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Intergenerational human capital evolution, local public good preferences, and stratification

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ABSTRACT

This paper considers heterogeneities in preferences over the local public good, human capital formation, and residential locations as primary underlying forces of economic stratification in an endogenously growing economy. We construct a two-period overlapping-generations model with two regions and various forms of human capital externalities where altruistic agents determine intertemporal allocation of time, investment in a child's education and residential location. We fully characterize a balanced growth equilibrium with no migration across generations to elaborate on how changes in preference, human capital accumulation, production, and interregional commuting parameters may affect the equilibrium stratification outcome in the long run.

1. Introduction

It has been recently documented that despite a decline in racial segregation, economic stratification in the U.S. has risen sharply over the past three decades (Jargowsky, 1996). This trend has led to various adverse socioeconomic consequences, particularly low intergenerational mobility of urban ghetto residents (Wilson, 1987). What are the underlying determinants causing economic stratification in the process of economic development? This paper attempts to address this important issue using a human capital-based endogenous growth model within the dynamic general equilibrium framework.

The study of dynamic process of economic stratification is still at its childhood stage. In his pivotal work, Benabou (1996a) constructs an overlapping-generations model in which strong complementarity between individual human capital and a local human capital aggregator encourages economic segregation whereas strong complementarity between individual human capital and a global human capital aggregator discourages it. Specifically, the local human capital aggregator is driven primarily by region-specific incomes and income taxes; the global human capital aggregator is in forms of economy-wide human capital spillovers *à la* Romer (1986) and Lucas (1988).[1]

Our paper extends Benabou (1996a) in three significant aspects. First, we consider three types of human capital spillovers that may be crucial for endogenous sorting: intergenerational spillovers, peer group externalities in schooling

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[1] In a companion paper, Benabou (1996b) illustrates that lack of access to capital markets can reinforce human capital-induced economic segregation.

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and peer group externalities in production. Second, we endogenize *both* parents' investment decision in children's education and students' education effort and allow these decisions to interact with each other. This interaction is crucial because the former will interact with peer group externalities in schooling, while the latter will only interact with peer group externalities in production via education-work trade-off. Both decisions will in turn affect the equilibrium sorting outcome. Third, we include heterogeneous preferences for the local public good (LPG) service as a source of economic segregation.[2] This inclusion is interesting because preferences for the LPG and human capital factors may reinforce each other, jointly influencing the likelihood of economic stratification in the long run.

Specifically, we construct a two-period overlapping-generations economy with two regions where altruistic agents differ *ex ante* in their preferences over the LPG, inherited human capitals, and initial residential locations. In addition to inherited human capital and parental investment in her education, a young agent may enhance her human capital by devoting greater effort to schooling, though it is at the expenses of reducing her work effort. An old agent allocates her after-tax income to consumption, investment in her child's education and commuting. The LPG facility is financed solely by local income taxes and established in one of the two regions; those who reside in another region must incur an interregional commuting cost in order to enjoy the LPG service. Thus, an agent optimizes to determine her consumption, educational choice, investment in her child's education, and her residential location when old. We will characterize a balanced growth equilibrium along which there is no migration across generations and consumption, human capital and output are all growing at constant rates.

We find that heterogeneous preferences for the LPG are important sources of economic stratification along a balanced growth path (BGP). Provided that LPG preferences are heterogeneous, the balanced growth equilibrium is completely integrated if the local income tax rate differential is sufficiently small or sufficiently large. When the tax rate differential takes intermediate values, agents are completely stratified where those with higher preference bias toward the LPG reside in the region containing the public facility. With weak parental altruism or intergenerational human capital spillovers, economic stratification is more likely to remain in the long run and intergenerational mobility is low. Similar results arise when the individual schooling effort effect in human capital accumulation is less essential and the schooling peer group effect in production is less important. Additionally, when interregional commuting is more costly, economic stratification is more likely to be sustained.

2. The model

Consider a two-period overlapping-generations economy with two goods, a composite consumption good, denoted by c, and an LPG, denoted by G. The LPG can be viewed as publicly provided local parks, museums, concert halls, or theaters. There are two regions, labelled by $\ell = A, B$. The LPG is in region A. There is a continuum of agents of mass one in each generation $t = 0, 1, \ldots$, where a continuum of initial old agents is populated in period 0. Agents are heterogenous only in their relative preferences away from the composite good towards the LPG, denoted by $\gamma = \gamma^H, \gamma^L$, where $\gamma^H > \gamma^L$. To ease the notation, let us use superscript to index generation/type/location, subscript to index time, and argument (i) to index family.

Agents when young undertake education, accumulating human capital to enhance their production when old. Without loss of generality, we assume that agents only consume when old. Each agent is endowed with one unit of time during the entire lifetime, which can be used for schooling when young (s) or for production when old ($1 - s$). To avoid unnecessary complexity from counting populations, the reader may imagine that each family has a single parent who gives birth to one child during her entire lifetime. Due to altruism, an agent when old (a parent) cares about her descendants: she looks after her single child's welfare both directly, via her investment effort devoted to her child's education (v), and indirectly, via intergenerational human capital spillovers and her own locational choices that affect the within-the-cohort human spillovers.

Consider a representative agent $a^t \equiv a^t(\gamma, h_t^{t-1}(i^{t-1}), z_t^{t-1}(i^{t-1}))$ of generation-t born in a family headed by i^{t-1}, whose inherited human capital from the parent is $h_t^{t-1}(i^{t-1})$ and whose residence pre-determined by the parent is $z_t^{t-1}(i^{t-1})$. While i^{t-1} represents the parent of agent a^t, we label the descendant of a^t by j^{t+1}. Thus, (i^{t-1}, a^t, j^{t+1}) summarizes the entire direct family tree of a^t. The lifetime utility of a representative agent a^t is given by

$$U_t(a^t) = c_{t+1}^t + \gamma(\zeta + \ln G_{t+1}) + \alpha h_{t+1}^{t+1}(j^{t+1}) \tag{1}$$

where $\zeta > 0$ is introduced to ensure normal locational choice outcome[3] and $\alpha \in (0, 1)$ is the intergenerational discounting factor. In the case where $\alpha = 0$, this utility functional form reduces to that in Berliant et al. (2006), which modifies the separable quasi-linear utility function originally proposed by Bergstrom and Cornes (1983).[4] Our setup extends the functional form used in the literature by considering *intergenerational altruism* (IA). With regard to IA, one may use the dynasty setup putting the descendant's utility into the parent's utility. However, for analytical simplicity, we follow Glomm and Ravikumar (1992), assuming instead that the parent only cares about the descendant's human capital (h_{t+1}^{t+1}).

[2] In Peng and Wang (2005), income and LPG preference heterogeneities are considered as the two driving forces of stratification in a static setup.

[3] In the endogenous-sorting framework, the cardinal properties of preferences are important. Particularly, in the case where $\ln G_{t+1} < 0$, one needs a sufficiently large value of ζ (such that $\zeta + \ln G_{t+1} > 0$) to guarantee that those with higher γ would prefer to reside in a location closer to the LPG site.

