clean-city residents' work time rises but dirty-city residents' work time remains ambiguous. Although the effect of higher work time is that it raises aggregate effective labor in the clean city, the reduction in the size of the clean city generates an ambiguous outcome.

4.2 Changes in Ability Distribution

Turning now to changes in the ability distribution, we examine how changes in the ability distribution may affect the equilibrium outcomes. In particular, we consider both a uniform rightward shift in the ability distribution and a mean-preserving spread of the ability distribution. We can establish parallel propositions as follows:

Proposition 3: (Effects of Ability Distribution Changes on the Equilibrium Value of Critical Ability)

(i) *In response to a uniform rightward shift in the ability distribution, the equilibrium value of critical ability increases.*

(ii) *In response to a mean-preserving spread, the equilibrium value of critical ability decreases if the competitive spatial equilibrium features a large dirty city (i.e., the equilibrium value of critical ability is high such that* $a_c > \frac{1}{2}(\bar{a}+\underline{a})$*) and increases if the competitive spatial equilibrium features a sufficiently large clean city (i.e., the equilibrium value of critical ability is high such that* $a_c < \sqrt{\bar{a}\underline{a}} < \frac{1}{2}(\bar{a}+\underline{a})$*).*

While it is obvious that a uniform rightward shift in the ability distribution increases the equilibrium value of critical ability, such an increase is less than one-for-one (i.e., $a_c - \underline{a}$ decreases in response). When the equilibrium value of critical ability is high (a large dirty city), a mean-preserving spread affects those residing in the clean city more than those in the dirty city, thus leading to an increase in the equilibrium value of critical ability.

Proposition 4: (Effects of Ability Distribution Changes on Equilibrium Population, Human Capital and Effective Labor)

(i) *In response to a uniform rightward shift in the ability distribution, the population, effective labor and human capital in a clean city all rise, whereas those in a dirty city all fall.*

(ii) *In response to a mean-preserving spread, the competitive spatial equilibrium features:*

- **a.** *with a large dirty city (i.e., the equilibrium value of critical ability is high such that* $a_c > \frac{1}{2}(\bar{a}+\underline{a})$*), the population, effective labor and human capital in the clean city all rise, whereas those in the dirty city all fall;*
- **b.** *with a sufficiently large clean city (i.e., the equilibrium value of critical ability is high such that* $a_c < \sqrt{\bar{a}\underline{a}} < \frac{1}{2}(\bar{a}+\underline{a})$*), the population, effective labor and human capital in the clean city all fall, whereas those in the dirty city all rise.*

A uniformly rightward shift in the ability distribution does not affect the range of the distribution but causes both the lower and the upper bounds of the distribution to increase more than the critical ability. Thus, the population, aggregate human capital and effective labor of the clean city all increase, but those of the dirty city all decrease.

The effects of a mean-preserving spread of the ability distribution depend crucially on the initial size of the two cities. Suppose the initial equilibrium features a large dirty city. Then the spread affects the larger dirty city less than proportionally but affects the smaller clean city more than proportionally. As a result, the population relocates from the dirty city to the clean city. Thus, both human capital and effective labor in the dirty city fall, while those in the clean city rise. When the initial equilibrium features a sufficiently large clean city, the reverse outcomes arise.

Due to analytic complexity, there are ambiguities in some comparative static results, particularly those concerning human capital, effective labor and working time, as well as aggregate pollution and aggregate output. To help remove such ambiguities and perform welfare analysis, we conduct numerical exercises to which we now turn.

5 Numerical Analysis

We set $\bar{a} = 1$ and $\underline{a} = 0$ (thus, $\Delta = \bar{a} - \underline{a} = 1$). We then normalize $Z = 1$, $D = 1$, and $w = 1$, and let $\phi = 1.5$ such that under our parameterization the value of output is sufficient to cover the wage payment. We now select the benchmark values of pollution cost per unit of output ($\theta = 5\%$), the unit commuting cost ($q = 4\%$) and the tax surcharge on those residing in a clean city ($\eta = 5\%$). Based on an annual discount rate of approximately 5% with 40 years per generation, we set $\beta = 0.15$. Based on the human capital-based endogenous growth literature, it is reasonable to choose the ratio of the external effect of human capital to the individual effect (measured by $\frac{1-\alpha}{\alpha}$) to fall in the range from 4% to 10%, implying $\alpha \in (0.85, 0.94)$. We thus take $\alpha = 0.9$. Finally, we select $\psi = 2$ and $Q = 5.156$ such that the equilibrium ratio of the aggregate pollution cost to aggregate output (denoted by R) is about 4% and the equilibrium fraction of the population in the clean city is about 1/3.

In the benchmark equilibrium, we have: $a_c = 0.667$ and the equilibrium distribution of the population in the clean city is $x^B = 0.333$. The aggregate human capital measures in dirty and clean cities are: $H^A = 0.4449$ and $H^B = 0.5551$. The fraction of time devoted to the production of individuals residing in the respective cities are: $v^A = 0.8778$ and $v^B = 0.8848$. As a consequence, the ratio of effective labor in the clean city to aggregate effective labor is $L^B/L = 0.5578$. The aggregate effective labor and aggregate output in this benchmark economy turn out to be $L = 0.8798$ and $Y = 4.5363$. Using (1) and (13), we can then obtain the aggregate level of pollution as $P = 0.3402$ and aggregate net income as $Y_N = 4.5230$.

