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LABOR-MARKET FRICTIONS, HUMAN CAPITAL ACCUMULATION, AND LONG-RUN GROWTH: POSITIVE ANALYSIS AND POLICY EVALUATION*

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We construct a search model with endogenous human capital and labor participation to study the growth effects of short-run frictions and the effectiveness of human capital policies. Employment, learning effort, and output growth increase with more effective learning, better labor-market matching, lower job separation, or less costly vacancy creation. Although output growth, employment, vacancy creation, and learning and search effort are most responsive to changes in a human capital policy that directly affects learning effort, such a policy need not be more beneficial for welfare. The effects of human capital policies become larger as the severity of labor-market frictions rises.

1. INTRODUCTION

Since the pivotal work by Romer (1986) and Lucas (1988), the endogenous growth framework has become a useful tool to evaluate the long-run growth consequences of public policy. A partial list of policy instruments evaluated by previous studies includes various forms of taxes, subsidies, and economic reforms—some of which focus upon human, physical, and research capital, whereas others focus on economic and political institutions. Following this convention, we reevaluate the effectiveness of some forms of human capital-related policies by developing an endogenous growth model in which the labor market is no longer frictionless. There are substantial informational and institutional barriers to labor search, recruiting, and job creation. Although it is well documented that these types of frictions can have important effects on individual decisions and economic performance in business cycles, the macroeconomic consequences and policy implications of labor-market frictions in a perpetually growing economy have not been fully explored. Our article attempts to fill this gap.

Specifically, we emphasize that *short-run* labor market frictions may have *long-run* growth and welfare implications. The issues concerning growth and unemployment have been studied by Aghion and Howitt (1994) and Laing et al. (1995). Both papers generalize the conventional Mortensen (1982)–Pissarides (1984) labor search framework to permit sustained growth. It is found that when labor markets are thicker, the unemployment rate is lower whereas the long-run growth rate is higher. We extend this line of research, establishing a two-sector endogenous growth model with physical and human capital accumulation in which the labor market is subject to search, matching, and entry frictions. As in their framework, both vacancy creation and job search are costly and vacancies and job seekers are brought together by a matching

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technology exhibiting constant returns. In contrast with theirs, we consider "large" firms and "large" households in the sense that each firm can create multiple vacancies and each household can choose labor-market participation endogenously. These features allow us to move one step closer to the canonical endogenous growth setup for conducting policy analysis quantitatively.[2]

Upon developing this search and growth framework, we calibrate the model to match the U.S. economy and then perform comparative-static analysis and policy evaluation. We find that employment, labor-market participation, vacancy creation, learning effort, and output growth rise with (i) an increase in the effectiveness of human capital accumulation or the degree of labor-market matching efficacy, or (ii) a reduction in the job separation rate and the vacancy creation cost. Moreover, any shift in these parameters fostering long-run growth is always accompanied by a higher unemployment rate. In response to such shifts in growth-enhancing parameters, the labor market also becomes tighter from the firm's viewpoint. Furthermore, our numerical experiments suggest that output growth, employment, vacancy creation, and learning and search effort are most responsive to changes in the human capital accumulation parameter that influences the intensive margin of learning and the labor-market participation decision, followed by the degree of labor-market matching efficacy and the job separation rate. Although an enhancement in human capital that influences learning is more effective in fostering growth, it is also associated with a larger decline in effective consumption and leisure, as well as a larger increase in the unemployment rate.

In terms of policy evaluation, we provide a quantitative assessment of the *relative effectiveness* of two human capital policy programs: one that does not directly affect households' learning effort and another that does. Although the former human capital enhancement policy may be experience accumulation on the job (henceforth referred to as an "experience enhancement policy"), the latter captures both on-the-job training and post-schooling executive learning that are more sensitive to job-related learning effort (henceforth referred to as an "on-the-job training policy").[3] Under the "tax incidence" exercises by maintaining a constant government budget, an on-the-job training policy is found more effective in promoting labor-market participation, learning, employment, and economic growth than an experience enhancement policy. However, an on-the-job training policy also leads to a larger drop in effective consumption and aggregate leisure for the employed, thereby reducing economic welfare despite its strong positive growth effect. As the severity of labor-market frictions diminishes, the effects of these human capital policy programs become smaller. This suggests that a quantitative evaluation of the effectiveness of labor-related policy in a frictionless Walrasian world is expected to be downward biased.

1.1. *Related Literature.* Specifically, in terms of primary methodological issues, our article is almost exactly the opposite to the conventional real business cycle (RBC) theory. The premise of the RBC theory is to argue that long-run technological changes can generate short-run fluctuations at the business cycle frequency. In this article, we instead hypothesize that short-run labor market frictions and the resulting temporary frictional unemployment can affect the long-run performance of the macroeconomy.

The main departure of our article from the labor search literature pioneered by Diamond (1982), Mortensen (1982), and Pissarides (1984) is the consideration of large firms and large households (see Rogerson et al., 2005, for a survey of the broader literature). In terms of this methodology of modeling labor-market frictions in a dynamic setting, our article is related to the RBC search model developed by Merz (1995) and Andolfatto (1996).[4] One major difference

[2] As pointed out by Shimer (2005), Mortensen (1982)–Pissarides (1984) based models are difficult to calibrate to match fundamental observations in the labor market.

[3] The reader is referred to Becker (1962) and Pencavel (1972) for a discussion on general versus job-specific training and to Werther et al. (1995) for issues concerning executive learning. Because our focus is on human capital accumulation on the job, our human capital policy should not be viewed as programs related to pre-employment formal education.

[4] There is a larger but only remotely related literature on growth and cycles. For example, Boldrin and Rustichini (1994) show that positive production externalities in Romer (1986)–Lucas (1988) convention can be sources of persistent

is that the rate of growth is driven by exogenous technological advancement in their models, although we allow the rate of growth to be *endogenously* determined by human capital investment decision. Moreover, their papers study how labor market frictions influence the propagation mechanism of technology shocks over the business cycle, whereas our article examines the interactions between short-run market frictions and *long-run* economic performance. Also in contrast to their setups, *both* vacancy creation and job search are modeled in terms of labor and time allocation.[5] This latter feature enables us to illustrate how labor–leisure–learning–search trade-offs and endogenous labor-market participation in the presence of labor-market frictions may influence the effectiveness of public policy in the long run.

In terms of policy analysis in optimal growth models with labor search, our paper is related to a recent paper by Mortensen (2005). Our framework is very different from Mortensen's, however. Although both papers allow for endogenous growth, Mortensen's is based on the quality ladder *without* physical or human capital accumulation. In contrast, we construct a two-sector endogenous growth framework in which both physical and human capital are endogenously accumulated and in which labor–leisure–learning–search trade-offs play central roles in our analysis. Our policy experiments also differ from Mortensen's. Specifically, Mortensen evaluates wage taxes and employment protection, whereas we assess two forms of *human capital policy* programs. Such a task is feasible and interesting because we model explicitly endogenous human capital accumulation and intratemporal/intertemporal time allocation trade-offs.

2. THE MODEL

Time is discrete. The basic economy features three theaters of economic activities: a continuum of identical infinitely lived competitive firms (of measure one), a continuum of identical infinitely lived households (of measure one), and a fiscal authority. All individual agents have perfect foresight. There are two productive factors: capital and labor, both owned by households. Firms and households exchange in both goods and factor markets. The goods market is Walrasian and the capital market is perfect, but the labor market exhibits search/entry frictions. Although each firm can create multiple vacancies and each household can choose search intensity endogenously, both vacancy creation and search intensity are costly.

To avoid unnecessary complexity involved in managing the distribution of the employed, the unemployed, and their respective human and non-human wealth, we adopt the "large households" framework proposed by Lucas (1990). Specifically, each household can be thought of as containing a continuum of "members" who are either employed (engaged in production, on-the-job learning, or leisure activity) or nonemployed (engaged in job seeking or leisure activity), with the sum of their mass normalized to unity (Figure 1). All members pool their income as well as their enjoyment of the fruit of employment (consumption) and unemployment (leisure). Thus, this structure eliminates the possibility of an endogenous distribution of human and physical capital due to idiosyncratic risk in the labor market. Vacancies and job seekers are brought together through a Diamond- (1982) type matching technology, where the flow matches depend on the masses of both matching parties. Each vacancy can be filled by exactly one searching workers. At an exogenous rate, filled vacancies and workers are separated every period, and separated workers immediately become job seekers.

Finally, the benevolent fiscal authority determines tax rates and human capital enhancement policies by maintaining periodic budget balance.

economic growth as well as endogenous fluctuations. Matsuyama (1999) formalizes the notion of Schumpeterian growth via creative destruction where innovation serves to promote future growth in the low-growth phase under monopolistic competition. In contrast to their technological considerations, our article focuses on search, matching, and entry frictions originated in the labor market.

[5] In Merz, both activities require only real resources of goods. In Andolfatto, vacancy creation requires only real resources of goods, where job search requires time.

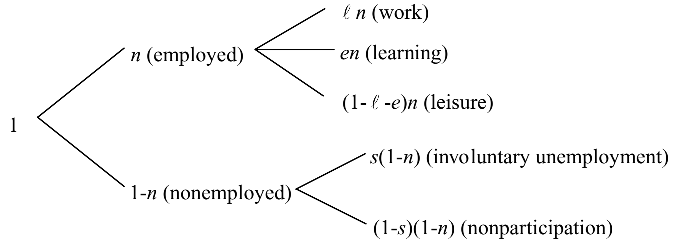

FIGURE 1

LABOR ALLOCATION FOR HOUSEHOLD

2.1. *Firms.* A representative firm, at a particular period t, rents capital k_t (beginning-of-period measure) from households at a gross rental rate r_t and employs labor of mass n_t with effort ℓ_t at a real market wage rate w_t to produce a single final good y_t under a constant-returns-to-scale Cobb-Douglas technology. However, not all employed workers at the representative firm are devoted to production. A mass of workers of measure Φ are employed solely to maintain the vacancies v_t, which can be thought of as covering the costs of posting vacancies, managing personnel-related documentations, as well as providing and maintaining the office space. This labor input will be shortly referred to as the vacancy creation cost. We postulate

$$\Phi(v_t) = \phi v_t^{\varepsilon},$$

where $\varepsilon > 1$ reflects the convexity of the vacancy creation cost and $\phi > 0$ captures any exogenous shift in such a cost. Accordingly, the measure of workers used for manufacturing is $n_t - \Phi(v_t)$, which is augmented by their corresponding effort ℓ_t and human capital h_t. The output of the representative firm can now be specified as

$$y_t = Ak_t^{\alpha}\left[(n_t - \Phi(v_t))\,\ell_t h_t\right]^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ is the output elasticity of capital and $A > 0$ denotes the scaling factor of the production technology.