[4] The specification of the separable quasi-linear utility function ensures that the willingness to pay for the LPG is free of wealth effects and yields clean analytic solution. Under this setup, the preference parameter γ affects individual decision only through locational choice.

When young, this agent accumulates human capital $h_t^t(a^t)$ according to

$$h_t^t(a^t) = \beta h_t^{t-1}(i^{t-1}) + \phi(s^t)^{\varepsilon}(v_t^{t-1}(i^{t-1}))^{1-\varepsilon}Q^t(z_t^{t-1}(i^{t-1})) \tag{2}$$

In this human capital evolution equation that generalizes the construction by Becker et al. (1990), $\beta > 0$ measures the degree of *intergenerational human capital spillovers* from a parent's human capital $h_t^{t-1}(i)$, $\phi > 0$ is the scaling factor for incremental human capital accumulation from schooling, and $\varepsilon \in (0,1)$ reflects the relative importance of their own education effort for enhancing human capital. We allow for *peer group externalities in schooling*, where the local within-the-cohort human spillovers are captured by the locally aggregate effective labor $Q^t(z_t^{t-1}(i^{t-1}))$ of cohort t in neighborhood $z_t^{t-1}(i^{t-1})$, given by

$$Q^t(z_t^{t-1}(i^{t-1})) = \int_{z_t^{t-1}(\tilde{a}^t)=z_t^{t-1}(i)} [s^t(z_t^{t-1}(i^{t-1}))h_t^t(\tilde{a}^t)]\,\mathrm{d}\tilde{a}^t \tag{3}$$

When old, agent a^t can fully utilize human capital $h_t^t(a^t)$ without depreciation[5]

$$h_{t+1}^t(a^t) = h_t^t(a^t) \tag{4}$$

This human capital stock can then be used with time effort $(1-s^t)$ to produce the single consumption good:

$$y_{t+1}^t(a^t) = \Psi(\bar{s}^t)[(1-s^t)h_{t+1}^t(a^t)]^{\theta}[K_{t+1}(z_{t+1}^t(a^t)]^{1-\theta} \tag{5}$$

where $\theta \in (0,1)$ denotes the output elasticity of effective labor $((1-s^t)h_t^t(a^t))$. We allow the production scaling factor to depend positively on generation t's average schooling effort $\bar{s}^t$: $\Psi(\bar{s}^t) = \overline{\psi}\psi(\bar{s}^t)$, where $\overline{\psi} > 0$ and ψ is increasing and strictly concave. The locally aggregate effective labor over generation t's colleagues at work in location $z_{t+1}^t(a^t)$ is denoted by $K_{t+1}(z_{t+1}^t(a^t))$, specified as

$$K_{t+1}(z_{t+1}^t(a^t)) = \int_{z_{t+1}^t(\tilde{a}^t)=z_{t+1}^t(a^t)} [(1-s^t(z_{t+1}^t(a^t)))h_{t+1}^t(\tilde{a}^t)]\,\mathrm{d}\tilde{a}^t \tag{6}$$

That is, while the scaling factor $\Psi(\bar{s}^t)$ captures the *direct external effect on productivity* via average schooling, K_{t+1} measures *peer group externalities in production*.

The aggregate income of the society is $Y_{t+1} = \sum_{\ell=A,B} Y_{t+1}^{\ell}$, where

$$Y_{t+1}^{\ell} = \Psi(\bar{s}^t)[K_{t+1}(\ell)]^{1-\theta}\int_{z_{t+1}^t(a^t)=\ell} [(1-s^t(a^t))h_{t+1}^t(a^t)]^{\theta}\,\mathrm{d}a^t \tag{7}$$

Assume that both the intraregional commuting cost and the inter-regional commuting cost are proportional to the aggregate income, measured by μY_{t+1} and $\mu(1+\delta)Y_{t+1}$, respectively, where $\mu, \delta > 0$. Since the LPG is in region A, only those residing in region B incur inter-regional commuting costs. Further assume that tuition is tax-exempt. Moreover, we assume that the cost incurred by a representative parent's investment effort devoted to her child's education is measured by vy.[6] Then the budget constraint facing this representative agent a^t is given by

$$c_{t+1}^t = (1-\tau^{\ell})[1 - v_{t+1}^t(a^t)]y_{t+1}^t(a^t) - I(z_{t+1}^t(a^t))\mu Y_{t+1} \tag{8}$$

where I is a locational choice indicator function

$$I = \begin{cases} 1 & \text{if } z_{t+1}^t(a^t) = A \\ 1+\delta & \text{if } z_{t+1}^t(a^t) = B \end{cases}$$

To close the model, we specify the government budget constraint as

$$\sum_{\ell=A,B} \tau^{\ell} Y_{t+1}^{\ell} = G_{t+1} \tag{9}$$

3. Equilibrium

We now solve the individual optimization problem and the long-run balanced growth equilibrium.

3.1. Individual optimization

Individual optimization includes (i) locational choice when old, (ii) the schooling decision when young (which subsequently pins down work effort when old), and (iii) the consumption–investment in a child's education trade-off when old.

5 Since our focus is on intergenerational human capital evolution as given by (2), this assumption is of no harm but simplifies our analysis greatly.

6 Since the utility function is quasi-linear, one may measure this education investment effort cost from either real resources or disutility.

Under perfect foresight, all decisions are made at the beginning of the first period of lifetime. In solving this optimization problem, one may recognize that locational choice is a discrete choice problem and hence can be separated from other decisions. That is, we can divide the individual optimization into two steps. In Step 1, we solve the individual optimizing problem concerning schooling/work effort decision (s) and the consumption–investment in a child's education trade-off (c and v), for each location $\ell = A, B$. In Step 2, we solve the locational choice problem by comparing the indirect utility between the two locations based on the optimization results in Step 1. Specifically, we have the following optimization problem:

(i) *Step* 1: at each location ℓ and taking aggregate variables, Q^t, Q^{t+1} and K_{t+1}, as given

$$V^t(\ell) = \max_{s^t, c^t_{t+1}, v^t_{t+1}} U^t(a^t)$$
$$\text{s.t.} \quad c^t_{t+1} = (1-\tau^\ell)[1 - v^t_{t+1}(a^t)]y^t_{t+1}(a^t) - \mu I(z^t_{t+1}(a^t))Y_{t+1}$$

(ii) *Step* 2:

$$z^t_{t+1}(a^t) = arg \max_{\ell \in \{A,B\}} V^t(\ell)$$

While the optimization problem in Step 2 is a straightforward discrete-choice problem, we can simplify the optimization problem in Step 1 by substituting (8) and (2) for the descendant into (1) to write,

$$\begin{aligned}&\max_{s^t, v^t_{t+1}} \{(1-\tau^\ell)[1 - v^t_{t+1}(a^t)]\Psi(\bar{s}^t)[(1-s^t(a^t))h^t_{t+1}(a^t)]^\theta [K_{t+1}(z^t_{t+1}(a^t)]^{1-\theta} \\ &\quad - \mu I(z^t_{t+1}(a^t))Y_{t+1}(z^t_{t+1}(a^t)) + \gamma(\zeta + \ln G_{t+1}) \\ &\quad + \alpha[\beta h^t_{t+1}(a^t) + \phi(s^{t+1}(j^{t+1}))^\varepsilon (v^t_{t+1}(a^t))^{1-\varepsilon} Q^{t+1}(z^t_{t+1}(a^t))]\}\end{aligned}$$

Notice that, in considering a child's human capital $h^{t+1}_{t+1}(j^{t+1})$, the representative agent a^t only decides the fraction of resources devoted to a child's education ($v^t_{t+1}(a^t)$) while taking the child's education effort ($s^{t+1}(j^{t+1})$) as given.