5.1 Quantitative Results

We now perform comparative-static analysis numerically. More specifically, we examine how these above-mentioned endogenous variables respond to changes in the pollution parameters (θ and D), the tax surcharge on those residing in the clean city (η), the unit commuting cost (q), the scaling factor for human capital formation (ψ), and the ability distribution. Since we normalized the range of the ability distribution to $[0, 1]$, we study the effect of the mean-preserving concentration (rather than the spread) of the ability distribution, changing the range of the ability distribution from $(0, 1)$ to $(\delta, 1 - \delta)$. We also examine how the economy responds to a uniform rightward shift in the ability distribution, from $(0, 1)$ to $(\varepsilon, 1 + \varepsilon)$.

We report the numerical results in Table 2. First, our numerical results help remove analytical ambiguity in the comparative statics. Under our benchmark parametrization, an increase in the human capital scaling factor reduces aggregate human capital in the dirty city. This is because its positive direct effect is dominated by the population size effect resulting from the reduction in critical ability. As a consequence, its effects on a dirty city's average working time and aggregate effective labor also become unambiguously negative. Moreover, since the per capita pollution subsidy effects are relatively small under our benchmark parametrization, an increase in the unit commuting cost or the tax surcharge on those residing in the clean city raises the dirty city's average working time and aggregate effective labor unambiguously. In addition, it creates a strong population size effect compared to the working time effect, thus causing the clean city's aggregate effective labor to decrease.

Second, the numerical analysis produces useful findings concerning how parameter shifts may affect aggregate effective labor and aggregate output in the entire economy. The general pollution scaling factor, the location-biased pollution parameter and the unit commuting cost all give rise to larger effects on aggregate effective labor in a dirty city than in a clean city, whereas the human capital scaling factor and the tax surcharge on those residing in a clean city affects aggregate effective labor in a clean city more. As a consequence, an increase in the location-biased pollution parameter, the human capital scaling factor or the unit commuting cost, or a decrease in the general pollution scaling factor or the tax surcharge on those residing in a clean city raises the society's aggregate effective labor and output. However, due to the offsetting effects, the magnitude of the increase in the society's aggregate effective labor and output is relatively small in response to any parameter shifts.

Third, our numerical results suggest that the population distribution, human capital and effective labor are very sensitive to changes in all but the unit commuting cost. One may further compare how the two pollution parameters affect the aggregate level of pollution and aggregate income differently. Consider a 20% increase in the general pollution scaling factor (θ) and a 20% decrease in the location-biased pollution parameter (D). The former causes aggregate pollution to rise by 19.99% and aggregate income to drop by 0.0088%, whereas the latter raises aggregate pollution by 20.36% and reduces aggregate income by 0.0110%. Intuitively, an increase in the location-biased pollution factor generates more aggregate pollution than a comparable increase in the general pollution factor, as a result of its more-than-proportional impact on the dirty city. Our numerical results therefore suggest that such a location-biased pollution factor not only leads to more aggregate pollution but is more harmful to the aggregate output of the local economy.

Fourth, it may be interesting to note that when the local tax differential is sufficiently large (about 0.05896), the economy degenerates to one city with no individuals willing to reside in the clean city, leading to a degenerate equilibrium.

Fifth, under our benchmark parametrization, we can conclude from the comparative static exercises reported in Table 2:

(i) aggregate pollution and aggregate net income are positively related in response to changes in the human capital scaling factor and the tax surcharge;

(ii) aggregate pollution and aggregate net income are negatively related in response to changes in the pollution parameters and the unit commuting cost.

These results may help explain the environmental Kuznets curve that displays an inverted-U relationship between pollution and income growth. Specifically, in the early stage of development, improvements in human capital may be the leading force driving income growth with more overall pollution; in the later stage, reduced pollution costs from abatement technology may lead to further growth with less overall pollution.

5.2 Welfare Outcomes

We inquire under which local tax differential the economy achieves the highest welfare. In this one-period non-overlapping generations framework with altruistic agents, a prototypical welfare measure would be simply the representative agent's dynastic value function if all agents were identical. With heterogeneous agents, we employ a utilitarian measure given as:[2]

$$\Omega = \int_0^{a_c} V^A(a)da + \int_{a_c}^1 V^B(a)da$$

Under the benchmark value, we find that the welfare-maximizing local tax differential is positive (precisely, $\eta^* = 3.1\%$). That is, it is socially optimal to impose a tax surcharge on those residing in the clean city.

One may quickly examine the local tax rate differentials between Seattle and Tacoma or between Ann Arbor and Detroit. Neither Seattle nor Tacoma has individual local income taxes. While Ann Arbor also has no individual local income taxes, Detroit imposes a 2.5% city tax rate on its residents. This may reflect, on the one hand, Detroit City's financing of local public infrastructure, and, on the other hand, the City's strategic fiscal competition. However, one may link individual income with real estate and compare instead the property taxes between different locations. The present property tax rates in these cities are: 1.115% (Seattle), 0.323% (Tacoma), 4.736% (Ann Arbor), and 3.385% (Detroit). Thus, in each region, the local tax differential is in favor of the dirty city, which is consistent with the welfare-maximizing tax system suggested above. In particular, the tax surcharge differential in Seattle (relative to Tacoma) is 0.792% whereas the comparable figure in Ann Arbor (relative to Detroit) is 1.351%.