The shadow rate of return on capital is defined as

$$r_{k_t} = A\alpha\left[\frac{k_t}{(n_t - \Phi(v_t))\ell_t h_t}\right]^{\alpha-1}, \tag{2}$$

which is a decreasing function of the effective capital–labor ratio alone. Because one can establish a one-to-one relationship between the shadow capital rental rate and the endogenously determined balanced growth rate of the economy, it will be convenient to invert (2) to write the effective capital–labor ratio q_t as a function of the shadow rental rate and then express other production-related terms in q_t (and hence as functions of the economic growth rate in balanced growth equilibrium)

$$q_t = \frac{k_t}{(n_t - \Phi(v_t))\ell_t h_t} = \left(\frac{A\alpha}{r_{k_t}}\right)^{\frac{1}{1-\alpha}}. \tag{3}$$

2.2. *Households.* Facing a pooled resource, a representative "large" household has a unified preference capturing enjoyment of all its members: the employed, whose fraction is n_t, and the nonemployed, whose fraction is $1 - n_t$. In order to simplify the analysis, we restrict our attention primarily to on-the-job learning. That is, only the employed will devote time to accumulating human capital. In Section 3.4, we will discuss the implications of a general setup that permits the unemployed to contribute to human capital accumulation.[6]

Thus, employed members divide their time into production ℓ_t (work effort), human capital investment e_t (learning effort), and leisure $1 - \ell_t - e_t$. Nonemployed members divide their time only into job search s_t (search effort or search intensity) and leisure $1 - s_t$. The search intensity augmented unemployment measure is defined as $u_t = s_t(1 - n_t)$. Figure 1 shows the time allocation for households.

In addition to leisure, members of the representative household also value their pooled consumption c_t. The representative household's periodic felicity function is given by

$$U(c_t, \ell_t, e_t, s_t, n_t) = u(c_t) + n_t\Lambda^1(1 - \ell_t - e_t) + (1 - n_t)\Lambda^2(1 - s_t),$$

where employed and nonemployed members need not value their leisure time equally (Λ^1 and Λ^2 may differ), particularly because the nonemployed need not voluntarily take leisure (e.g., a nonemployed member may be involuntarily unemployed as a result of job separation). Functions u and Λ^1 and Λ^2 are strictly increasing and concave. Accordingly, the representative household's preference can be written in a standard time-additive form as

$$\Omega = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(c_t, \ell_t, e_t, s_t, n_t),$$

where Ω is the lifetime utility and $\rho > 0$ is the subjective rate of time preference.

Finally, we extend Lucas (1988, 1993) to specify human capital evolution as

$$h_{t+1} = (1 + \zeta + Dn_t e_t)\, h_t, \tag{4}$$

where $\zeta > 0$ denotes the exogenous component of the rate at which human capital is accumulated, $D > 0$ measures the maximum rate of endogenous human capital accumulation (i.e., the endogenous human capital accumulation rate with maximal learning and full employment, $n_t e_t = 1$), and $h_0 > 0$ represents initial human capital prior to entry to the labor market after completing mandatory formal schooling (K–12). Although the endogenous choice of e resembles the labor–human capital investment trade-off in Lucas (1988), the positive dependence of incremental human capital on n captures learning on the job postulated by Lucas (1993).

In our policy analysis, we shall refer to any policy to increase ζ as an experience enhancement policy and that to raise D as an on-the-job training policy. Although the former does not directly affect learning effort, the latter does; thus, these policy programs are expected to affect human capital accumulation differently. Moreover, as argued by Heckman (1976), better formal education not only leads to a higher *level* of initial human capital before entering the labor market (h_0) but also raises the *rate* at which human capital is accumulated. One may thus regard mandatory K–12 education as to increase ζ and college education as to increase D.

2.3. *The Aggregate Economy.* Because there is only a single good in the economy, the resource constraint requires that aggregate goods supply must be equal to aggregate goods demand, which is the sum of households' consumption and gross investment:

$$c_t + [k_{t+1} - (1 - \delta)k_t] = Ak_t^{\alpha}\left[(n_t - \Phi(v_t))\,\ell_t h_t\right]^{1-\alpha}, \tag{5}$$

where $\delta \in (0, 1)$ denotes the constant rate of capital depreciation.

[6] We will illustrate that allowing the unemployed to learn at a different effort will increase the complexity without generating additional insights toward understanding the long-run growth and welfare effects of labor-market frictions.

Although the capital market is perfect as in the conventional Walrasian models, the labor market exhibits search frictions. Similar to Diamond (1982), the aggregate flow matches depend on the masses of both matching parties, namely, search intensity augmented job seekers, $s_t(1 - n_t)$, and vacancies, v_t. Assume the matching technology exhibits constant returns, as suggested by the empirical evidence in Blanchard and Diamond (1990) using the U.S. data. We can specify

$$m_t = B\,[s_t(1 - n_t)]^{\beta}\, v_t^{1-\beta}, \tag{6}$$

where $B > 0$ measures the degree of matching efficacy and $\beta \in (0, 1)$.

Let ψ be the (exogenous) job separation rate, $\eta_t = m_t/v_t$ be the firm recruitment rate, and $\mu_t = m_t/[s_t(1 - n_t)]$ be the job finding rate. Because each vacancy can be filled by only one worker, the inflow of workers to employment is m_t and the outflow is ψn_t. Employment within the economy thus evolves according to the following birth–death process: $n_{t+1} - n_t = m_t - \psi n_t$, or, by using (6),

$$n_{t+1} = (1 - \psi)n_t + B\,[s_t(1 - n_t)]^{\beta}\, v_t^{1-\beta}. \tag{7}$$

3. OPTIMIZATION AND EQUILIBRIUM

Following the arguments in Merz (1995) and Andolfatto (1996), we assume that a decentralized economy will have the same outcome as the pseudo social planner's problem. As shown in the Appendix, this requires the supporting wage to have the households' bargaining share equal to the corresponding matching elasticity, β; that is, Hosios' (1990) rule holds. With this equivalence property, we can therefore focus on the pseudo social planner's problem to which we now proceed.

Notably, the policy evaluation herein is on contrasting two human capital policy programs, where one favors those who devoted more time to learning and another treats everyone identically. Because both policy instruments only affect the technology of human capital accumulation (i.e., Equation (4)) rather than households' budget constraints or firms' flow profits, it is valid to conduct equilibrium and welfare analysis based exclusively on the pseudo social planner's problem.[7] Notice also that the optimization problem to be solved is a "pseudo" social planner's problem in the sense that the social planner cannot fully coordinate search/matching and that the social planner takes prices as given when considering policy programs.

We will proceed as follows in the next three subsections. To begin, we will derive the pseudo social planner's optimizing conditions. Then, we will define the dynamic search equilibrium as well as the balanced growth equilibrium. Finally, we will illustrate how to determine the balanced growth values of the key macroeconomic variables such as employment, output, capital, and variables related to search such as job matching rates, search intensity, and vacancies.

3.1. *Optimization.* This dynamic programming problem can be specified in the Bellman equation form as

$$\Omega(k_t, h_t, n_t) = \max_{c_t,\, \ell_t, e_t,\, s_t, v_t} U\,(c_t, \ell_t, e_t, s_t, n_t) + \frac{1}{1+\rho}\Omega(k_{t+1}, h_{t+1}, n_{t+1}) \tag{8}$$

subject to constraints (4), (5), and (7) and nonnegativity constraints and initial conditions.

In the Appendix, we present the first-order conditions with respect to consumption (c), work effort (ℓ), learning (e), and search intensity (s) and vacancy creation (v), as well as the

[7] Should one intend to study distortionary factor income taxes/subsidies, a decentralized optimization problem must be used.

Benveniste–Scheinkman conditions governing the two capital stocks and the level of employment (k, h, n). Let us suppress the time subscripts and use "prime" to indicate the next period values. Further denote the marginal valuation of additional human capital accumulated for the next period and the marginal valuation of additional employment to be used in next period production as $MVH' = \Omega_h(k', h', n')/(1+\rho)$ and $MVN' = \Omega_n(k', h', n')/(1+\rho)$, respectively, where all subscripts attached to functionals are derivatives. As shown in the Appendix, one can manipulate the first-order conditions to obtain the following intratemporal and intertemporal trade-off relationships:

$$-\frac{U_\ell}{U_c} = (1-\alpha)Aq^\alpha(n-\Phi)h, \tag{9}$$

$$MVH' \cdot (Dnh) = -U_e, \tag{10}$$

$$MVN' \cdot [\beta\mu(1-n)] = -U_s, \tag{11}$$

$$MVN' \cdot [(1-\beta)\eta] = U_c[(1-\alpha)Aq^\alpha \ell h \Phi_v(v)]. \tag{12}$$

Although Equation (9) displays a standard consumption–leisure trade-off by equating the marginal rate of substitution with the marginal product of labor, others require further elaboration. Concerning the other relationships, we begin by noting that Dnh measures incremental human capital accumulated as a result of learning. Moreover, $\beta\mu\,(1-n)$ and $(1-\beta)\,\eta$ represent the incremental employment as a consequence of, respectively, more effort devoted to finding a job and more vacancy created to recruit workers. Furthermore, $(1-\alpha)Aq^\alpha \ell h \Phi_v(v)$ is the marginal cost of vacancy in units of goods due to a loss of labor productivity. The intuition underlying the remaining three equations is now clear-cut. Equation (10) requires that the future net gain from learning, by enhancing human capital and hence productivity, be equal to the current loss from a reduction in leisure. Equation (11) states that the employment gain next period from a marginal increase in search intensity this period equals the disutility from the corresponding reduction in leisure. Equation (12) indicates that the marginal benefit of vacancy as a result of a successful recruitment equals the sacrifice in the labor used for production in order to maintain the additional vacancy created.

Also as shown in the Appendix, we can manipulate the first-order and the Benveniste–Scheinkman conditions to obtain the following intertemporal trade-off relationships[8]:

$$(1+\rho)\frac{U_c}{U_{c'}} = (1-\delta) + \alpha A(q')^{\alpha-1}, \tag{13}$$

$$MVH \cdot h = -U_\ell \ell + \left(1 + \frac{1+\zeta}{Dne}\right)(-U_e e), \tag{14}$$

$$MVN \cdot n = U_n n - U_e e + \frac{n}{n-\Phi}(-U_\ell \ell) + \frac{n}{1-n}\frac{1-\psi-\beta\mu s}{\beta\mu s}(-U_s s). \tag{15}$$

[8] As shown in the Appendix, the second-order conditions are met. Thus, the first-order conditions and the Benveniste–Scheinkman conditions, together with the transverality conditions associated with the three state variables, are necessary and sufficient for the interior solution(s) to be the maximum.