We derive the first-order condition with respect to s^t as

$$\begin{aligned}&\Psi(\bar{s}^t)\theta(1-\tau^\ell)[1 - v^t_{t+1}(a^t)]\left[\frac{h^t_{t+1}(a^t)}{Q^t(z^{t-1}_t(i^t))} - (1-s^t)\phi\varepsilon(s^t)^{\varepsilon-1} v^t_{t+1}(a^t)^{1-\varepsilon}\right] \\ &\quad = \alpha\beta\phi\varepsilon(s^t)^{\varepsilon-1} v^{t-1}_t(a^t)^{1-\varepsilon}\left[\frac{(1-s^t)h^t_{t+1}(a^t)}{K_{t+1}(z^t_{t+1}(a^t))}\right]^{1-\theta}\end{aligned} \tag{10}$$

Using (4), the first-order condition with respect to v^t_{t+1} can be reduced to

$$\Psi(\bar{s}^t)(1-\tau^\ell)(1-s^t)^\theta\left[\frac{h^t_{t+1}(a^t)}{Q^{t+1}(z^t_{t+1}(a^t))}\right] = \alpha\phi(1-\varepsilon)(s^{t+1})^\varepsilon[v^t_{t+1}(a^t)]^{-\varepsilon}\left[\frac{h^t_{t+1}(a^t)}{K_{t+1}(z^t_{t+1}(a^t))}\right]^{1-\theta} \tag{11}$$

Eqs. (10) and (11) simultaneously determine the paths of $\{s^t, v^t_{t+1}\}$ for given location, z^t_{t+1}, a child's schooling decision, s^{t+1}, and aggregate spillover variables, $\bar{s}^t$, Q^t, Q^{t+1} and K_{t+1}. It may be noted that these various externalities affect the decision for parental investment in a child's education and individual schooling effort via very different channels. From (2), peer group externalities in schooling increases the marginal benefit of parental investment as well as the marginal benefit of schooling. From (5), both peer group externalities in production and the direct external effect on productivity via average schooling raises the marginal cost of parental investment and the marginal cost of schooling. Intergenerational human capital spillovers affect education decisions only via its positive effect on the shadow price of human capital investment.

3.2. Balanced growth equilibrium

A *dynamic competitive equilibrium* (DCE) is one such that (i) each representative agent of every generation chooses her schooling effort when young, her consumption and investment in her child's education when old, and her residential location when old, to maximize her utility subject to the budget constraint (8), production technology (5), and the human capital evolution process given by (2) and (4); (ii) the government budget is balanced as in (9); and, (iii) the average schooling effort and individual schooling effort are identical (i.e., $\bar{s}^t = s^t$ for all t). It is not difficult to verify that combining individual and government budget constraints will imply a periodic goods market clearance.[7]

A BGP can now be defined as a DCE along which (i) children's schooling effort and the share of parents' resources invested in children's education are constant; (ii) human capital, output and consumption grow at constant rates; and, (iii) there are no migrations across generations (i.e., $z^t_{t+1}(a^t) = z^{t-1}_t(i^{t-1}) = z(a)$). Under constant returns, one can easily see that, along a BGP, human capital, output and consumption all grow at the same balanced growth rate g, which can be computed

[7] Our DCE can thus be viewed as a dynamic version of the multi-class competitive spatial equilibrium defined in Hartwick et al. (1976).

from (2) as

$$1+g=\frac{h_{t+1}^{t}}{h_{t}^{t-1}}=\beta\left(1-\frac{\phi s^{\varepsilon}v^{1-\varepsilon}}{h/Q}\right)^{-1} \tag{12}$$

where we denote the balanced growth value of h_t^t as h (and hence the balanced growth value of h_{t+1}^t is $(1+g)h$). A BGP is *nondegenerate* if the common balanced growth rate is positive (i.e., $g>0$).

Substituting the above relationships into (10) and (11) yields:

$$\Psi(s)\theta(1-\tau^{\ell})(1-v)(1-s)^{\theta-1}\left(\frac{h}{Q}\right)^{\theta-1}\left(\frac{K}{Q}\right)^{1-\theta}\left[\frac{h}{Q}-(1-s)\phi\varepsilon s^{\varepsilon-1}v^{1-\varepsilon}\right]=\alpha\beta\phi\varepsilon s^{\varepsilon-1}v^{1-\varepsilon} \tag{13}$$

$$\Psi(s)(1-\tau^{\ell})(1-s)^{\theta}\left(\frac{h}{Q}\right)^{\theta}\left(\frac{K}{Q}\right)^{1-\theta}=\alpha\phi(1-\varepsilon)s^{\varepsilon}v^{-\varepsilon} \tag{14}$$

Combining (13) and (14), we have

$$(1-\varepsilon)\theta(1-v)(1-s)^{-1}\left(\frac{h}{Q}\right)^{-1}\left[\frac{h}{Q}-(1-s)\phi\varepsilon s^{\varepsilon-1}v^{1-\varepsilon}\right]=\beta\varepsilon s^{-1}v \tag{15}$$

or, after simplification,

$$(1-\varepsilon)\theta\left(\frac{s}{1-s}-\frac{\varepsilon}{h/Q}\right)=\beta\varepsilon\frac{v}{1-v} \tag{16}$$

Under a uniform distribution of human capital (and without locational choice), we can use (3) and (6) to compute

$$Q=sh \quad \text{and} \quad K=(1-s)h$$

which implies

$$h/Q=1/s \quad \text{and} \quad K/Q=(1-s)/s \tag{17}$$

While the cohort-wide aggregate human capital rises with the schooling effort, the colleague-wide aggregate human capital is increasing in the work effort and hence decreasing in the schooling effort. Thus, along a BGP, the schooling effort affects the two peer group factors oppositely.