6 Concluding Remarks

In this paper, we have developed an equilibrium sorting model in which individuals decide their residential locations depending on crucially local environmental considerations. Workers of different abilities may choose different locations in which to reside, thereby generating a segregated spatial configuration. For analytical convenience, the model is highly stylized. Nonetheless, the results obtained serve to provide a better understanding regarding the role that environmental concerns play in locational choice, labor-market outcomes and aggregate income.

Along these lines, one natural extension is to incorporate a travel-for local public good following the setup in Peng and Wang (2005). In particular, a public good such as central-city

[2] We note that the dynastic value function is generically dynamic as it includes both the agent's own welfare and his/her descendent's welfare.

parks and museums may neutralize the disutility from pollution in the dirty city and reduce economic stratification. There are two other possible extensions that may be of interest. One is to consider the political economy aspect of environmental protection in a way similar to police protection in İmrohoroğlu, *et al.* (2000). Another is to consider a more sophisticated human capital accumulation process following Benabou (1996a) and more expanded local public finance instruments following Benabou (1996b). Of course, such extensions would substantially increase the model's complexity and it is thus likely that the analysis would rely exclusively on numerical exercises.

Appendix

In this appendix, we provide detailed mathematical proofs of the existence and the interiority of the steady-state equilibrium established in Theorem 1 as well as the comparative-static results presented in Propositions 1-3.

Proof of Theorem 1:

In our model economy, it is clear that once critical ability is pinned down, the remaining 16 endogenous variables can be obtained uniquely in a recursive manner. Thus, the key task is to determine critical ability using (20), which equates NMB with NMC. Substituting (2), (3), (4) and (17) into (20), we obtain:

$$NMB = (1\text{-}\alpha)\beta\ln(a_c^2\text{-}\underline{a}^2) - (1\text{-}\alpha)\beta\ln(\bar{a}^2\text{-}a_c^2) - \frac{\theta ZY}{D(D{+}Z)} \tag{A1}$$

$$NMC = \text{-}\alpha\beta\ln(1\text{-}\eta){+}\ln[(1\text{-}\eta)\psi a_c\text{-}\Gamma qZ]{+}\alpha\beta\ln[(1\text{-}\eta)\psi a_c\text{-}qZ]\text{-}(1{+}\alpha\beta)\ln\Lambda \tag{A2}$$

where $\Lambda = \psi a_c + \eta\frac{1}{1+\alpha\beta}\frac{\bar{a}-a_c}{a_c-\underline{a}}[\frac{\psi}{2}(\bar{a}+a_c) + \frac{\alpha\beta qZ}{1-\eta}]$. It is straightforward to verify the following properties:

(i) If $a_c = \underline{a}$, then $NMB(\underline{a}) = (1-\alpha)\beta\ln(0) - (1-\alpha)\beta\ln(\bar{a}^2 - (a^*)^2) - \frac{\theta ZY}{D(D+Z)} \to -\infty$ and

$$NMC(\underline{a}) = -\alpha\beta\ln(1\text{-}\eta){+}\ln[(1\text{-}\eta)\psi\underline{a}{-}\Gamma qZ]{+}\alpha\beta\ln[(1\text{-}\eta)\psi\underline{a}{-}qZ]{-}(1{+}\alpha\beta)\ln(0) \to \infty.$$

where $\Gamma = 1 - \eta(1+\alpha\beta) > 0$ under Condition 1. Thus, $NMB - NMC \to -\infty$.

(ii) If $a_c = \bar{a}$, then $NMB = \infty$, $\Lambda = \psi\bar{a}$ and then,

$$NMC(\bar{a}) = -\alpha\beta\ln(1\text{-}\eta) + \ln[(1\text{-}\eta)\psi\bar{a} - \Gamma qZ] + \alpha\beta\ln[(1\text{-}\eta)\psi\bar{a} - qZ] + \ln(\psi\bar{a}) < \infty.$$

Thus, $NMB - NMC \to \infty$.

(iii) The slope of the NMB locus is:

$$\frac{\partial NMB}{\partial a_c} = (1-\alpha)\beta\frac{2a_c}{a_c^2 - \underline{a}^2} + (1-\alpha)\beta\frac{2a_c}{\bar{a}^2 - a_c^2} > 0.$$

(iv) The slope of the NMC locus is:

$$\frac{\partial NMC}{\partial a_c} = \frac{(1-\eta)\psi}{(1-\eta)\psi a_c - \Gamma qZ} + \alpha\beta\frac{(1-\eta)\psi}{(1-\eta)\psi a_c - qZ} - (1+\alpha\beta)\frac{1}{\Lambda}\frac{\partial\Lambda}{\partial a_c}$$

where

$$\frac{\partial\Lambda}{\partial a_c} = \psi + \eta\frac{1}{1+\alpha\beta}\frac{\psi}{2}\frac{1}{a_c - \underline{a}}\left\{(\bar{a} - a_c) - \frac{\bar{a} - \underline{a}}{a_c - \underline{a}}\left[(\bar{a} + a_c) + \frac{\alpha\beta qZ}{1-\eta}\right]\right\}.$$

Thus, the $NMB - NMC$ locus starts from $-\infty$ as $a_c \to \underline{a}$ and approaches $+\infty$ as $a_c \to \bar{a}$, being positively sloped toward either boundary. In the intermediate range, the slope of the $NMB - NMC$ locus is ambiguous.