Equation (13) is a standard intertemporal consumption-saving trade-off condition, equating the marginal rate of intertemporal substitution with the rate of returns on capital. While (14) governs the evolution of human capital, (15) governs the evolution of employment. These relationships equate next period's marginal valuation of incremental human capital and incremental employment, respectively, with the corresponding net marginal opportunity cost from the productivity loss today. It should be noted that, if the employed value leisure more than the nonemployed, the marginal opportunity cost of incremental employment is dampened by an increase in the marginal utility of leisure resulting from having more employed members in the large household (measured by $U_n n$).

3.2. *Equilibrium.* A *dynamic search equilibrium* is a tuple of individual choice variables, $\{c_t, \ell_t, e_t, s_t, v_t, y_t\}_{t=0}^{\infty}$, state variables, $\{k_{t+1}, h_{t+1}, n_{t+1}\}_{t=0}^{\infty}$, and aggregate variables, $\{m_t, r_{kt}, q_t\}_{t=0}^{\infty}$, such that

(i) all individuals optimize, i.e., (9)–(12) and (13)–(15) are met;
(ii) human capital and employment evolve according to (4) and (7), respectively;
(iii) goods production is given by (1) and the effective capital–labor ratio satisfies (3);
(iv) labor-market matching satisfies (6); and
(v) the goods market clears, i.e., (5) holds.[9]

The model economy exhibits perpetual growth, and hence we cannot simply analyze the economic aggregates without transforming perpetually growing quantities into stationary ratios. Throughout the remainder of the article, we focus on a *balanced growth path* (BGP) along which consumption, physical and human capital, and output all grow at positive constant rates. Since the production function is homogeneous of degree one in reproducible factors (k and h) and the human capital accumulation equation is linear (in h), these quantities (c, k, h, and y) must all grow at a common rate, g, on a BGP, whereas other quantities are all constant.

Along a BGP, the labor market must satisfy the steady-state matching (Beveridge curve) relationships given by

$$\psi n = \mu s(1-n) = \eta v = B[s(1-n)]^{\beta} v^{1-\beta}. \tag{16}$$

That is, the equilibrium outflows from the matched pool (ψn) must equal the inflows from either the unmatched worker pool ($\mu s(1-n)$) or the unmatched job vacancy pool (ηv).

For analytical convenience, we assume the felicity function to take the following form: $u(c) = \ln c$, $\Lambda^1(1-\ell-e) = \gamma_1(1-\ell-e)^{1-\sigma}/(1-\sigma)$, and $\Lambda^2(1-s) = \gamma_2(1-s)^{1-\sigma}/(1-\sigma)$, where $\gamma_i > 0$ and $\sigma > 0$. Although log utility in consumption ensures bounded lifetime utility, employed and nonemployed members value leisure differently only by a scaling factor of γ_1 versus γ_2. For a reason to be seen in the calibration analysis later, it is convenient to write the ratio of the marginal utility of leisure of employed to unemployed members as $R = \gamma_1(1-\ell-e)^{-\sigma}/[\gamma_2(1-s)^{-\sigma}]$. Hence the marginal utility of additional employment can be calculated as $U_n = \Lambda^1 - \Lambda^2 = \gamma_2(1-s)^{-\sigma}[(1-\ell-e)R - (1-s)]/(1-\sigma)$, which is expected to be positive in our benchmark economy.

Along a BGP, we can rewrite the two evolution equations (4) and (5), as

$$e = \frac{g-\zeta}{Dn}, \tag{17}$$

$$\frac{c}{h} = [Aq^{\alpha} - (\delta+g)q](n-\Phi)\ell. \tag{18}$$

[9] Notably, there are 13 equations every period, determining 12 endogenous variables. One can easily verify that the goods market clearance condition is automatically met once (9), (12), (13), (14), and (15) are met. Thus, Walras' law holds in our economy.

Next, we show in the Appendix that

$$g = \frac{r_k - (\delta + \rho)}{1 + \rho}, \tag{19}$$

$$\rho(1 + g) = Dn\ell, \tag{20}$$

$$\frac{\rho + \psi}{\beta\mu} + \frac{1 - \sigma s}{1 - \sigma} = R\left(\frac{n\ell}{n - \Phi} + \frac{1 - \ell - \sigma e}{1 - \sigma}\right), \tag{21}$$

$$\frac{\Phi_v n\ell R}{n - \Phi} = \frac{(1 - \beta)\eta}{\beta\mu}. \tag{22}$$

Equation (19) gives the prototypical Keynes–Ramsey relationship that governs consumption growth. While (20) is a relationship based upon intertemporal human capital accumulation, (21) is one based on intertemporal employment evolution and (22) is one based on the vacancy creation trade-off.

Using (19) and (3), we have

$$r_k = (\delta + \rho) + (1 + \rho)g, \tag{23}$$

$$q = \left[\frac{A\alpha}{(\delta + \rho) + (1 + \rho)g}\right]^{\frac{1}{1-\alpha}}. \tag{24}$$

Both relationships are standard in discrete-time optimal growth models with a Cobb–Douglas production technology. As shown in the Appendix, we can substitute out c/h and q in (18) to yield

$$\gamma_1(1 - \ell - e)^{-\sigma} n = \frac{1}{\ell}\frac{(1 - \alpha)[(\delta + g) + \rho(1 + g)]}{(1 - \alpha)(\delta + g) + \rho(1 + g)}, \tag{25}$$

where the right-hand side is increasing in g and the left-hand side may also be locally increasing in g. One may then see that the fixed point mapping may lead to multiple solutions for the balanced growth rate of the economy. In practice, reducing the system to one dimension will not only be overly complicated but also lose economics insights for explaining the underlying results. We will therefore try to reduce the system to two dimensions to which we now turn.

3.3. *Reducing the System to Two-by-Two.* The equations determining the BGP can be rearranged in a recursive fashion that is conducive to performing comparative statics. Essentially, we can reduce the system to 2×2 in (μ, n) space. Once the BGP values of (μ, n) are pinned down, the rest of endogenous variables can then be derived recursively.

In order to see this, we use (16) to derive

$$\eta = B^{\frac{1}{1-\beta}}\mu^{\frac{-\beta}{1-\beta}} = \eta(\mu; B), \tag{26}$$

$$v = B^{\frac{-1}{1-\beta}}\mu^{\frac{\beta}{1-\beta}}\psi n = v(\mu, n; B, \psi), \tag{27}$$

(28) $$s = \frac{\psi n}{(1-n)\mu} = s(\mu, n; \psi),$$

where it is clear that $\eta_\mu < 0$, $\eta_B > 0$, $v_\mu > 0$, $v_n > 0$, $v_B < 0$, $v_\psi > 0$, $s_\mu < 0$, $s_n > 0$, and $s_\psi > 0$. The properties regarding (26) are standard: Although an increase in B represents an outward shift in the Beverage Curve that tends to raise both job finding rate and firm recruitment rate, any other parameter changes cause a movement along the Beverage Curve in (μ, η) space and hence affect the job finding rate and firm recruitment rate differently. Accordingly, an increase in B fosters more matches and hence reduces unfilled vacancies; however, an increase in the job finding rate is associated with a reduction in the firm recruitment rate, leading to more unfilled vacancies. In addition, a higher job separation rate raises unfilled vacancies whereas an increase in employment requires creation of more vacancies to match. The last relationship is a direct consequence of the first equality in (16): A higher job finding rate enables workers to devote less effort to job search and a higher job separation rate requires workers to spend more search effort.

Then, from (20) and (17), we can write learning effort e as

(29) $$e = \frac{\ell}{\rho} - \frac{1+\zeta}{Dn},$$

which is positively related to both employment and work effort. We then show in the Appendix to pin down work effort as

$$\ell\left(1 + \frac{1+\zeta}{Dn} - \frac{1+\rho}{\rho}\ell\right)^{-\sigma} = \frac{(1-\beta)\eta}{\beta\mu}\frac{n-\Phi}{n\Phi_v}\frac{\gamma_2(1-s)^{-\sigma}}{\gamma_1},$$

which can be rewritten as an implicit function

(30) $$\ell = \ell(\mu, n; B, \psi, \phi, D, \zeta),$$

where $\ell_\mu < 0$, $\ell_n \lesseqgtr 0$, $\ell_B > 0$, $\ell_\psi \lesseqgtr 0$, $\ell_\phi < 0$, $\ell_D > 0$, and $\ell_\zeta < 0$.[10] That is, work effort can be expressed as a function of (μ, n) alone. A higher job finding rate fosters more matches and, as a result of diminishing returns, lowers the marginal benefit of additional employment (measured by $\Omega_n(\mathcal{H}')$). In our production function specification, employment and work effort are Pareto complements, so the marginal benefit of work effort decreases. This explains why work effort is negatively related to the job finding rate. An increase in employment creates two opposing effects. It, on the one hand, lowers the marginal benefit of employment (by diminishing returns) and hence the marginal benefit of work effort. On the other, it increases the marginal benefit of work effort as a result of Pareto complementarity. On balance, we have an ambiguous relationship between work effort and employment. Since the effects of exogenous parameters are all partial effects for given values of (μ, n), we will not devote our time to discussing the details but will return to these issues in the numerical analysis after solving each of the endogenous variables in terms of exogenous parameters.

We next substitute (30) into (20) and then (23) and (24) to derive

(31) $$g = g(\mu, n; B, \psi, \phi, D, \zeta),$$

(32) $$r_k = r_k(\mu, n; B, \psi, \phi, D, \zeta),$$

(33) $$q = q(\mu, n; B, \psi, \phi, D, \zeta),$$

[10] In addition to endogenous variables, we have only written down a function in terms of parameters of interest.

where $g_\mu < 0, g_n \lesseqgtr 0, g_B > 0, g_\psi \lesseqgtr 0, g_\phi < 0, g_D > 0, g_\zeta < 0$, as do the functions r_k and q. We would like to restrict our attention to the balanced growth rate that is of greater interest. Since the growth rate is positively related to work effort and work effort is negatively related to the job finding rate, we immediately establish the relationship between the growth rate and the job finding rate for a given level employment. The ambiguity between work effort and employment is also carried over, leading to an ambiguous relationship between growth and employment.