Substituting (17) into (14) and (16) we obtain two fundamental balanced growth relationships:

$$\frac{s^{1+\varepsilon}}{(1-s)\Psi(s)}=\frac{(1-\tau^{\ell})}{\alpha\phi(1-\varepsilon)}v^{\varepsilon} \tag{18}$$

$$s\left(\frac{1}{1-s}-\varepsilon\right)=\frac{\beta\varepsilon}{(1-\varepsilon)\theta}\frac{v}{1-v} \tag{19}$$

For illustrative purposes, we shall refer to (18) as the *IA* locus and to (19) as the *equilibrium choice* (*EC*) locus. The *IA* locus highlights parents' trade-off between their own consumption and investment in their children's education, whereas the *EC* locus summarizes both equilibrium consumption–children education investment choice and equilibrium schooling-work effort choice. While it is clear that s and v are positively related along the *EC* locus, such a relation is ambiguous for the *IA* locus. When the direct external effect on productivity (via Ψ) dominates the peer group effects in production (via Q and K) and the complementarity effect in human capital accumulation, s and v are negatively related along the *IA* locus; otherwise, they are positively related.

We rewrite (18) to get:

$$v=\left[\frac{\alpha\phi(1-\varepsilon)}{1-\tau^{\ell}}\frac{s^{1+\varepsilon}}{(1-s)\Psi(s)}\right]^{1/\varepsilon} \tag{20}$$

which can be substituted into (19) to derive the BGP value of the schooling effort:

$$\Lambda(s)\equiv s\left(\frac{1}{1-s}-\varepsilon\right)\left\{\left[\frac{(1-\tau^{\ell})}{\alpha\phi(1-\varepsilon)}\frac{(1-s)\Psi(s)}{s^{1+\varepsilon}}\right]^{1/\varepsilon}-1\right\}-\frac{\beta\varepsilon}{(1-\varepsilon)\theta}=0 \tag{21}$$

Then, substituting (17) into (12) gives the balanced growth rate,

$$g=\beta(1-\phi s^{1+\varepsilon}v^{1-\varepsilon})^{-1}-1 \tag{22}$$

Thus, the balanced growth rate of the economy is increasing in both children's schooling effort and parents' investment in children's education.

It is useful to observe that under our separable quasi-linear utility function specification, both (20) and (21) are not affected by the LPG preference parameter, γ, or the commuting cost parameter, μ. As a consequence, schooling/work effort, education spending share, individual/aggregate human capital, and output are all independent of γ and μ. However,

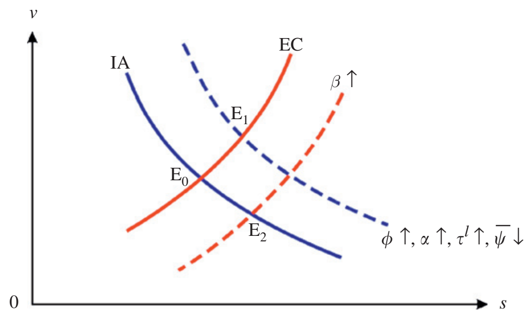

Fig. 1. Comparative statics – Case 1.

through locational choice and the commuting cost incurred, μ, will have a long-run effect on individual consumption, as it can be seen from (8). Moreover, both parameters will affect the balanced growth value of the lifetime utility, while γ will affect the equilibrium configuration as a result of individual locational choice.

3.3. Comparative statics

Utilizing (18), (19) and (22), we can perform a comparative-static analysis. More specifically, we examine how children's schooling effort (s), parents' education spending share (v), and the balanced growth rate (g) responds to changes in: (i) the degree of altruism (α), (ii) the rate of intergenerational human capital spillover (β), (iii) the scaling factor for incremental human capital accumulation from schooling (ϕ), (iv) the exogenous component of the productivity scaling factor ($\overline{\psi}$), and (v) the income tax rate (τ^{ℓ}).

It is useful to plot (18) and (19) in (v, s) space, labelled as the *IA* and the *EC* loci, respectively, in Fig. 1. As discussed above, the *EC* locus is always upward-sloping, whereas the *IA* locus need not be. When both loci are upward-sloping, the *EC* locus must be steeper than the *IA* locus, to satisfy the Samuelson Correspondence Principle (in the sense that an autonomous increase in the net marginal benefit of v raises its balanced growth value). Thus, we have only two possible outcomes: (Case 1) downward-sloping *IA* locus and (Case 2) upward-sloping *IA* locus whose slope is flatter than that of the *EC* locus. We perform a comparative-static analysis and report and the results under Cases 1 and 2 in Figs. 1 and 2, respectively.

Consider, for example, an increase in the scaling parameter in human capital formation (ϕ): it raises the marginal benefit of investment in a child's education, shifting the *IA* locus upward without affecting the *EC* locus. In both cases, s and v rise (see Figs. 1 and 2; the BGP equilibrium shifts from E_0 to E_1). It is obvious that an increase in the marginal benefit of investment in a child's education raises such investment. Because a parent's investment in her child's education and her child's own schooling effort are complementary in human capital formation, the equilibrium level of schooling is higher as well. Because g depends positively on v and s and because ϕ has a direct positive effect on g, the rate of growth increases unambiguously. An increase in α or τ^{ℓ}, or a decrease in $\overline{\psi}$, produces similar effects on s, v and g along the BGP (though these parameters have no direct growth effect). Intuitively, higher α encourages intergenerational human capital accumulation, whereas lower $\overline{\psi}$ decreases the marginal benefit to work and hence the opportunity cost of schooling. Since tuition is tax-exempt, a higher τ^{ℓ} reduces the opportunity cost of a parent's investment in a child's education. This outweighs the disincentive effect on work effort, leading to higher growth.[8]

In contrast, the balanced growth effects of intergenerational human capital spillover parameter (β) depend crucially on the slope of the *IA* locus. An increase in β raises the marginal benefit of schooling, shifting the *EC* locus rightward without affecting the *IA* locus. In Case 1 where the *IA* locus is downward-sloping, s rises but v falls (see Fig. 1; the BGP equilibrium shifts from E_0 to E_2). While its positive effect on schooling is straightforward, the response of a parent's investment in her child's education is generally ambiguous. When the direct external effect on productivity dominates the peer group effects in production and the complementarity effect in human capital accumulation, a parent's investment in her child's education and her child's schooling effort become gross substitutes in aggregate production (Case 1). A higher schooling effort is therefore accompanied by less investment in a child's education. Conversely, in Case 2 where the *IA* locus is upward-sloping, a parent's investment in her child's education and her child's schooling effort are gross complements, so both s and v increase. A stronger intergenerational human capital spillover also has a direct positive effect on economic growth. When this direct effect is strong, one expects a positive balanced growth effect of β regardless of the slope of the *IA* locus.

So far, we have obtained some useful analytical results to guide our intuition. To further characterize the equilibrium, we will conduct numerical analysis to which we now turn.

[8] The reader is reminded that, even with this positive growth effect, higher τ^{ℓ} lowers consumption. These opposing effects give the possibility for an interior welfare-maximizing tax rate (see Section 4.1).