We next examine the second-order condition that is associated with the locational stability of the steady-state equilibrium. Specifically, the second-order condition for an interior steady-state equilibrium requires: $\frac{\partial(NMB-NMC)}{\partial a_c} < 0$, or,

$$\frac{\psi}{\psi a_c - \frac{\Gamma qZ}{1-\eta}} + \frac{\alpha\beta\psi}{\psi a_c - \frac{qZ}{1-\eta}} > \frac{2(1\text{-}\alpha)\beta\,(\bar{a}^2\text{-}\underline{a}^2)\,a_c}{(a_c^2\text{-}\underline{a}^2)\,(\bar{a}^2\text{-}a_c^2)} + \frac{(1+\alpha\beta)\,\psi + \eta\frac{\psi}{2}\frac{1}{a_c\text{-}\underline{a}}\{(\bar{a}\text{-}a_c)\text{-}\frac{\bar{a}\text{-}\underline{a}}{a_c\text{-}\underline{a}}[(\bar{a}+a_c)+\frac{\alpha\beta qZ}{1\text{-}\eta}]\}}{\psi a_c + \eta\frac{1}{1+\alpha\beta}\frac{\bar{a}-a_c}{a_c-\underline{a}}[\frac{\psi}{2}(\bar{a}+a_c)+\frac{\alpha\beta qZ}{1-\eta}]}$$

Observe that $(\bar{a} - a_c)\,(a_c - \underline{a}) < (\bar{a} - \underline{a})\,(\bar{a} + a_c)$ and so $(\bar{a} - a_c) < \frac{\bar{a}-\underline{a}}{a_c-\underline{a}}[(\bar{a} + a_c) + \frac{\alpha\beta qZ}{1-\eta}]$. Then, a sufficient condition, by taking a smaller value of the LHS and a larger value of the RHS of the inequality, is

$$1 + \alpha\beta > 2(1-\alpha)\beta\frac{(\bar{a}^2 - \underline{a}^2)\,a_c^2}{(a_c^2 - \underline{a}^2)\,(\bar{a}^2 - a_c^2)} + \frac{1+\alpha\beta}{1 + \eta\frac{1}{1+\alpha\beta}\frac{\bar{a}-a_c}{a_c-\underline{a}}[\frac{\bar{a}+a_c}{2a_c} + \frac{\alpha\beta qZ}{(1-\eta)\psi a_c}]}$$

Thus, steady-state equilibrium critical ability can be classified into three categories:

(i) Case I: As depicted in Figure 2(a), there exists a unique interior steady-state equilibrium critical ability (point B), which satisfies the second-order condition.

(ii) Case II: As depicted in Figure 2(b), there exist multiple interior steady-state equilibrium critical abilities, satisfying the second-order condition; for illustrative purposes, we plot the case with two (labelled as points B and D).

(iii) Case III: As depicted in Figure 2(c), there exists no interior steady-state equilibrium critical ability to meet the second-order condition (i.e., point A is locationally unstable). In this case, if every agent expects critical ability to be below his/her own ability, then $a_c = \underline{a}$; on the contrary, if every agent expects critical ability to be above his/her own ability, then $a_c = \bar{a}$.

This completes the proof. ■

Proof of Proposition 1: An increase in D or a reduction in θ does not directly affect x^i, H^A, H^B, L^A, L^B or NMC (see equations (2), (3), (4), (16), (17) and (20), respectively). It, however, increases NMB (see equation (20)). As a result, it raises critical ability a_c. Next, an increase in ψ raises H^A and H^B proportionately without changing MB^A, but leads to a higher NMC. It therefore lowers a_c. Finally, an increase in q or η does not directly affect H^A, H^B, or NMB; yet, it decreases NMC (under Condition 1), thus raising a_c. ■

Proof of Proposition 2:

From (2), x^B is completely driven by a_c; hence, the comparative statics are trivial given Proposition 1. Using (3) and (4), H^A and H^B depend only on a_c and ψ. So the comparative statics with respect to changes in other parameters are direct consequences of Proposition 1. An increase in ψ has a positive direct effect on H^A and H^B; however, it also reduces a_c, thus causing H^A to fall and H^B to rise. Its net effect on H^B is therefore positive, but the net effect on H^A is ambiguous.

Next, consider the comparative statics with regard to L^A and L^B. From (16) and (17), θ and D do not affect L^A and L^B directly, so their effects on L^A and L^B are only through a_c. An increase in ψ has a positive direct effect on L^B and a positive indirect effect via a reduction in a_c. Since its direct effect on L^A is ambiguous (depending on the relative size of the two cities), its net effect is ambiguous. In response to an increase in q or η, the direct effects are to reduce L^A and to raise L^B, but the indirect effects via a higher a_c are exactly the opposite. Thus, the comparative statics are ambiguous.