To the end, we substitute (30)–(33) into (21) and (25), which constitute two fundamental relationships to jointly pin down (μ, n). The relationship derived from (21) can be referred to as the *pseudo labor supply locus* (LS) and the relationship obtained from (25) can be called the *pseudo labor demand locus* (LD). Intuitively, the LS locus represents how labor supply responds to a better labor market condition as a result of a higher job finding rate (higher μ), whereas the LD locus indicates how labor demand changes in response to a tighter labor market from the viewpoint of employers (higher μ or lower η). These schedules are named as "pseudo" demand and supply because both schedules are in terms of a job matching probability μ in lieu of labor wages and because both relationships have incorporated goods market clearance and labor-matching equilibrium conditions. Although the direct effects are to yield an upward-sloping LS locus and a downward-sloping LD locus, there are several indirect effects present in our dynamic general equilibrium models, making the net effects ambiguous. The ambiguity of the underlying indirect effects include the potential conflicts between (i) the substitution and the wealth effects, (ii) the employed and the nonemployed within each households, and (iii) households and firms. Of course, the elastic work effort and learning effort as well as the variable vacancies created by each firm lead to further complexity and ambiguity. Nonetheless, one assumes log-linear utility to remove the first potentially conflicting forces and restricts the nonemployed to have less marginal enjoyment in leisure to remove the second ambiguity. If some forms of normality in matching and in labor allocation are further imposed, one may then expect an upward-sloping LS locus in conjunction with a downward-sloping LD locus.

Because of the aforementioned complication in general, we will not perform any further analytic characterization, but instead defer the comparative static analysis to the next section using a numerical method by calibrating the model based on the U.S. data.[11] As will be illustrated, our calibrations will reconfirm the benchmark case with well-behaved upward-sloping LS locus and downward-sloping LD locus.

REMARK: As shown in the Appendix, the decentralized supporting prices, capital rental (r) and wage rate (w), take the following forms:

$$1 + r = 1 + r_k = (1+\rho)\frac{U_c}{U_{c'}}, \tag{34}$$

$$w = \left[\beta + (1-\beta)\left(1 - \frac{\Gamma}{1-\beta}\right)\right]\overline{w} > \overline{w}, \tag{35}$$

where competitive wage is $\overline{w} = [(n - \Phi)/n] \cdot MPL$ and the wage discount is

$$\Gamma = \frac{1-\beta}{\beta}\frac{1+\rho}{R\ell\mu}\frac{r_k + \psi - g(1-\psi)}{1+r_k} > 0.$$

Thus, the supporting wage with frictional labor markets is lower than the competitive wage.

3.4. *Further Discussion*. Before departing for quantitative analysis, we would like to take two theoretical considerations. On the one hand, we eliminate endogenous search intensity to

[11] It is also difficult to prove analytically the existence of a balanced growth path with positive growth, though our calibration exercises ensure such a property.

enable the production of two explicit expressions governing the balanced growth path. On the other, we generalize the human capital accumulation process by allowing the unemployed to contribute with a different effort level from the employed.

3.4.1. *Exogenous labor-market participation.* By eliminating endogenous search intensity, the labor-market participation decision becomes exogenous. Technically, this is equivalent to setting $s = 1$ and $\Lambda^2(1-s) = \gamma_2$ while eliminating optimization condition (11). In the Appendix, we show that the system along the BGP can be reduced to a 2×2 system in (n, g) explicitly. Notice that although the first expression replaces (21), the second is identical to (25).

Thus, reducing the system to 2×2 is now straightforward without requiring the great effort described in Section 3.2. Although such simplification may be mathematically desirable, this system is unfortunately less intuitive than the benchmark one in (μ, n) space, which captures pseudo labor demand and pseudo labor supply schedules. This is because without voluntary unemployment, μ and n are immediately pinned down by steady-state matching relationship, $\mu = \psi n/(1-n)$. As a result, we can no longer separate the changes in employment from the perspectives of household (labor supply) and firms (labor demand).

3.4.2. *Generalized human capital accumulation.* We next turn to a generalized human capital accumulation setup given by

$$h_{t+1} = \left[1 + \zeta + D(ne_1)^a((1-n)e_2)^{1-a}\right]h_t, \tag{36}$$

where $a \in (0, 1)$. That is, the unemployed also contribute to human capital accumulation by devoting an effort e_2 different than that by the employed e_1. Three remarks are now in order. First, the contributions by the employed and the unemployed are Pareto complements, where the share of contribution by the employed to human capital accumulation is measured by a. Second, we expect $e_1 > e_2$; when $a \to 1$, $e_1 \to e$, $e_2 \to 0$, and the model is reduced to the basic setup in Section 2. Third, under this generalized setting, we can no longer call D an on-the-job training parameter because the unemployed also learn to enhance the household's stock of human capital. Rather, D may now be referred to as a general training parameter regardless of one's employment status.

Straightforward optimization leads to the intratemporal trade-off between two effort decisions:

$$\frac{a}{1-a}\frac{e_2}{e_1} = \frac{n\gamma_1(1-\ell-e_1)^{-\sigma}}{(1-n)\gamma_2(1-\ell-e_2)^{-\sigma}}. \tag{37}$$

This expression equates the marginal rate of substitution between the two effort variables on the human capital accumulation side with that on the utility side. Along the BGP, (17) is replaced by

$$g = \zeta + D(ne_1)^a((1-n)e_2)^{1-a}. \tag{38}$$

In the Appendix, we show that (20) and (21) now become

$$\rho(1+g) = Da\left[\frac{ne_1}{(1-n)e_2}\right]^{a-1} n\ell, \tag{39}$$

$$\frac{\rho+\psi}{\beta\mu} + \frac{1-\sigma s - e_2}{1-\sigma} = R\left[\frac{n\ell}{n-\Phi} + \frac{1-\ell-e_1}{1-\sigma} + \frac{(1+\rho)(a-n)}{an(1-n)}e_1\right] \tag{40}$$

while (18), (22), and (25) remain unchanged. Thus, not only must we now add a trade-off relationship (37), but three fundamental relationships (38)–(40) also become more complicated than their counterparts in the benchmark setup.

4. NUMERICAL ANALYSIS

We now turn to quantifying our results in the previous section by calibration analysis. Moreover, we provide a policy analysis by assessing the growth effects and the welfare consequences of an array of labor-market related subsidies.

4.1. *Calibration.* We calibrate parameter values to match the U.S. quarterly data over the period of 1951–2003. In particular, the quarterly per capita real GDP growth rate is set to $g = 0.45\%$ and the quarterly depreciation rate of capital is set to 2% to match the annual per capita real GDP growth rate of 1.8% and the annual depreciation rate of capital in the range of 5−10%, respectively. The rate of time preference is assigned to 1% (which is equivalent to an annual time preference rate of 4%, as used by Kydland and Prescott, 1991)[12]. Then we can calculate from (23) the shadow capital return as $r_k = 0.0345$, along the balanced growth path. Set the capital share to the commonly used value $\alpha = 0.36$, which gives the calibrated capital-real GDP ratio (k/y) of 10.4 and the calibrated consumption-real GDP ratio (c/y) of 0.745, both are very close to the observed value in quarterly data. Based on Kendrick (1976), human capital is as large as physical capital, so we set the physical to human capital ratio at $k/h = 1$.

Based on the study by Shimer (2005), the monthly separation rate is 0.034, the monthly job finding rate is 0.45, and the elasticity parameter of matching is $\beta = 0.72$. Therefore, the quarterly separation rate $\psi = 1 - (1 - 0.034)^3$ and the quarterly job finding rate $\mu = 1 - (1 - 0.45)^3$ are computed as 0.0986 and 0.834, respectively. We calibrate the search intensity augmented unemployment measure $(u = s(1 - n))$ to 0.065, and the employment rate can be calibrated to 0.55 to match the labor force participation rate of 61.5% (by setting $1 - (1 - s)(1 - n) = n + u = 0.615$) and the steady-state matching condition $(\mu u = \psi n)$. We then apply the first equality of (16) to set $v = (1 - n)s = 0.065$. Using (26) and (27), we calibrate $\eta = B = 0.834$. From (28), we have the value of search intensity: $s = 0.145$.

Although Andolfatto (1996) set the average work time as 1/3, the average figure based on 1965 and 2003 American Time Use Survey for an average man with 13–15 years of schooling is about 28.8%. Thus, we set $\ell = 0.32$. Since the observed fraction of time for leisure by the employed is about 60%, we set $e = 1 - 0.32 - 0.60 = 0.08$, which is largely consistent with the observed time allocation that an average worker spends 5−10% of time for advanced learning (including all postmandatory schooling learning, both on the job and at home, and training, both on-the-job training and self-training). Substituting these into (20) and (17), we get $D = 0.0571$ and $\zeta = 0.0020$. So the exogenous rate of human capital accumulation is at a low rate just about 0.1%. Shimer (2005) normalizes the vacancy-searching worker ratio $(\frac{v}{u})$ as one, which we follow. Thus, although employed members allocate about 60% of their time $(1 - \ell - e = 0.6)$ to leisure, the comparable figure for nonemployed members is about 85% $(1 - s = 0.855)$.

We then assign a reasonable labor cost of vacancy creation and management as a percentage of employment (Φ/n) at 2.5%. This gives $\Phi = 0.025 \cdot 0.55 = 0.0138$, which can be plugged into (2) to obtain $A = 0.297$. Since learning effort is nonseparable from work effort, we cannot compute directly the labor supply elasticity, but the learning-augmented labor supply elasticity is given by $(1/\ell - 1)/\sigma$. Although the labor literature estimates the labor supply elasticity around 0.5, the home production literature gets a higher value at 1.7. We select $\sigma = 1.93$, which yields a reasonable learning-augmented labor supply elasticity about 1.1.[13] Now, we can use (22)

[12] In Kydland and Prescott (1991), the quarterly time preference rate is 0.01.

[13] It is difficult to conclude whether the learning-augmented elasticity should be larger or smaller—it all depends on whether education effort is more or less sensitive to market wages.