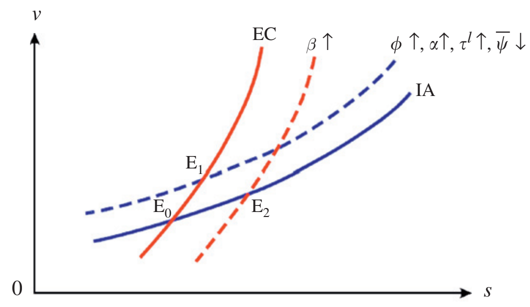

Fig. 2. Comparative statics – Case 2.

Table 1
Parametrization under homogeneous preference.

Parameter		Benchmark value
A. Selected		
LPG Preference	γ	8
Utility constant adjustment	ζ	8
Human capital formation scaling factor	ϕ	0.0225
Human capital accumulation elasticity of schooling	ε	0.8
Production scaling factor	$\overline{\psi}$	5
Schooling spillover elasticity	η	0.85
Output elasticity of effective labor	θ	0.75
Unit commuting cost	μ	0.05
Income tax rate	τ	0.2
B. Calibrated		
Degree of altruism	α	0.125
Intergenerational human capital spillovers	β	1.447
Autonomous schooling spillover parameter	ψ_0	0.2
Schooling spillover scaling factor	ψ_1	1.4433

4. Characterization of balanced growth equilibrium

To perform numerical analysis, we further specify the functional form of $\psi : \psi(\bar{s}) = \ln(\psi_0 + \psi_1 \bar{s}^{\eta})$, where $\psi_0 > 0$, $\psi_1 > 0$, and $\eta \in (0, 1)$. The assumption of positive ψ_0 is to guarantee positive output even without direct productivity externality, whereas ψ_1 measures the degree of the external productivity effect of schooling.

In the benchmark case, the unit commuting cost μ is assumed to be 5%. We set the utility constant adjustment term at $\zeta = 8$, which is sufficient for $\zeta + \ln G_{t+1} > 0$ in all numerical exercises. We choose children's human capital accumulation elasticity of their own schooling effort to be $\varepsilon = 0.80$, implying that children's human capital accumulation uses their own schooling effort more intensively. We select the output elasticity of effective labor as $\theta = 0.75$, implying that goods production uses workers' own effective labor more intensively. We further set the external productivity scaling parameter, the schooling spillover elasticity, and the human capital accumulation scaling factor at $\overline{\psi} = 5$, $\eta = 0.85$ and $\phi = 0.0225$, respectively, to ensure the existence of a unique BGP.

In the case where all agents are homogeneous, we choose $\gamma = 0.000023849$ such that the community welfare-maximizing income tax rate is $\tau^{\ell} = \tau = 20\%$ (recall that the existence of a positive welfare maximizing tax rate is due to its encouragement effect for parents to invest in children's education). We then select the degree of altruism $\alpha = 0.125$ to yield a consumption share about 70% ($c/y = 0.6923$ in the benchmark case). Finally, we choose $\beta = 1.447$ (the degree of intergenerational human capital spillovers), $\psi_0 = 0.2$ and $\psi_1 = 1.4433$ so that the balanced growth rate is about 45.25% (implying an annual rate $\widehat{g} = 1.5\%$ based on 25 years per generation), children's schooling effort is about 0.5 (with effort equally distributed over the entire lifetime, this implies the young only undertake education whereas the old only work), and parents' education spending share falls in the range of 5 to 10% ($v = 0.0721$ in the benchmark case).

All benchmark parameter selections are listed in Table 1.

We begin by illustrating the special case with homogeneous agents. We then allow for two types of agents differing in their preferences toward the LPG and determine whether the equilibrium is *integrated* (different agents mixed in one location) or *stratified* (different agents segregated by locations).

4.1. Homogeneous preferences and integrated equilibrium

With homogeneous preferences, the BGP equilibrium is integrated where agents are mixed in regions A and B with population in each region equal to $\frac{1}{2}$. To ensure symmetry in this integrated equilibrium, we assume that the LPG is at the border between the two regions and there are only intraregional commuting costs incurred (i.e., $\delta = 0$).

From (5) and (7), we have

$$\frac{Y^\ell}{h} = \frac{1}{2}\Psi(s)(1-s) = \frac{1}{2}\frac{Y}{h} = \frac{1}{2}\frac{y}{h}$$

which can be substituted into (8) to obtain

$$\frac{c}{h} = [(1-\tau)(1-v) - \mu]\Psi(s)(1-s)$$

Also, the LPG-human capital ratio is

$$\frac{G}{h} = \tau\Psi(s)(1-s)$$

Using the benchmark parameter values, the annual balanced growth rate ($\widehat{g}$) is 1.504477%, children's schooling effort (s) is 49.9957%, parents' education spending share (v) is 7.2123%, output per effective unit of human capital (Y/h) is 0.165444%, and the consumption–output ratio (c/y) is 69.2302%. As mentioned above, the community welfare maximizing income tax rate under the benchmark parameter values is 20%. In this benchmark case, the IA locus is downward-sloping (i.e., Case 1 as illustrated in Fig. 1).[9]

In order to gain insights toward understanding the working of the model, we characterize how changes in preference (α), human capital accumulation (β, ϕ), production ($\overline{\psi}$, ψ_0, ψ_1), and policy (τ) parameters affect a selected array of endogenous variables of interest: (i) children's schooling effort (s), (ii) parents' education spending share (v), (iii) (annual) growth rate ($\widehat{g}$), (iv) effective output (Y/h), (v) the consumption–output ratio (c/y), and (vi) effective LPG provision (G/h). For completeness, we also examine the effects of these parameter changes on lifetime utility evaluated at $h = 1$ (denoted U). Table 2 summarizes these comparative-static results, in which we consider changes of parameter values by ±50% and ±25% (with percentage changes reported in parentheses).[10]

Recall that an increase in α, ϕ, τ, or a decrease in $\overline{\psi}$ (or ψ_0 or ψ_1 under our functional form specification) create positive long-run effects on s, v and g. Our numerical analysis shows that v and s respond more sensitively to changes in ψ_0 and ψ_1, while ϕ and ψ_1 have stronger growth effects (on g or $\widehat{g}$) and α has stronger welfare effects. Theoretically, a lower β creates a positive long-run effect on s but its effect on v and g is ambiguous. Our numerical results suggest that, under Case 1, v responds negatively though it does not dominate the direct growth effect of β – so a higher degree of intergenerational human capital spillovers is growth-enhancing. Generally speaking, most of the endogenous variables (except s) are very sensitive to changes in β.

Furthermore, the effects of an increase in α, ϕ, τ, $\overline{\psi}$, or β are all to increase effective output and effective LPG provision. An increase in α or ϕ encourages intergenerational savings in kind, thus lowing effective consumption. However, higher $\overline{\psi}$ or β results in a lower parents' education spending share (v) and reduces human capital proportionately more than consumption, thus raising effective consumption.

Finally, it is informative to examine how the community welfare maximizing income tax rate (τ*) changes in response to changes in the preference toward the LPG (γ) from one-tenth to ten times of its benchmark value $\gamma_0 = 0.000023849$:

γ	$0.1\gamma_0$	$0.5\gamma_0$	γ_0	$5\gamma_0$	$10\gamma_0$
τ*	2.4%	11.1%	20.0%	55.6%	71.4%

Thus, the more agents prefer to consume the LPG, the more willing they are to pay the local income tax and hence the higher the welfare maximizing tax rate is.