In order to perform comparative statics with respect to individual working time (v^A and v^B), we evaluate an individual whose ability is a and whose residential location remains the same before and after a parameter shift. We have:

$$v^A(a) = \frac{1}{1+\alpha\beta}\left(1 - \frac{\alpha\beta\eta\Delta L^B(a_c)}{a_c - \underline{a}}\frac{1}{\psi a}\right)$$
$$v^B(a) = \frac{1}{1+\alpha\beta}\left(1 + \frac{\alpha\beta q Z}{1-\eta}\frac{1}{\psi a}\right)$$

Again, θ and D do not affect v^A and v^B directly and their effects on v^A and v^B are only driven by changes in a_c. Since $v^B(a)$ does not depend on a_c, neither θ nor D will affect v^B. From (A2), an increase in ψ or a decrease in q or η generates a negative direct effect and hence reduces v^B (recalling that no indirect effect exists). An increase in ψ has a positive direct effect on v^A and negative effects via $L^B(a_c)$ and a_c (see (A1)), thereby leading to ambiguous comparative-static outcomes. Since the effects of q and η on L^B are ambiguous, their net effects on v^A cannot be signed. ■

Proof of Proposition 3:

We begin by examining the comparative-static results with respect to a uniformly rightward shift in the ability distribution. Note that such a shift will not affect the range of the distribution Δ. The results are driven crucially by three key terms that are involved with a_c: $\bar{a} - a_c$, $a_c^2 - \underline{a}^2$, and $\bar{a}^2 - a_c^2$. This rightward shift raises $\bar{a}$ and $\underline{a}$ more than a_c, thus raising $\bar{a} - a_c$ and $\bar{a}^2 - a_c^2$ but reducing $a_c^2 - \underline{a}^2$. Utilizing these results and equations (2), (3), (4), (16), and (17), we immediately obtain comparative statics with regard to x^B, L^A, L^B, H^A, and H^B.

We then consider the comparative-static results with respect to a mean-preserving spread. For convenience, denote $\acute{\bar{a}} = \bar{a} + \zeta$, $\acute{\underline{a}} = \underline{a} - \zeta$, and $a_c(\zeta) = a_c + \sigma\zeta$, where $\zeta > 0$ measures the spread and $\sigma \in (-1, 1)$ is endogenously determined. It is clear that as $\zeta \to 0$, $\sigma \to 0$.

Using (2), we obtain:

$$\lim_{\zeta\to 0,\sigma\to 0} \frac{dx^A}{d\zeta} = \frac{1}{2}(\bar{a} + \underline{a}) - a_c$$

Let $\kappa_1 = \frac{(\acute{\bar{a}})^2 - a_c(\zeta)^2}{\acute{\bar{a}} - \acute{\underline{a}}}$, $\kappa_2 = \frac{\acute{\bar{a}} - a_c(\zeta)}{\acute{\bar{a}} - \acute{\underline{a}}}$ and $\kappa_3 = \frac{a_c(\zeta)^2 - (\acute{\underline{a}})^2}{\acute{\bar{a}} - \acute{\underline{a}}}$, which are the key terms involved with $a_c(\zeta)$ in equations (3), (4), (16), (17), (18) and (20). Then we can show:

$$\begin{aligned}
\lim_{\zeta\to 0,\sigma\to 0} \kappa_1 &= \lim_{\zeta\to 0,\sigma\to 0} \frac{2\{\zeta(1-\sigma^2)(\Delta+\zeta) - \acute{\bar{a}}[\acute{\underline{a}} + \sigma a_c(\zeta)] + \sigma\acute{\underline{a}} a_c(\zeta) + a_c(\zeta)^2\}}{(\Delta + 2\zeta)^2} = a_c^2 - \bar{a}\underline{a} \\
\lim_{\zeta\to 0,\sigma\to 0} \kappa_2 &= \lim_{\zeta\to 0,\sigma\to 0} \frac{(1-\sigma)(\acute{\bar{a}} - \acute{\underline{a}} + 2\zeta) - 2[\acute{\bar{a}} - a_c(\zeta)] + (1-\sigma)\zeta}{\acute{\bar{a}} - \acute{\underline{a}} + 2\zeta} = 2a_c - (\bar{a} + \underline{a}) \\
\lim_{\zeta\to 0,\sigma\to 0} \kappa_3 &= \bar{a}\underline{a} - a_c^2
\end{aligned}$$

Therefore, under a mean-preserving spread of size ζ, we can show:

(i) with a large dirty city (i.e., $a_c > \frac{1}{2}(\bar{a} + \underline{a}) > \sqrt{\bar{a}\underline{a}}$), x^B, L^B and H^B increase, but x^A, L^A, H^A, and NMC decrease. Moreover, a higher H^B and a lower H^A lead to a lower NMB. A lower NMB and a lower NMC imply that there is an ambiguous effect on a_c.

(ii) with a sufficiently large clean city (i.e., $a_c < \sqrt{\bar{a}\underline{a}} < \frac{1}{2}(\bar{a} + \underline{a})$), x^B, L^B and H^B decrease, but x^A, L^A, H^A, and NMC increase. A lower H^B and a higher H^A result in a higher NMB, which together with a higher NMC lead to an ambiguous effect on a_c.