TABLE 1
BENCHMARK PARAMETER VALUES AND CALIBRATION

Benchmark parameters and observables		
Per capita real economic growth rate	g	0.0045
Capital's depreciation rate	δ	0.0200
Time preference rate	ρ	0.0100
Physical capital–human capital ratio	k/h	1.0000
Fraction of time devoted to work	ℓ	0.3200
Fraction of time devoted to education	e	0.0800
Capital's share	α	0.3600
Labor searcher's share in matching production	β	0.7200
Job separating rate	ψ	0.0986
Job finding rate	μ	0.8336
Labor force participation rate	$n+u$	0.6150
Vacancy–searching worker ratio	v/u	1.0000
Labor supply elasticity	$(1/\ell - 1)/\sigma$	1.1000
Vacancy creation cost per employment	Φ/n	0.0250
Calibration		
Coefficient of goods technology	A	0.2965
Coefficient of matching technology	B	0.8336
Capital–output ratio	k/y	10.4212
Aggregate consumption–aggregate output ratio	c/y	0.7447
Consumption–human capital ratio	c/h	0.0715
Coefficient of the cost of vacancy creation and management	ϕ	6.0729
Exogenous human capital accumulation rate	ζ	0.0020
Maximum rate of endogenous human capital accumulation	D	0.0571
Rate of return of capital	r_k	0.0345
Elasticity of substitution of leisure	σ	1.9318
Unemployment measure	u	0.0650
Fraction of time devoted to employment	n	0.5500
Search intensity	s	0.1445
Vacancy creation	v	0.0650
Cost elasticity of vacancy creation and management	ε	2.2286
Employee recruitment rate	η	0.8336
Coefficient in the utility function	γ_1	1.8203
Coefficient in the utility function	γ_2	1.4365

to calibrate $\varepsilon = 2.229$, and from the definition of Φ, we obtain $\phi = 6.073$. Next, we use (21) to compute the BGP value of R at 2.515. We then apply (25) to calculate $\gamma_1 = 1.820$, which together with the definition of R implies $\gamma_2 = 1.437$. That is, the employed value their leisure time more than the nonemployed, an intuitive result due to the fact that the nonemployed may be forced to take leisure involuntarily. Finally, these calibrated parameters can be substituted into (35) to obtain $w = 0.323$ and $\Gamma = 0.073$. Thus, the wage discount from its competitive counterpart ($\overline{w} = 0.349$), as a consequence of labor-market frictions, is about 7.3%, which seems quite reasonable.

We summarize the observables, benchmark parameter values, and calibrated values of key endogenous variables in Table 1.

4.2. *Numerical Results*. We are now ready to simulate the model to examine quantitatively the effects of two human capital accumulation parameters (ζ and D) and labor-market parameters (B, ψ, and ϕ) on an array of endogenous variables of interest, including the balanced growth rate (g), effective consumption (c/h), physical–human capital ratio (k/h), effective output (y/h), employment (n), unemployment (measured by search intensity augmented job seekers, $s(1-n)$), work effort (ℓ), learning effort (e), search effort (s), workers' job finding rate (μ), firms' employee recruitment rate (η), and firms' vacancies (v). The results are reported in Table 2.

TABLE 2
NUMERICAL RESULTS

	g	c/h	k/h	y/h	n	ℓ	e	s	μ	η	v	u	$n+u$
Benchmark	0.004500	0.071458	1.000000	0.095958	0.549969	0.320000	0.080000	0.144503	0.833625	0.833625	0.065031	0.065031	0.615000
ζ up by 1%	0.005482	−0.000491	−0.001114	−0.000393	0.000344	−0.000319	0.001560	0.000755	0.000009	−0.000022	0.000366	0.000335	0.000343
D up by 1%	0.210375	−0.029885	−0.052548	−0.026324	0.055146	−0.060763	0.292090	0.127404	0.003539	−0.009043	0.064776	0.051425	0.054753
B up by 1%	0.048535	−0.004524	−0.009987	−0.003365	0.017059	−0.016559	0.068740	0.026034	0.012359	0.003960	0.013048	0.004643	0.015746
ψ up by 1%	−0.047869	0.004507	0.010005	0.003644	−0.016834	0.016904	−0.070124	−0.024810	−0.002265	0.005849	−0.012776	−0.004748	−0.015556
ϕ up by 1%	−0.006256	0.000588	0.001300	0.000476	−0.002200	0.002176	−0.009031	−0.003338	−0.001541	0.003975	−0.006150	−0.000659	−0.002037

NOTE: Numbers reported in rows 3–7 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous shift.

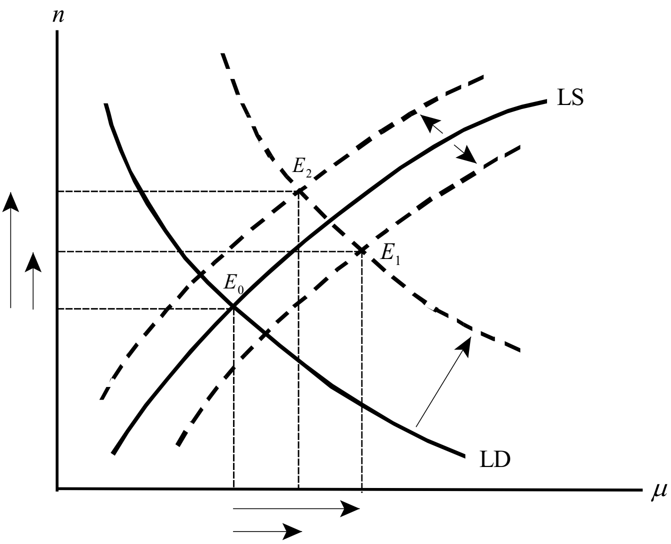

FIGURE 2

BALANCED GROWTH EQUILIBRIUM AND EFFECTS OF HUMAN CAPITAL/LABOR-MARKET IMPROVEMENTS

Under the benchmark parameterization, the value function ($\Omega(\mathcal{H})$) is strictly increasing and strictly concave in each argument (see the Appendix). Also, the LD locus is downward-sloping and the LS locus is upward-sloping (Figure 2). Although there are many underlying forces driving this outcome, one may identify the dominant forces to gain some intuition. When the job finding rate is higher, the marginal benefit of employment is lower, thereby leading to a downward-sloping pseudo labor demand locus. Turning next to the pseudo labor supply locus, one can see from (30) work effort decreases, which causes leisure to rise. A dominant force to offset this effect to restore the equilibrium is to increase investment in human capital, which can be accomplished by raising employment according to (29). Thus, the LS locus slopes upward. In response to human capital accumulation and labor market-improving parameter shifts, there is a large outward shift in the LD locus that outweighs the shift in the LS locus (the BGP equilibrium shifts from E_0 to E_1 or E_2 in Figure 2), thus raising both the job finding rate and employment.

Moreover, it is noted that in the calibrated equilibrium, Pareto complementarity between employment and work effort in production is a dominant force; as a consequence, the relationship between growth and employment given in the human capital envelope condition, (31), is always positive. We may therefore characterize the growth effects of parameter changes based on their direct effects through (31) as well as their indirect effects via the job finding rate and employment in (31). The numerical results suggest that any shift in human capital-enhancing and labor market-improving parameters always create a negative *free-rider effect from thick matching* (through μ) and a positive *employment creation effect* (through n): The former reduces growth whereas the latter raises it. All but a shift in ζ also generates a positive *direct human capital effect* through (31). On balance, each of such shifts affects the growth rate positively. That is, in response to an increase in the experience enhancement parameter, the positive employment creation effect dominates the negative direct human capital effect and the negative free-rider effect from thick matching. In response to other human capital-enhancing and labor market-improving parameter shifts, the positive employment creation effect and the positive direct human capital effect together dominate the negative free-rider effect from thick matching.

An increase in either the experience enhancement parameter (ζ) or the on-the-job training parameter (D) raises learning effort, thus raising employment and economic growth. Not surprisingly, the on-the-job training parameter creates stronger employment and growth effects compared to the experience enhancement parameter. Since both parameters raise labor productivity, they also induce labor-market participation and encourage workers to devote greater effort to job search and firms to create more vacancies. Whereas higher search effort raises the unemployment rate, higher employment lowers it.[14] Around the calibrated equilibrium, the search effort effect dominates, and hence the unemployment rate is higher in response to an increase in either human-capital enhancing parameter. Because of these offsetting forces, the net increase in unemployment is not as much as the increase in vacancies, thus leading to a higher job finding rate and a lower firm recruitment rate. In addition, as a result of higher learning and search effort, work effort decreases. Since accumulating human capital is more profitable, there is a factor substitution from physical to human capital. This latter outcome, together with lower work effort and higher vacancy costs, causes the level of effective output to fall, despite a positive growth effect. The fall in effective output subsequently leads to a decrease in effective consumption.

An increase in the degree of labor-market matching efficacy (B) or a reduction in the separation rate (ψ) or the vacancy creation cost (ϕ) raises employment and job finding rates. Although the induced wage incentive effect encourages labor-market participation, learning and search effort, individual workers may free-ride on the thickness of the labor market that in turns reduces learning and search effort. In the calibrated BGP equilibrium, the wage incentive effect dominates the free-rider effect and, as a result, both output growth and unemployment rates are higher. Moreover, an increase in B shifts the Beverage Curve outward but a decrease in ψ and ϕ induces a downward movement along the Beverage Curve in (μ, η) space. Thus, the former results in higher job finding rate and firm recruitment rate whereas the latter raises job finding rate but reduces firm recruitment rate. Although it is obvious that more effective matching or less costly vacancy creation induces more vacancies, a lower separation rate implies that firms retain current employees without the need for creating more vacancies. Similar to the increase in human capital accumulation parameters, these labor-market improvements also cause work effort to fall as a result of higher learning and search effort. For similar arguments, the levels of effective output and effective consumption decrease as well.

Generally speaking, economic growth, employment, labor-market participation, vacancy creation, and learning and search effort are most responsive to changes in the on-the-job training parameter (D), followed by job matching and separation rates (B and ψ). The positive growth effect of an increase in the experience enhancement parameter (ζ) is by far the smallest, which is not surprising because of the presence of a negative direct human capital effect. Although an increase in the on-the-job training parameter is most effective in fostering growth, it is also associated with the largest decline in work effort, effective output, effective consumption, and leisure, as well as the largest increase in the unemployment rate. Although job finding rate responds most sensitively to the job matching rate followed by the on-the-job training parameter, firm recruitment rate responds most sensitively to the on-the-job training parameter followed by the job separation rate and the vacancy creation cost parameter (ϕ).

It is worth noting that, in response to shifts in any learning and labor-matching parameters, growth and employment always move in the same direction, as do growth and the search intensity augmented unemployment measure (u). Thus, if one measures unemployment by purely head counts ($1 - n$), there is a negative relationship between growth and unemployment in the long run. If one instead measures unemployment by taking search intensity into account, the long-run relationship between growth and unemployment becomes positive.

4.3. *Human Capital Policy*. In our economy, we consider two labor-related public policy programs of particular interest:

[14] The positive effect of training on job search intensity is consistent with empirical findings in Barron et al. (1989).

- an experience enhancement policy enhancing the exogenous component of human capital growth (ζ), which is uniform to all agents (e.g., basic mandatory education);
- an on-the-job training policy raising the marginal benefit of human capital accumulation (D), which favors agents devoting more effort to learning (e.g., on-the-job training and executive learning).