4.2. Heterogeneous preferences as the source of stratification

We now return to the case of interest, with two types of agents ($m = H, L$) of equal size differing in their preferences toward the LPG provided only in region A, where $\gamma^H = 10\gamma_0$ and $\gamma^L = 0.1\gamma_0$. The BGP equilibrium may be completely stratified with each region populated by only one type of agents, or completely integrated with all agents mixed in one of the two regions (A or B). Further assume that the inter-regional unit commuting cost is twice as much as the intraregional

[9] We may change some parameter values to produce an upward-sloping IA locus (i.e., Case 2). However, the BGP value of schooling effort in this case is too high (usually 90% or above). We therefore regard Case 1 as our benchmark.

[10] Such large ranges of parameter values are chosen in part to serve for the purposes of sensitivity analysis – to illustrate the robustness of our numerical findings.

Table 2
Comparative statics of balanced growth equilibrium.

Parameter	s	v	$\hat{g}$	Y/h	c/y	G/h	U
Benchmark	0.499957	0.072123	0.01504477	0.00165444	0.692302	0.000330888	0.1832406
$\alpha \cdot 0.5$	0.499713	0.072036	0.01504460	0.00082729	0.692371	0.000165459	0.0916108
	(−0.0488)	(−0.1206)	(−0.0011)	(−49.9958)	(0.0100)	(−49.9955)	(−50.00052)
$\alpha \cdot 0.75$	0.499835	0.072080	0.01504469	0.00124089	0.692336	0.000248177	0.1374255
	(−0.0244)	(−0.0596)	(−0.0005)	(−24.9964)	(0.0049)	(−24.9967)	(−25.0027)
$\alpha \cdot 1.25$	0.500078	0.072167	0.01504486	0.00206796	0.692267	0.000413592	0.2290542
	(0.0242)	(0.0610)	(0.0006)	(24.9946)	(−0.0051)	(24.9946)	(25.0019)
$\alpha \cdot 1.5$	0.500200	0.072210	0.01504495	0.00248145	0.692232	0.000496289	0.2748669
	(0.0486)	(0.1206)	(0.0012)	(49.9873)	(−0.0101)	(49.9870)	(50.0033)
$\beta \cdot 0.5$	Degenerate balanced growth equilibrium						
$\beta \cdot 0.75$	0.499864	0.093864	0.00343959	0.00133957	0.674908	0.000267914	0.1371861
	(−0.0186)	(30.1443)	(−77.1376)	(−19.0318)	(−2.5125)	(−19.0318)	(−25.1333)
$\beta \cdot 1.25$	0.500045	0.058569	0.02413900	0.00195486	0.703145	0.000390972	0.2294384
	(0.0176)	(−18.7929)	(60.4478)	(18.1584)	(1.5662)	(18.1584)	(25.2116)
$\beta \cdot 1.5$	0.500130	0.049310	0.03163012	0.00224406	0.710552	0.000448813	0.2757753
	(0.0346)	(−31.6307)	(110.2400)	(35.6386)	(2.6361)	(35.6389)	(50.4990)
$\phi \cdot 0.5$	0.499713	0.072036	0.01496695	0.00082729	0.692371	0.000165459	0.1820430
	(−0.0488)	(−0.1206)	(−0.5173)	(−49.9958)	(0.0100)	(−49.9955)	(−0.6536)
$\phi \cdot 0.75$	0.499835	0.072080	0.01500582	0.00124089	0.692336	0.000248177	0.1826426
	(−0.0244)	(−0.0596)	(−0.2589)	(−24.9964)	(0.0049)	(−24.9967)	(−0.3263)
$\phi \cdot 1.25$	0.500078	0.072167	0.01508381	0.00206796	0.692267	0.000413592	0.1838383
	(0.0242)	(0.0610)	(0.2595)	(24.9946)	(−0.0051)	(24.9946)	(0.32623)
$\phi \cdot 1.5$	0.500200	0.072210	0.01512293	0.00248145	0.692232	0.000496289	0.1844363
	(0.0486)	(0.1206)	(0.5195)	(49.9873)	(−0.0101)	(49.9870)	(0.6525)
$\psi_0 \cdot 0.5$	0.574267	0.103346	0.01510380	0.00159222	0.667323	0.000318444	0.1833858
	(14.8633)	(43.2913)	(0.3924)	(−3.7608)	(−3.6081)	(−3.7608)	(0.0792)
$\psi_0 \cdot 0.75$	0.536918	0.086408	0.01507262	0.00162788	0.680874	0.000325577	0.1833121
	(7.3928)	(19.8064)	(0.1851)	(−1.6054)	(−1.6507)	(−1.6051)	(0.0390)
$\psi_0 \cdot 1.25$	0.463403	0.060032	0.01502001	0.00167130	0.701974	0.000334259	0.1831695
	(−7.3114)	(−16.7644)	(−0.1646)	(1.0191)	(1.3971)	(1.0188)	(−0.0388)
$\psi_0 \cdot 1.5$	0.427279	0.049772	0.01499809	0.00167774	0.710182	0.000335480	0.1830972
	(−14.5369)	(−30.9901)	(−0.3103)	(1.4083)	(2.5827)	(1.3878)	(−0.0783)
$\psi_1 \cdot 0.5$	Degenerate balanced growth equilibrium						
$\psi_1 \cdot 0.75$	0.701619	0.188287	0.01523662	0.00141306	0.599371	0.000282612	0.1836692
	(40.3359)	(161.0637)	(1.2752)	(−14.5898)	(−13.4235)	(−14.5898)	(0.2339)
$\psi_1 \cdot 1.25$	0.384454	0.039457	0.01497518	0.00167053	0.718434	0.000334106	0.1830064
	(−23.1026)	(−45.2921)	(−0.4626)	(0.9725)	(3.7747)	(0.9725)	(−0.1278)
$\psi_1 \cdot 1.5$	0.310200	0.025450	0.01494280	0.00161227	0.729640	0.000322453	0.1828252
	(−37.9547)	(−64.7131)	(−0.6778)	(−2.5489)	(5.3933)	(−2.5492)	(−0.2267)
$\overline{\psi} \cdot 0.5$	0.500444	0.072297	0.01504512	0.00165415	0.692162	0.000330830	0.1832415
	(0.0974)	(0.2413)	(0.0023)	(−0.0175)	(−0.0202)	(−0.0175)	(0.0005)
$\overline{\psi} \cdot 0.75$	0.500119	0.072181	0.01504489	0.00165435	0.692255	0.000330869	0.1832409
	(0.0324)	(0.0804)	(0.0008)	(−0.0054)	(−0.0068)	(0.0057)	(0.0002)
$\overline{\psi} \cdot 1.25$	0.499859	0.072088	0.01504470	0.00165450	0.692329	0.000330910	0.1832404
	(−0.0196)	(−0.0485)	(−0.0005)	(0.0036)	(0.0039)	(0.0066)	(−0.0001)
$\overline{\psi} \cdot 1.5$	0.499795	0.072065	0.01504466	0.00165454	0.692348	0.000330907	0.1832403
	(−0.0324)	(−0.0804)	(−0.0007)	(0.0060)	(0.0066)	(0.0057)	(−0.0002)
$\tau \cdot 0.5$	0.499903	0.072104	0.01504474	0.00147064	0.785107	0.000147064	0.1832344
	(−0.0108)	(−0.0263)	(−0.0002)	(−11.1095)	(13.4053)	(−55.5547)	(−0.00034)
$\tau \cdot 0.75$	0.499928	0.072113	0.01504475	0.00155714	0.738704	0.000233571	0.1832393
	(−0.0058)	(−0.0139)	(−0.0001)	(−5.8811)	(6.7026)	(−29.4109)	(−0.007)
$\tau \cdot 1.25$	0.499989	0.072135	0.01504480	0.00176472	0.645899	0.000441179	0.1832395
	(0.0064)	(0.0166)	(0.0002)	(6.6657)	(−6.7027)	(33.3318)	(−0.0006)
$\tau \cdot 1.5$	0.500026	0.072148	0.01504482	0.00189074	0.599496	0.000567223	0.1832364
	(0.0138)	(0.0347)	(0.0003)	(14.2828)	(−13.4054)	(71.4245)	(−0.0023)