This completes the proof. ■

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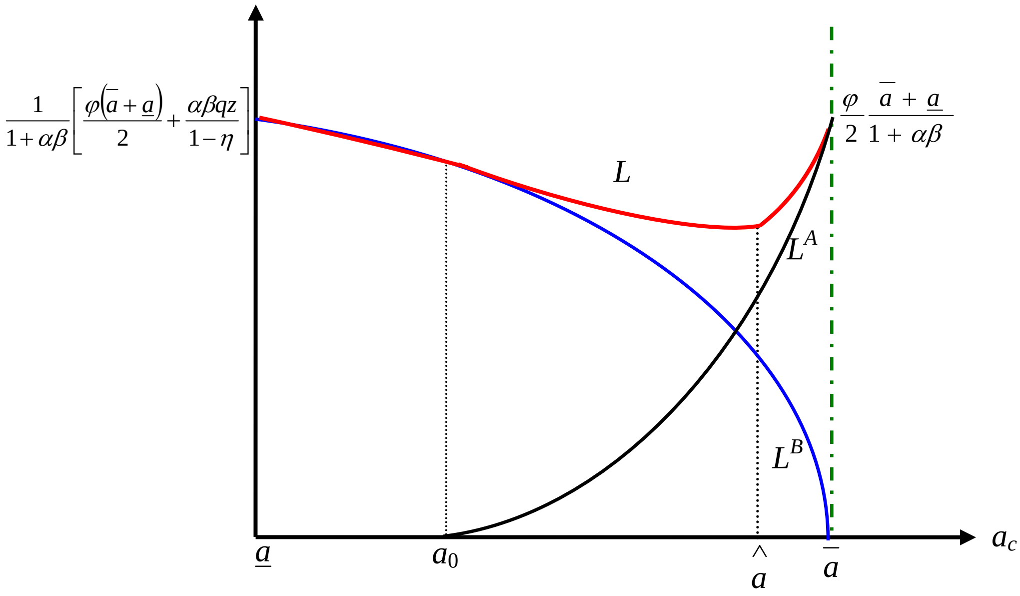