Such training programs have been commonly employed in practice and some programs may be intense.[15]

Notably, in order to highlight the role of labor-market frictions, we have abstracted from considering any other imperfections or distortions such as human capital externalities or factor tax distortions. Thus, when the Hosios rule of efficiency bargaining holds, it is expected that any public policy will not improve upon the decentralized market equilibrium. Nonetheless, one may still compare the growth and welfare effects of the two above-mentioned policies in the revenue-neutral tax-incidence context.

Denote the rate of the respective human capital "subsidy" as a. In each of the policy experiments, the subsidy is financed by a lump-sum tax whose value in effective unit is fixed at 1% of the benchmark value of effective output (this effective lump-sum tax turns out to be $T/h = 0.00096$).

Two preliminary tasks are now in order prior to policy evaluation. First, we must compute the welfare along the BGP. Setting $h_0 = 1$, we can derive the welfare measured by the lifetime utility,

$$\Omega = \frac{1+\rho}{\rho}\left[\ln\left(\frac{c}{h}\right)^* + \frac{1}{\rho}\ln(1+g) + n\gamma_1\frac{(1-\ell-e)^{1-\sigma}}{1-\sigma} + (1-n)\gamma_2\frac{(1-s)^{1-\sigma}}{1-\sigma}\right], \tag{41}$$

where we need to modify (5), using (3), (23), (24) and the definition of BGP, to derive the after-tax effective consumption

$$\left(\frac{c}{h}\right)^* = Aq^\alpha(n-\Phi)\ell - (\delta+g)\frac{k}{h} - \frac{T}{h}. \tag{42}$$

Second, we must compute the relative price of human capital investment in order to compute the rate of subsidy for the two human capital policy experiments. Notice that individual optimization implies that the relative price of human capital investment in unit of outputs (P_h) multiplied by the marginal utility of consumption must be equal to the marginal valuation of human capital, which can be used to derive $P_h = MVH'/U_c$. By utilizing (14) and (A.9), this reduces to

$$P_h = \frac{\overline{w}}{D}. \tag{43}$$

Table 3A summarizes the results of our key endogenous variables in response to each of the two human capital policies subject to the government budget constraint at a given effective value of lump-sum tax. More specifically, the government budget constraint in each case is given by

- an experience enhancement policy that increases ζ to $(1+b)\zeta$: $b\zeta P_h h = T$;
- an on-the-job training policy that increases D to $(1+b)D$: $bP_h hDne = T$.

The required rates of subsidy for the two experiments are about 7.9% and 3.3%, respectively. Notice that these policies can be evaluated based only on the relative price of human capital investment.

[15] For example, in terms of firm training alone, Barron et al. (1989) document that it takes about 29% of the total employment hours for new hires over the first three months since the start of the jobs.

TABLE 3A

POLICY EXPERIMENTS: PERCENTAGE CHANGES IN KEY VARIABLES

	b	g	$(c/h)^*$	$(y/h)^*$	n	ℓ	e	$1-\ell-e$	s	μ	η	v	$n+u$	Ω
Benchmark	NA	0.004500	0.071458	0.095958	0.549969	0.320000	0.080000	0.600000	0.144503	0.833625	0.833625	0.065031	0.615000	−476.856006
Subsidizing human capital uniformly: ζ	0.079181	0.043300	−0.017287	−0.013091	0.002684	−0.002484	0.012168	−0.000298	0.005916	0.000068	−0.000174	0.002858	0.002677	−0.000271
Subsidizing human capital discretionarily: D	0.032870	0.592740	−0.097991	−0.085480	0.141204	−0.149366	0.749491	−0.020270	0.363742	0.011336	−0.028570	0.174768	0.139852	−0.004683

NOTE: Variables $(c/h)^*$ and $(y/h)^*$ represent after-tax effective consumption and output, respectively; see also Table 2.

TABLE 3B

POLICY EXPERIMENT: DECOMPOSITION OF CHANGES IN WELFARE

Welfare Decomposition	Ω	(1) Effective Consumption	(2) Human Capital Growth	(3) Leisure of the Employed	(4) Leisure of the Unemployed
Subsidizing human capital uniformly: ζ	−0.000271	−0.003693	0.004108	−0.001085	0.000400
Subsidizing human capital discretionarily: D	−0.004683	−0.021843	0.056167	−0.059773	0.020767

NOTE: See Table 2.

Overall, under the benchmark parameterization, an on-the-job training policy is more effective in promoting human capital accumulation and economic growth. In particular, such a subsidy amounted to 1% of effective output evaluated at the benchmark value can raise output growth by 59.3% (which is about a 0.267 percentage point increase). This is far more than the effect of an experience enhancement policy (4.3%). Of course, this stronger welfare-enhancing growth effect is accompanied by a larger drop in effective consumption, which is welfare-reducing.

However, due to its encouragement for households to participate in the labor market, to seek jobs, and to spend time on learning, an on-the-job training policy also generates larger drops in leisure for each of the employed and the unemployed members of the large household ($1-\ell-e$ and $1-s$). Since the calibrated value of σ exceeds one, the "aggregate value" of leisure of the employed ($n\gamma_1(1-\ell-e)^{1-\sigma}/(1-\sigma)$) is decreasing in n, whereas the "aggregate value" of leisure of the unemployed ($(1-n)\gamma_2(1-s)^{1-\sigma}/(1-\sigma)$) is increasing in n. Thus, the aggregate leisure effect for the employed in response to an on-the-job training policy is negative, but that for the unemployed is ambiguous. Around the calibrated equilibrium with public policies (see the last column of Table 3B), it turns out that the aggregate leisure effect for the unemployed is positive.

We summarize in Table 3B the four components of changes in welfare according to (41): that due to changes in effective consumption, that due to changes in the rate of human capital accumulation, that due to changes in the aggregate leisure effect for the employed, and that due to changes in the aggregate leisure effect for the unemployed. In response to an on-the-job training policy, the negative welfare effect via the aggregate leisure effect for the employed is large, which in conjunction with the negative welfare effect via effective consumption dominates the positive welfare effects via the accumulation of human capital and the aggregate leisure effect for the unemployed. As a result, an on-the-job training policy reduces economic welfare despite its stronger positive effect on the balanced growth rate. For similar arguments, an experience enhancement policy also generates qualitatively similar component effects on welfare, leading to a net reduction in our benchmark economy.[16] Quantitatively, the growth-promoting policy instrument by subsidizing human capital discretionarily is associated with higher welfare cost than subsidizing human capital uniformly.

To highlight the role played by labor-market frictions, we repeat the policy experiments presented in Table 3A in an alternative economy in which such frictions are less severe. We do so by raising the degree of labor-market matching efficacy (B) by 5% while maintaining a constant government budget at the value computed from the benchmark economy. The results are summarized in Table 4. By comparing the results with their counterparts in Table 3A, a strong conclusion arrives. That is, as the severity of labor-market frictions diminishes, the effects of these human capital policy programs on key variables all become smaller.[17] Quantitatively, such policy consequences are noticeably smaller even with only a moderate improvement in the job-matching conditions. For example, in this alternative economy with 5% less severe labor-market frictions, the effects of an experience enhancement policy on learning, output growth, and employment reduce by about 50% and 20% and 30%, respectively, whereas the comparable figures for an on-the-job training policy are about 40% and 30% and 30%, respectively. Thus, a quantitative evaluation of the effectiveness of public policy in a frictionless Walrasian world is expected to be biased downward severely. This finding is noteworthy because there is a call for reevaluating such human capital policies when the labor market is not frictionless.

[16] Recall that the dynamic search equilibrium features efficient wage bargaining. In the absence of preference/production externalities, distortionary taxes, or other imperfections, education and investment subsidies are not expected to improve welfare. Should one include uncompensated human capital spillovers (cf. Lucas, 1988) or factor income taxation (cf. Bond et al., 1996), these subsidy programs may become welfare enhancing. Thus, our discussion here only focuses on relative welfare comparisons between different policies, rather than the absolute welfare gains/losses associated with each policy.

[17] This conclusion applies to all individual macroeconomic variables. Here, we exclude the welfare measure because it is an aggregator of several macroeconomic variables.

TABLE 4
POLICY EXPERIMENTS FOR AN ALTERNATIVE ECONOMY: PERCENTAGE CHANGES IN KEY VARIABLES

Equilibrium	b	g	$(c/h)^*$	$(y/h)^*$	n	ℓ	e	$1-\ell-e$	s	μ	η	ν	Ω
($B = 0.8336 \cdot 1.05$)	NA	0.005488	0.070034	0.094406	0.592388	0.297378	0.103502	0.599119	0.161794	0.885418	0.849831	0.068711	−476.457493
Subsidizing human capital uniformly: ζ	0.080544	0.034317	−0.017286	−0.013046	0.001815	−0.001625	0.006221	−0.000268	0.004398	0.000067	−0.000172	0.001987	−0.000253
Subsidizing human capital discretionarily: D	0.030040	0.399985	−0.085479	−0.074664	0.099939	−0.115446	0.436289	−0.018069	0.273534	0.010452	−0.026382	0.129744	−0.003533

NOTE: Numbers reported in rows 3–4 are percentage changes of key variables from their equilibrium values with B = 0.8336 · 1.05 (presented in row 2) due to each educational subsidy.

TABLE 5
SENSITIVITY ANALYSIS—POLICY ANALYSIS

Human Capital Enhancement Policy	Percentage Change in g in Response to		Percentage Change in Ω in Response to	
	ζ	D	ζ	D
Benchmark	0.043300	0.592740	−0.000271	−0.004683
$k/h = 0.50$	0.086641	0.413193	−0.000507	−0.006529
$k/h = 2.00$	0.021645	0.381868	−0.000153	−0.003290
$n = 0.50$	0.051609	0.366929	−0.000306	−0.004371
$n = 0.60$	0.039363	0.454832	−0.000256	−0.004207
$\ell = 0.20$	0.035712	0.189110	−0.000058	−0.000543
$\ell = 0.45$	0.052114	0.219506	−0.000091	−0.001153
$e = 0.06$	0.040778	0.559854	−0.000246	−0.004556
$e = 0.10$	0.048462	0.596902	−0.000313	−0.005020
$v/u = 0.85$	0.043300	0.592740	−0.000271	−0.004683
$v/u = 1.15$	0.043300	0.592740	−0.000271	−0.004683
$\Phi/n = 0.02$	0.042357	0.565817	−0.000265	−0.004543
$\Phi/n = 0.03$	0.044443	0.562975	−0.000277	−0.004762
LSE = 0.5	0.035285	0.209979	−0.000225	−0.002816
LSE = 1.7	0.041718	0.572079	−0.000091	−0.001537

4.4. *Sensitivity Analysis.* In the earlier calibration exercises, all the preset parameters are well justified. However, one may argue that some of the calibration criteria are possibly questionable. Thus, we conduct a sensitivity analysis, examining the qualitative as well as quantitative implications of taking alternative calibration criteria in fairly wide ranges. Specifically, we consider the following perturbations:

- We allow the amount of physical capital to be twice as large as or half of the amount of human capital.
- We allow the employment ratio to fall in [0.5, 0.6], the work effort in [0.20, 0.45], the learning effort in [0.06, 0.10], the vacancy-unemployment ratio in [0.85, 1.15], and the cost of vacancy creation and management as a percentage of employment in [0.02, 0.03].
- We also consider a wide range ([0.5,1.7]) of labor supply elasticities to encompass both micro labor and macro literature.