Note: Percentage changes (in %) are reported in parentheses.

Table 3
Balanced growth equilibrium with heterogeneous preferences.

Parameter	s	v	$\hat{g}$	Y/h	c/y	G/h	U
Benchmark	0.499957	0.072123	0.015044770	0.00165444	0.692302	0.000330888	0.1832406
Value of $\tau^A-\tau^B$							
15%	0.500007	0.072141	0.014044775	0.00166911	0.622709	0.000666725	0.1832057
	0.499915	0.072108					
20%	0.500026	0.072148	0.014044780	0.00168069	0.599510	0.000488257	0.1831847
	0.499902	0.072104					
25%	0.500047	0.072155	0.014044783	0.00160583	0.576311	0.000184745	0.1831630
	0.499891	0.072100					

Note: For s and v, we report two equilibrium values with those of the high-type above and those of the low-type below. See also Table 2.

unit commuting cost (i.e., $\delta = 1$). Consider the income tax rates prevailed in the two regions as

$$\tau^A = (1+\rho)\times 0.2 \quad \text{and} \quad \tau^B = (1-\rho)\times 0.2$$

where $\rho \in (0.025, 0.75)$, implying that the range of tax rate differentials between the two regions is from 1% to 30% (which covers most of commonly observed tax rate differentials across different locations).

By evaluating the representative cohort's human capital when young at $h = 1$, the individual human capital stock of an agent of type-m when old is $1+g$, where $g = \beta(1-\phi s^{1+\varepsilon}v^{1-\varepsilon})^{-1} - 1$ is constant across *types* because γ has no effect on s or v. However, income tax rates are different across *regions*, so the balanced growth rates prevailed in the two regions are different (denoted by g^A and g^B). The society's aggregate human capital stock is therefore given by

$$H = \frac{1}{2}\sum_{\ell=A,B}(1+g^\ell) \equiv 1+\overline{g}$$

where $\overline{g}$ measures the average growth rate of the society along the BGP.

There are three possible equilibrium configurations:

(Case I) completely stratified equilibrium: all high-type agents reside in region A and all low-type in B;
(Case II) completely integrated equilibrium:
  (A) all agents, regardless of their types, reside in region A,
  (B) all agents, regardless of their types, reside in region B.

It is noted that in the absence of land usage and population crowding, there does not exist a completely integrated equilibrium with half of the agents residing in each region, or an incompletely mixed equilibrium where one region is populated with both types of agents while another is with only one type of agents.[11]

In Table 3, we summarize equilibrium values of key indicators under each representative case with $\tau^A-\tau^B = 15\%, 20\%, 25\%$ that correspond to Case IIA, Case I, and Case IIB, respectively.

4.2.1. Case I

In this case, regional and aggregate outputs in effective units can be derived as

$$\frac{Y^\ell}{h^\ell} = \frac{1}{2}\psi(s^\ell)(1-s^\ell)$$
$$\frac{Y}{H} = \sum_{\ell=A,B}\frac{h^\ell}{H}\frac{Y^\ell}{h^\ell} = \frac{1}{2}\sum_{\ell=A,B}\left(\frac{1+g^\ell}{1+\overline{g}}\right)\psi(s^\ell)(1-s^\ell)$$

The provision of the LPG in effective units is

$$G = \sum_{\ell=A,B}\tau^\ell Y^\ell = (\tau^A+\tau^B)(1+\overline{g})\frac{Y}{H}$$

[11] The reader may refer to Peng and Wang (2005) for a discussion of such cases.

Individual consumption for agents residing in region ℓ becomes

$$\frac{c^\ell}{h^\ell} = \frac{1}{2}\left\{\left[(1-\tau^\ell)(1-v^\ell) - \frac{\mu \cdot I}{2}\right]\psi(s^\ell)(1-s^\ell) - \frac{\mu \cdot I}{2}\left(\frac{1+g^{\ell'}}{1+g^\ell}\right)\psi(s^{\ell'})(1-s^{\ell'})\right\}$$

where $\ell' \neq \ell$. The corresponding lifetime utility is hence given by

$$U^\ell = (1+g^\ell)\frac{c^\ell}{h^\ell} + \gamma^\ell(\zeta + \ln G) + \alpha(1+g^\ell)$$

4.2.2. Case IIA

In this case, no one resides in region B, so regional and aggregate outputs in effective units become

$$\frac{Y^A}{h^A} = \frac{Y}{H^A} = \frac{1}{2}\sum_{m=H,L}\left(\frac{1+g^m}{1+\overline{g}^A}\right)\psi(s^m)(1-s^m)$$

where $1+\overline{g}^A = \frac{1}{2}\sum_{m=H,L}(1+g^m)$ and $H^A = 1+\overline{g}^A$. The provision of the LPG in effective units is

$$G = \tau^A(1+\overline{g}^A)\frac{Y}{H^A}$$

Since no one commutes interregionally, individual consumption for agents of type m can be derived as

$$\frac{c^m}{h^m} = \frac{1}{2}\left\{\left[(1-\tau^A)(1-v^m) - \frac{\mu}{2}\right]\psi(s^m)(1-s^m) - \frac{\mu}{2}\left(\frac{1+g^{m'}}{1+g^m}\right)\psi(s^{m'})(1-s^{m'})\right\}$$

where $m' \neq m$.