Figure 1: Determination of Critical Ability

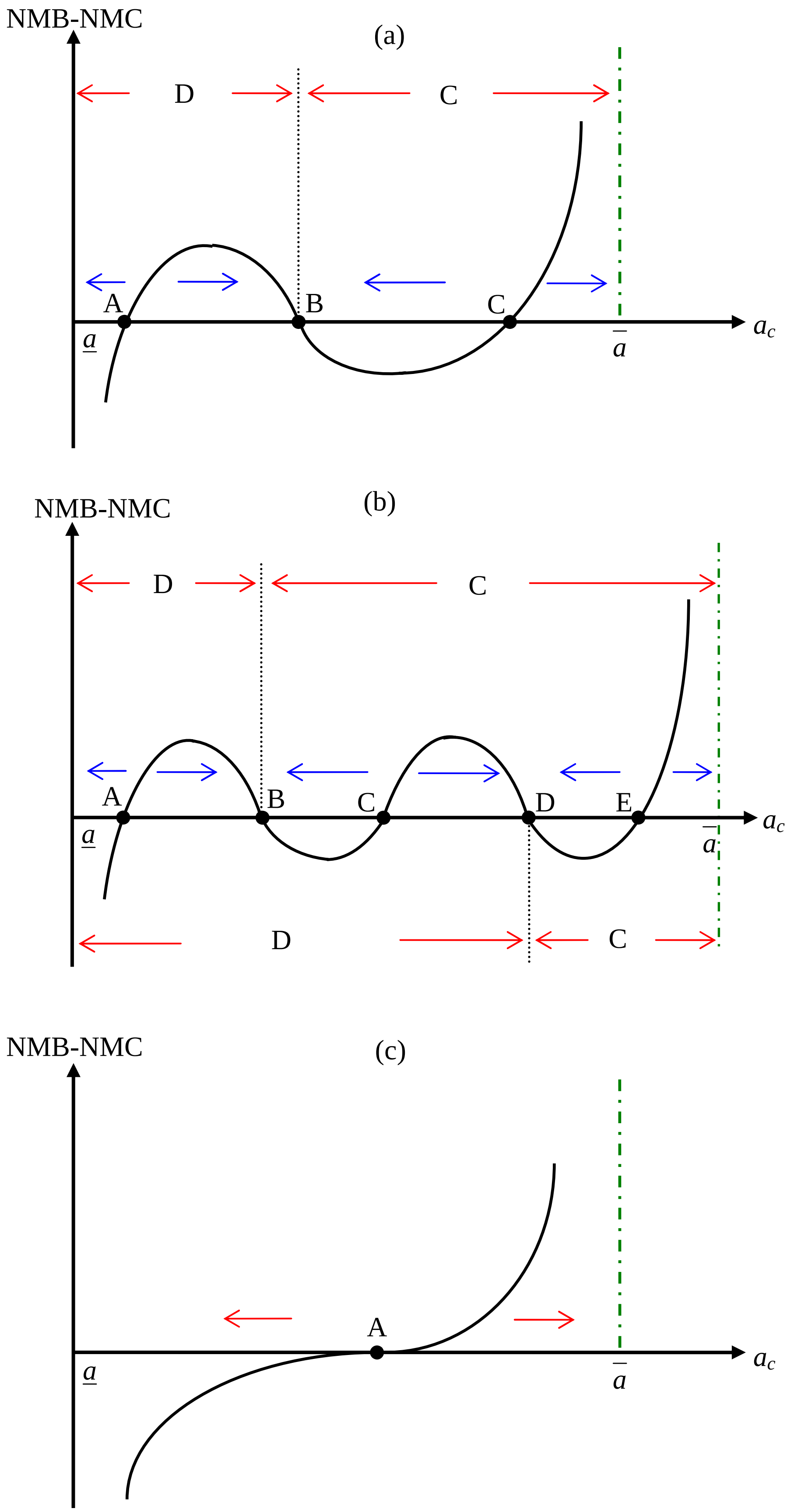

Figure 2: Steady-State Equilibrium Configuration

Table 1: Comparative Statics

Parameter	a_c	x^B	H^A	H^B	L^A	L^B	v^A	v^B
$\theta\uparrow$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	0
$D\uparrow$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	0
$\psi\uparrow$	$-$	$+$	?	$+$	?	$+$	?	$-$
$q\uparrow$	$+$	$-$	$+$	$-$	?	?	?	$+$
$\eta\uparrow$	$+$	$-$	$+$	$-$	?	?	?	$+$

Table 2: Comparative Statics

(A) **Responses to Pollution and Local Tax Parameters**

Parameter	x^B	H^A	H^B	v^A	v^B	L^B/L	L	Y	R
Benchmark	0.333	0.4449	0.5551	0.8778	0.88481	0.5578	0.8798	4.5363	0.0417
$\theta \cdot 0.8$	0.2834	0.5135	0.4865	0.8784	0.88481	0.4887	0.8799	4.5369	0.0408
	(-14.9)	(15.4)	(-12.4)	(0.1)	(0.0)	(-12.4)	(0.0)	(0.0)	(-2.2)
$\theta \cdot 0.9$	0.3105	0.4754	0.5246	0.8781	0.88481	0.5271	0.8799	4.5365	0.0412
	(-6.8)	(6.9)	(-5.5)	(0.0)	(0.0)	(-5.5)	(0.0)	(0.0)	(-1.2)
$\theta \cdot 1.1$	0.3525	0.4193	0.5807	0.8775	0.88481	0.5835	0.8798	4.5361	0.0422
	(5.9)	(-5.8)	(4.6)	(0.0)	(0.0)	(4.6)	(0.0)	(0.0)	(1.2)
$\theta \cdot 1.2$	0.3697	0.3973	0.6027	0.8773	0.88481	0.6057	0.8797	4.5359	0.0428
	(11.0)	(-10.7)	(8.6)	(-0.1)	(0.0)	(8.6)	(0.0)	(0.0)	(2.6)
$D \cdot 0.8$	0.3878	0.3747	0.6253	0.8770	0.88481	0.6284	0.8797	4.5358	0.0422
	(16.5)	(-15.8)	(12.6)	(-0.1)	(0.0)	(12.7)	(0.0)	(0.0)	(1.2)
$D \cdot 0.9$	0.3621	0.407	0.593	0.8774	0.88481	0.596	0.8798	4.536	0.0419
	(8.7)	(-8.5)	(6.8)	(0.0)	(0.0)	(6.8)	(0.0)	(0.0)	(0.5)
$D \cdot 1.1$	0.2989	0.4916	0.5084	0.8782	0.88481	0.5108	0.8799	4.5367	0.0416
	(-10.2)	(10.5)	(-8.4)	(0.0)	(0.0)	(-8.4)	(0.0)	(0.0)	(-0.2)
$D \cdot 1.2$	0.2546	0.5557	0.4443	0.8787	0.88481	0.4463	0.88	4.5373	0.0417
	(-23.5)	(24.9)	(-20.0)	(0.1)	(0.0)	(-20.0)	(0.0)	(0.0)	(0.0)
$\psi \cdot 0.8$	0.264	0.5147	0.4353	0.8786	0.88501	0.4604	0.836	4.3106	0.0434
	(-20.7)	(15.7)	(-21.6)	(0.1)	(0.0)	(-17.5)	(-5.0)	(-5.0)	(4.1)
$\psi \cdot 0.9$	0.3029	0.4738	0.5012	0.8782	0.88491	0.5165	0.8579	4.4234	0.0424
	(-9.0)	(6.5)	(-9.7)	(0.0)	(0.0)	(-7.4)	(-2.5)	(-2.5)	(1.7)
$\psi \cdot 1.1$	0.358	0.4224	0.6026	0.8774	0.88472	0.5908	0.9017	4.6492	0.041
	(7.5)	(-5.1)	(8.6)	(0.0)	(0.0)	(5.9)	(2.5)	(2.5)	(-1.7)
$\psi \cdot 1.2$	0.3796	0.4041	0.6459	0.8771	0.88463	0.6182	0.9236	4.7622	0.0405
	(14.0)	(-9.2)	(16.4)	(-0.1)	(0.0)	(10.8)	(5.0)	(5.0)	(-2.9)