Our sensitivity analysis suggests that, by changing the calibration criteria and recalibrating the model, the changes in equilibrium outcomes are either nonexistent or inessential.[18] Moreover, by performing policy exercises (Table 5), we establish that our findings concerning the long-run growth and welfare effects of the two human capital policy programs remain robust.

One may also inquire whether the role played by market frictions in the growth effects of human capital policies may be overstated in our benchmark model where the nonemployed do not accumulate human capital. In order to address this, we let the labor force participation rate take a high value of 76% (at which all model restrictions still remain valid). In this case, the growth effect of a discretionary on-the-job training policy is still stronger than that of a uniform experience enhancement policy, and the former again leads to a larger drop in economic welfare. With 5% less severe labor-market frictions, the growth effects of an experience enhancement policy and an on-the-job training policy reduce by about 8% and 15%, respectively. Thus, although the role played by market frictions in the growth effects of human capital policies is not as strong quantitatively when the labor force participation rate is higher and the employment pool is larger (with a larger fraction of household members accumulating human capital), it remains quite noticeable.

[18] For brevity, we do not report the results in the article, but will have them available upon request.

TABLE 6A

QUANTITATIVE ANALYSIS WITH GENERALIZED HUMAN CAPITAL ACCUMULATION: NUMERICAL RESULTS

	g	c/h	k/h	y/h	n	ℓ	e_1	e_2	s	μ	η	ν	u	$n+u$
Benchmark	0.004500	0.071458	1.000000	0.095958	0.549969	0.320000	0.080000	0.020000	0.144503	0.833625	0.833625	0.065031	0.065031	0.615000
ζ up by 1%	0.010284	−0.000866	−0.002033	−0.000683	0.002246	−0.002072	0.008339	0.011316	0.004589	0.000414	−0.001064	0.003314	0.001832	0.002203
D up by 1%	0.313196	−0.034938	−0.068123	−0.029724	0.094615	−0.089158	0.389065	0.568624	0.213213	0.020207	−0.050142	0.152398	0.072934	0.092322

NOTE: Numbers reported in rows 3–4 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous shift.

TABLE 6B

QUANTITATIVE ANALYSIS WITH GENERALIZED HUMAN CAPITAL ACCUMULATION: POLICY EXPERIMENTS

	b	g	$(c/h)^*$	$(y/h)^*$	n	ℓ	e_1	e_2	s	μ	η	ν	$n+u$	Ω
Benchmark	NA	0.004500	0.071458	0.095958	0.549969	0.320000	0.080000	0.020000	0.144503	0.833625	0.833625	0.065031	0.615000	458.661788
Subsidizing human capital uniformly: ζ	0.101372	0.079154	−0.020092	−0.015272	0.013607	−0.012334	0.050479	0.069462	0.028185	0.002493	−0.006382	0.020118	0.013341	−0.000447
Subsidizing human capital discretionarily: D	0.035157	0.681145	−0.099012	−0.085256	0.174243	−0.165724	0.785332	1.224152	0.431767	0.042022	−0.100438	0.305350	0.169236	−0.011574

NOTE: Variables $(c/h)^*$ and $(y/h)^*$ represent after-tax effective consumption and output, respectively; see also Table 2.

TABLE 6C
QUANTITATIVE ANALYSIS WITH GENERALIZED HUMAN CAPITAL ACCUMULATION: DECOMPOSITION OF CHANGES IN WELFARE

Welfare Decomposition	Ω	(1) Effective Consumption	(2) Human Capital Growth	(3) Leisure of the Employed	(4) Leisure of the Unemployed
Subsidizing human capital uniformly: ζ	−0.000447	−0.004470	0.007807	−0.005237	0.001452
Subsidizing human capital discretionarily: D	−0.011574	−0.022959	0.067092	−0.073265	0.017559

NOTE: Subsidize ξ-policy by $b = \tau(y/h)/[\xi(\overline{w}/D)]$ and D-policy by $b = \tau(y/h)/[\overline{w}(ne_1)^a ((1-n)e_2)^{1-a}]$; see also Table 2.

Turning now to the exogenous labor-market participation case, we note that from $\mu = \psi n/(1-n)$ and the data on job finding and separation rates, $n = 89\%$ and $u = 11\%$. The unemployment rate is obviously too high, compared to the U.S. data. Moreover, by giving up a behavioral equation governing s, we are now short of one equation to pin down all the parameters. Specifically, (A.19), (A.20), and $\Phi/n = 0.025$ are used to pin down four parameters, $(\gamma_1, \gamma_2, \varepsilon, \phi)$, leaving one indeterminate.

Finally, we consider the case with generalized human capital accumulation. We set $e_2 = e_1/4 = 0.02$. Although most of the calibration steps can be carried through, (37) and (40) can now be combined to calibrate $a = 0.939$ and (38) and (39) are used to calibrate $D = 0.0669$ and $\zeta = 0.00183$. Although the calibrated value of D is higher, that of ζ is lower. Interestingly, about 94% of the accumulation of human capital is due to the contribution by the employed. This conclusion is obtained with much small differences in size and effort between the employed and the unemployed. By examining the numerical results reported in Tables 6A–C, we find our main conclusion remains unchanged. In summary, these findings suggest that our simplified setup is a good benchmark capturing the essence of the real world observations.

5. CONCLUDING REMARKS

In this article, we develop an endogenous growth model where sustained human capital accumulation and labor search, matching, and entry frictions are integral parts of the economy. Our analysis demonstrates the significant role of labor market frictions in assessing macroeconomic performance and policy effectiveness. We find that an increase in the effectiveness of human capital accumulation or a reduction in the job separation rate or the vacancy creation cost will raise employment, vacancy creation, learning effort, and output growth. By conducting two policy experiments that enhance human capital accumulation, we find that an on-the-job training policy is more growth promoting than an experience enhancement policy, though the on-the-job training policy is also associated with a higher welfare cost. Our numerical results also suggest that the effects of these public policy programs become larger as the severity of labor-market frictions increase, which reconfirms the important role of labor market frictions.

Our model is subject to several qualifications that calls for future research. For brevity, we would only mention four possible extensions. First, to simplify the analysis, we assume that the accumulation of human capital depends only on learning effort. It would be interesting to consider the case in which physical capital also contributes to human capital accumulation as modeled by Bond et al. (1996). One may then conduct a full tax-incidence analysis on labor and capital income taxes in the presence of labor-market frictions and compare the results with findings obtained in canonical growth models without frictions. Second, in the present framework, job separation is assumed to be exogenous. It may be extended to allow the separation rate to depend on on-the-job learning effort, as postulated by Mortensen (1988). Such generalization yields an additional margin that may differentiate experience enhancement and on-the-job training policy via endogenous layoff. Third, our framework is ready to be extended to one with credit-market imperfections. The resulting credit constraints may affect human capital investment financing and/or vacancy creation, so the effectiveness of subsidies to learning and vacancy creation need be reevaluated. Finally, in this study, we focus on the long-run implications of an endogenously growing economy with labor market frictions. Our model may be modified to include technological shocks, as in Merz (1995) and Andolfatto (1996), for quantifying the short-run effects of education and labor-market policies over the business cycle.

APPENDIX

A.1. *Optimization Conditions.* Let $\mathcal{H}$ denote the vector of state variables this period, namely, $\mathcal{H} = (k, h, n)$, and $\mathcal{H}'$ denote the triplets in the next period. The first-order conditions with respect to c, ℓ, e, s, and v are

$$U_c = \frac{1}{1+\rho}\Omega_k(\mathcal{H}'), \tag{A.1}$$

$$\frac{1}{1+\rho}\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}(n-\Phi)h = -U_{\ell}, \tag{A.2}$$

$$\frac{1}{1+\rho}\Omega_h(\mathcal{H}')Dnh = -U_e, \tag{A.3}$$

$$\frac{1}{1+\rho}\Omega_n(\mathcal{H}')\beta\mu(1-n) = -U_s, \tag{A.4}$$

$$\Omega_n(\mathcal{H}')(1-\beta)\eta = \Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}\ell h\Phi_v(v). \tag{A.5}$$

Combining (A.1) and (A.2) gives (9), and rewriting (A.3)–(A.5) yields (9)–(12).

The Benveniste–Scheinkman conditions governing (k, h, n) are given as follows (after making use of the first-order conditions earlier):

$$\Omega_k(\mathcal{H}) = \frac{1}{1+\rho}\Omega_k(\mathcal{H}')\left[(1-\delta) + \alpha Aq^{\alpha-1}\right], \tag{A.6}$$

$$\Omega_h(\mathcal{H}) = \frac{1}{1+\rho}\left[\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}(n-\Phi)\ell + \Omega_h(\mathcal{H}')\left(1+\zeta+Dne\right)\right], \tag{A.7}$$

$$\begin{aligned}\Omega_n(\mathcal{H}) = U_n + \frac{1}{1+\rho}[&\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}\ell h + \Omega_h(\mathcal{H}')Deh \\ &+ \Omega_n(\mathcal{H}')(1-\psi-\beta\mu s)].\end{aligned} \tag{A.8}$$

Manipulating (A.6) and using (A.1) give (13). Substituting (4), (A.2), and (A.3) into (A.7), one obtains (14). Similarly, we can substitute (7) and (A.2)–(A.4) into (A.8) to yield (15).

A.2. *Second-Order Conditions.* We next turn to the second-order conditions with respect to choice variables $\{c, \ell, e, s, v\}$. Denote MP as marginal product. By differentiating (A.1)–(A.5) and using the following relationships,

$$\begin{aligned}
\frac{1}{1+\rho}\Omega_k(\mathcal{H}') \cdot MP_\ell &= -U_\ell \\
\Omega_n(\mathcal{H}')(1-\beta)\eta &= \Omega_k(\mathcal{H}') \cdot (-MP_v) \\
MP_v &= Ak^\alpha(\ell h)^{1-\alpha}(1-\alpha)(n-\Phi)^{-\alpha}(-\phi\varepsilon v^{\varepsilon-1}) \\
MP_{vv} &= Ak^\alpha(\ell h)^{1-\alpha}(1-\alpha)\phi\varepsilon v^{\varepsilon-1}(n-\Phi)^{-\alpha-1}\left[\frac{\varepsilon-1}{v} + \alpha\phi\varepsilon v^{\varepsilon-1}\right] \\
\mu &= \frac{m}{s(1-n)} = \frac{B[s(1-n)]^\beta v^{1-\beta}}{s(1-n)} \\
\eta &= \frac{m}{v} = \frac{B[s(1-n)]^\beta v^{1-\beta}}{v},
\end{aligned}$$

we can prove that the second-order conditions with respect to $\{c, \ell, e, s, v\}$ are all met.

A.3. *Fundamental Equilibrium Relationships.* By applying the specific functional forms, (9) becomes

$$\text{(A.9)} \qquad \frac{c}{h} = (1-\alpha)Aq^\alpha\left(\frac{n-\Phi}{n}\right)[\gamma_1(1-\ell-e)^{-\sigma}]^{-1}.$$

Since the felicity function is separable in consumption and leisure, we can see from (A.1), (A.3), and (A.4) that, along a BGP, $\Omega_n(\mathcal{H}')$ is constant whereas $\Omega_k(\mathcal{H}')$ and $\Omega_h(\mathcal{H}')$ are decreasing at the common growth rate. In turn, we can utilize these equations to derive

$$\begin{aligned}
\Omega_k(\mathcal{H}') &= \frac{1+\rho}{c} = \frac{\Omega_k(\mathcal{H})}{1+g}, \\
\Omega_h(\mathcal{H}') &= \frac{(1+\rho)\,\gamma_1(1-\ell-e)^{-\sigma}}{Dnh} = \frac{\Omega_h(\mathcal{H})}{1+g}, \\
\Omega_n(\mathcal{H}') &= \frac{(1+\rho)\,\gamma_2(1-s)^{-\sigma}}{\beta\mu} = \Omega_n(\mathcal{H}).
\end{aligned}$$

These, together with (2), (10), (11), (17), and (A.9), can be substituted into (12) and (13)–(15) to obtain (19)–(22). We can then substitute (17), (20), and (26)–(28) into (22) to yield (30). Finally, manipulation of (18), (A.9), and (24) leads to (25).

A.4. *Equilibrium Price Support.* We now derive equilibrium price support. Since the capital market is perfect, the capital rental r is equal to its shadow rate of return and the intertemporal relative price of consumption. In contrast, the labor market is frictional; thus, the wage rate w derived from efficient bargaining that supports the pseudo social planner's solution is generally different from the competitive wage $\overline{w}$.

To derive this wage support, it is convenient to write out the marginal product of labor, $MPL = dy/d[(n-\Phi(v))\,\ell h] = (1-\alpha)Aq^\alpha$, and the competitive wage rate, $\overline{w} = [(n-\Phi)/n] \cdot MPL$. We turn now to deriving firms' unmatched value (Π^U) and matched value (Π^M) accrued from a successful bargain with their employees. Consider a representative firm that is currently unmatched. Its flow profit is negative due to costly vacancy creation and maintenance (VC). However, at probability η, it will change the state from unmatched to matched next period with a value $\Pi^{M'}$; at probability $1-\eta$, it will remain unmatched next period with a value $\Pi^{U'}$. Thus, the value of an unmatched firm is given by

$$\Pi^U = -VC + \frac{1}{1+r}[\eta\Pi^{M'} + (1-\eta)\Pi^{U'}],$$

where the marginal vacancy cost is $VC = -dy/dv = \Phi_v \ell h \cdot MPL$. In the absence of entry costs, we must have $\Pi^{U'} = \Pi^U = 0$ in balanced growth equilibrium. We can therefore use (34) to rewrite the Bellman equation above as

$$\Pi^{M'} = \left(\frac{1+r_k}{\eta}\right)\Phi_v \ell h \cdot MPL. \tag{A.10}$$

We can also specify the Bellman equation concerning the value of a matched firm later:

$$\Pi^M = \pi + \frac{1}{1+r}[(1-\psi)\Pi^{M'} + \psi\Pi^{U'}], \tag{A.11}$$

where the flow profit per vacancy is governed by $\pi = \max_{k/n}\{y/n - r_k k/n - w\ell h\}$. By the constant-returns property of the production function, we have

$$\pi = \left(MPL \cdot \frac{n-\Phi}{n} - w\right)\ell h. \tag{A.12}$$

The matching surplus accrued from a successful hire of an additional employment is Ω_n/U_c (which is in unit of outputs). Denoting the share of this surplus to workers as ξ (to be determined later) and hence the remaining $(1-\xi)$ to firms, we can write $\Pi^M - \Pi^U = (1-\xi)\Omega_n/U_c$. With $\Pi^U = 0$, this implies

$$\Pi^M = (1-\xi)\frac{\Omega_n}{U_c}. \tag{A.13}$$

Also, updating (A.13) by one period and using (34) and (A.10) to eliminate $\Pi^{M'}$ and $U_{c'}$, we get

$$\Omega_n(\mathcal{H}')(1-\xi)\eta = \Omega_k(\mathcal{H}')(1-\alpha)Aq^\alpha \ell h \Phi_v(v).$$

Comparing this expression with (A.5), one can easily solve

$$\xi = \beta, \tag{A.14}$$

which implies that Hosios' rule holds in our model economy.

From (A.13), we know that $\Pi^M = (1-\xi)\Omega_n(\mathcal{H})/U_c = (1-\beta)c\Omega_n$ and $\Pi^{M'} = (1-\xi)\Omega_n(\mathcal{H}')/U_{c'} = (1-\beta)c\Omega_n(1+g)$. Substituting these into (A.11), we have

$$\pi = (1-\beta)\frac{r_k + \psi - g(1-\psi)}{1+r_k}c\Omega_n. \tag{A.15}$$

Combining (A.12) and (A.15) to eliminate π, we derive the wage support given by (35), which is a weighted average of the competitive wage and the outside option facing each worker, parallel to one obtained by Merz (1995) and Andolfatto (1996).

A.5. *Properties of the Value Function.* Using (A.1)–(A.8) and the evolution equations of $\{k, h, n\}$, we can express choice variables c, ℓ, e, s, v as functions of $\mathcal{H}$. These, together with the co-state variables $\lambda^k = \Omega_k$, $\lambda^h = \Omega_h$, and $\lambda^n = \Omega_n$, enable us to calculate, under our calibrated benchmark economy, $\Omega_{kk}(\mathcal{H}) < 0$, $\Omega_{hh}(\mathcal{H}) < 0$, and $\Omega_{nn}(\mathcal{H}) < 0$.

A.6. *Exogenous Labor-Market Participation.* In this special case with $s = 1$, (15) becomes

$$\Omega_n \cdot n = U_n n - U_e e + \frac{n}{n-\Phi}(-U_\ell \ell)\left[1 + \frac{1-\psi-\beta\mu}{(1-\beta)\psi n}\Phi_v v\right], \tag{A.16}$$

where from (16) $\mu = \psi n/(1-n)$ and from (A.5) $\Omega_n = (1+\rho)(-U_\ell \ell)(\Phi_v v)/[(1-\beta)\psi n \times (n - \Phi)]$. Thus, (A.16) can be rewritten as

$$\frac{-U_\ell \ell}{n-\Phi}\left[\frac{\rho + \psi + \beta\psi\dfrac{n}{1-n}}{(1-\beta)\psi}\Phi_v v - n\right] = U_n n - U_e e. \tag{A.17}$$

Combining (12) and (18) and using the expression for Ω_n, we obtain

$$-U_\ell \ell = \frac{(1-\alpha)Aq^\alpha}{Aq^\alpha - (\delta+g)q}. \tag{A.18}$$

Substituting the utility functional form into (A.17) and (A.18) and using (17), (20), and (24) to eliminate e, ℓ, and q, we then obtain a 2×2 system in (n, g):

(A.19)

$$\gamma_1\left[1 - \frac{\rho - \varsigma + (1+\rho)g}{Dn}\right]^{-\sigma}\frac{\rho(1+g)}{Dn}\left[\frac{\rho+\psi+\beta\psi\dfrac{n}{1-n}}{(1-\beta)\psi}\frac{\Phi_v(v(n))v(n)}{n-\Phi} - \frac{n}{n-\Phi} - \frac{g-\varsigma}{\rho(1+g)}\right]$$
$$= \left\{\frac{\gamma_1}{1-\sigma}\left[1 - \frac{\rho-\varsigma+(1+\rho)g}{Dn}\right]^{1-\sigma} - \gamma_2\right\}n,$$

$$\gamma_1\left[1 - \frac{\rho-\varsigma+(1+\rho)g}{Dn}\right]^{-\sigma}\frac{\rho(1+g)}{Dn} = \frac{(1-\alpha)[(\delta+\rho)+(1+\rho)g]}{(1-\alpha)(\delta+g)+\rho(1+g)}, \tag{A.20}$$

where $v(n) = (\psi n/B)^{1/(1-\beta)}(1-n)^{-\beta/(1-\beta)}$ is increasing in n. Although the first expression replaces (21), the second is identical to (25).

A.7. *Generalized Human Capital Accumulation.* In this generalized framework, (A.7) becomes

$$\rho(1+g) = \frac{h}{c}\frac{(1-\alpha)Aq^\alpha(n-\Phi)\ell}{\gamma_1(1-\ell-e_1)^{-\sigma}}Da\left(\frac{ne_1}{(1-n)e_2}\right)^{a-1}, \tag{A.21}$$

whereas (A.9) is unchanged. Combining (A.9) and (A.21) yields (39). Next, (A.8) is now modified as

$$\Omega_n(\mathcal{H}) = U_n + \frac{1}{1+\rho}\left[\Omega_k(\mathcal{H}')(1-\alpha)Aq^{\alpha}\ell h + \Omega_n(\mathcal{H}')(1-\psi-\beta\mu s)\right] + \frac{1}{1+\rho}\Omega_h(\mathcal{H}')Dh(e_1)^a(e_2)^{1-a}\left(\frac{n}{1-n}\right)^{a-1}\frac{a-n}{1-n}. \tag{A.22}$$

Along the BGP, we can use (A.1), (A.2), and (A.5) (all unchanged) to derive

$$\Omega_k(\mathcal{H}') = \frac{1+\rho}{c},$$

$$\Omega_n(\mathcal{H}') = \Omega_n(\mathcal{H}) = \frac{(1+\rho)\gamma_2(1-s-e_2)^{-\sigma}}{\beta\mu}.$$

Substituting these expressions as well as (22) (which is also unchanged) into (A.22), we obtain (40). Since (22), (18), and (A.9) are unchanged, (25) is unchanged as well.

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