Table 2: Comparative Statics

(B) **Responses to Transportation and Human Capital Parameters**

Parameter	x^B	H^A	H^B	$v^A(a_c)$	$v^B(a_c)$	L^B/L	L	Y	R
Benchmark	0.333	0.4449	0.5551	0.8778	0.88481	0.5578	0.8798	4.5363	0.0417
$q \cdot 0.8$	0.3455	0.4283	0.5717	0.8776	0.88462	0.5744	0.8797	4.5357	0.0414
	(3.8)	(-3.7)	(3.0)	(0.0)	(0.0)	(3.0)	(0.0)	(0.0)	(-0.7)
$q \cdot 0.9$	0.3394	0.4365	0.5635	0.8777	0.88472	0.5663	0.8798	4.536	0.0415
	(1.9)	(-1.9)	(1.5)	(0.0)	(0.0)	(1.5)	(0.0)	(0.0)	(-0.5)
$q \cdot 1.1$	0.3265	0.4536	0.5464	0.8779	0.88491	0.5491	0.8799	4.5366	0.0418
	(-2.0)	(2.0)	(-1.6)	(0.0)	(0.0)	(-1.6)	(0.0)	(0.0)	(0.2)
$q \cdot 1.2$	0.3199	0.4626	0.5374	0.8779	0.88500	0.54	0.8799	4.5368	0.042
	(-3.9)	(4.0)	(-3.2)	(0.0)	(0.0)	(-3.2)	(0.0)	(0.0)	(0.7)
$\eta \cdot 0.8$	0.3623	0.4066	0.5934	0.8776	0.88480	0.5962	0.8799	4.5368	0.0409
	(8.8)	(-8.6)	(6.9)	(0.0)	(0.0)	(6.9)	(0.0)	(0.0)	(-1.9)
$\eta \cdot 0.9$	0.348	0.4251	0.5749	0.8777	0.88481	0.5776	0.8799	4.5365	0.0413
	(4.5)	(-4.5)	(3.6)	(0.0)	(0.0)	(3.5)	(0.0)	(0.0)	(-1.0)
$\eta \cdot 1.1$	0.3173	0.466	0.534	0.8779	0.88482	0.5366	0.8798	4.5361	0.0421
	(-4.7)	(4.7)	(-3.8)	(0.0)	(0.0)	(-3.8)	(0.0)	(0.0)	(1.0)
$\eta \cdot 1.2$	0.3007	0.489	0.511	0.8780	0.88482	0.5135	0.8797	4.536	0.0425
	(-9.7)	(9.9)	(-7.9)	(0.0)	(0.0)	(-7.9)	(0.0)	(0.0)	(1.9)

Table 2: Comparative Statics

(C) **Responses to Ability Distribution Parameters**

Parameter	x^B	H^A	H^B	v^A	v^B	L^B/L	L	Y	R
Benchmark	0.333	0.4449	0.5551	0.8778	0.88481	0.5578	0.8798	4.5363	0.0417
δ=0.05	0.3189	0.4856	0.5144	0.8781	0.88481	0.5169	0.8799	4.5370	0.0420
	(-4.2)	(9.1)	(-7.3)	(0.0)	(0.0)	(-7.3)	(0.0)	(0.0)	(0.7)
δ=0.10	0.3017	0.5298	0.4702	0.8784	0.88481	0.4724	0.8801	4.5378	0.0425
	(-0.1)	(0.2)	(-0.2)	(0.1)	(0.0)	(-0.2)	(0.0)	(0.0)	(0.0)
δ=0.15	0.2790	0.5801	0.4199	0.8788	0.88481	0.4218	0.8802	4.5386	0.0430
	(-16.2)	(30.4)	(-24.4)	(0.1)	(0.0)	(-24.4)	(0.0)	(0.1)	(3.1)
δ=0.20	0.2453	0.6437	0.3563	0.8792	0.88481	0.3580	0.8804	4.5394	0.0439
	(-26.3)	(44.7)	(-35.8)	(0.2)	(0.0)	(-35.8)	(0.1)	(0.1)	(5.3)
ε=0.10	0.4692	0.3879	0.8121	0.8750	0.88481	0.6802	1.0553	5.4414	0.0383
	(40.9)	(-12.8)	(46.3)	(-0.3)	(0.0)	(21.9)	(19.9)	(20.0)	(-8.2)
ε=0.20	0.5563	0.3743	1.0257	0.8719	0.88481	0.7364	1.2309	6.3464	0.0361
	(0.7)	(-0.2)	(0.8)	(-0.7)	(0.0)	(0.3)	(0.4)	(0.4)	(-0.1)
ε=0.30	0.6213	0.3706	1.2294	0.8683	0.88481	0.7724	1.4063	7.2511	0.0345
	(86.6)	(-16.7)	(121.5)	(-1.1)	(0.0)	(38.5)	(59.8)	(59.8)	(-17.3)
ε=0.40	0.6724	0.3694	1.4306	0.8639	0.88481	0.799	1.5818	8.1555	0.0332
	(101.9)	(-17.0)	(157.7)	(-1.6)	(0.0)	(43.2)	(79.8)	(79.8)	(-20.4)