4.2.3. Case IIB

Since no one resides in region A, regional and aggregate outputs in effective units can then be specified as

$$\frac{Y^B}{h^B} = \frac{Y}{H^B} = \frac{1}{2}\sum_{m=H,L}\left(\frac{1+g^m}{1+\overline{g}^B}\right)\psi(s^m)(1-s^m)$$

where $1+\overline{g}^B = \frac{1}{2}\sum_{m=H,L}(1+g^m)$ and $H^B = 1+\overline{g}^B$. The provision of the LPG in effective units becomes

$$G = \tau^B(1+\overline{g}^B)\frac{Y}{H^B}$$

Now, everyone commutes interregionally, so individual consumption for agents of type m is

$$\frac{c^m}{h^m} = \frac{1}{2}\left\{\left[(1-\tau^B)(1-v^m) - \frac{\mu(1+\delta)}{2}\right]\psi(s^m)(1-s^m) - \frac{\mu(1+\delta)}{2}\left(\frac{1+g^{m'}}{1+g^m}\right)\psi(s^{m'})(1-s^{m'})\right\}$$

where $m' \neq m$.

4.2.4. Equilibrium configuration

Given $(\gamma^H, \gamma^L) = (10\gamma_0, 0.1\gamma_0)$, we can now inquire under what tax rate differential a particular equilibrium configuration emerges. Our numerical analysis implies

	$\rho < 0.4573$	$0.4573 < \rho < 0.5612$	$\rho > 0.5612$
Tax rate differential	$\tau^A - \tau^B < 18.29\%$	$18.29\% < \tau^A - \tau^B < 22.45\%$	$\tau^A - \tau^B > 22.45\%$
Equilibrium configuration	completely integrated with all agents in A	completely stratified	completely integrated with all agents in B

When the tax rate differential is sufficiently small, the utility cost of the extra tax payment by residing in the LPG site is less than that of the interregional commuting cost, thus resulting a completely integrated equilibrium with all agents residing in region *A*. When the tax rate differential is sufficiently large, the converse is true where all agents reside in region *B*. When the tax rate differential takes intermediate values, agents are completely stratified where those with higher valuation of the LPG reside in region *A* and those with lower valuation in *B*. We find that as the tax rate differential increases, the differentials in parental investment in child's education and in child's own schooling effort between the high and the low-type are both widened. The case with all agents residing in region *A* features the lowest economy-wide growth but the highest values of effective output, effective consumption and effective provision of the LPG, as well as the highest welfare. The case with all agents residing in region *B* attains the highest growth but the lowest welfare. The completely stratified case has intermediate value in both growth and welfare, compared to the two completely integrated cases.

Table 4
Comparative statistics under heterogeneous preferences.

Parameter	ρ_1	$\tau_1^A-\tau_1^B$	ρ_2	$\tau_2^A-\tau_2^B$
Benchmark	0.457264	0.182906	0.561236	0.224494
$\alpha \cdot 0.9$	0.457380	0.182952	0.610382	0.244153
	(0.0254)	(0.0251)	(8.7567)	(8.7570)
$\alpha \cdot 1.1$	0.457171	0.182868	0.530680	0.212272
	(−0.0203)	(−0.0208)	(−5.4444)	(−5.4442)
$\beta \cdot 0.9$	0.461705	0.184682	0.695941	0.278376
	(0.9712)	(0.9710)	(24.0015)	(24.0015)
$\beta \cdot 1.1$	0.453657	0.181463	0.507784	0.203113
	(−0.7888)	(−0.7889)	(−9.5240)	(−9.5241)
$\phi \cdot 0.9$	0.457380	0.182952	0.610611	0.244244
	(0.0254)	(0.0251)	(8.7975)	(8.7976)
$\phi \cdot 1.1$	0.457170	0.182868	0.530574	0.212229
	(−0.0206)	(−0.0208)	(−5.4633)	(−5.4634)
$\psi_0 \cdot 0.9$	0.460363	0.184145	0.569412	0.227765
	(0.6777)	(0.6774)	(1.4568)	(1.4571)
$\psi_0 \cdot 1.1$	0.454403	0.181761	0.554141	0.221657
	(−0.6257)	(−0.6260)	(−1.2642)	(−1.2637)
$\psi_1 \cdot 0.9$	0.473260	0.189304	0.609291	0.243717
	(3.4982)	(3.4980)	(8.5624)	(8.5628)
$\psi_1 \cdot 1.1$	0.447845	0.179138	0.539643	0.215857
	(−2.0599)	(−2.0601)	(−3.8474)	(3.8473)
$\overline{\psi} \cdot 0.9$	0.457272	0.182909	0.561270	0.224508
	(0.0017)	(0.0016)	(0.0061)	(0.0062)
$\overline{\psi} \cdot 1.1$	0.457257	0.182903	0.561209	0.224483
	(−0.0015)	(−0.0016)	(−0.0048)	(−0.0049)
$\delta \cdot 0.9$	0.405654	0.162261	0.499490	0.199796
	(−11.2867)	(−11.2872)	(−11.0018)	(−11.0016)
$\delta \cdot 1.1$	0.510654	0.204262	0.628751	0.251500
	(11.6760)	(11.6759)	(12.0297)	(12.0297)

Note: See Table 2.

We turn next to studying how changes in preference, human capital accumulation, production, and interregional commuting parameters affect the underlying equilibrium configurations. We summarize the results below.

Increase in	Likelihood of equilibrium configuration		
	Completely integrated with all agents in A	Completely stratified	Completely integrated with all agents in B
α, β, ϕ, ψ_0, ψ_1 or $\overline{\psi}$	Decrease	Decrease	Increase
δ	Increase	Increase	Decrease

Thus, an economy is more likely to be stratified if (i) the degree of altruism (α) is low, (ii) intergenerational human capital spillovers (β) are small, (iii) human capital accumulation responds to children's schooling effort (ϕ) less sensitively, (iv) the direct productivity effect is low or less sensitive to schooling (ψ_0, ψ_1 or $\overline{\psi}$ low), and (v) interregional commuting (δ) is more costly. From Table 4, one can see that the stratification ranges (both critical values) are very sensitive to changes in the sensitivity of productivity to schooling and the cost of interregional commuting. On the contrary, changes in the degree of altruism, intergenerational human capital spillovers and the response of human capital formation to schooling affect the likelihood of stratification only through their effects on the upper critical value that is associated with the high-type's locational choice. Furthermore, while changes in the direct productivity scaling factor ($\overline{\psi}$) generate a qualitatively similar stratification effect to the sensitivity parameter (ψ_1), such an effect is quantitatively negligible. Overall, stronger intergenerational human capital spillovers or location-specific within-the-cohort spillovers helps reducing economic stratification, whereas stronger location-specific production spillovers result in more segregation.

Intuitively, an increase in α or ϕ encourages parents' investment in a child's education and children's schooling effort, both helping intergenerational mobility and thus reducing the likelihood of economic stratification. An increase in β raises schooling effort but lowers parents' investment in children's education. Since the former effect dominates under the benchmark parametrization, the likelihood of economic stratification decreases. While an increase in $\overline{\psi}$ discourages parents' investment in children's education and children's schooling effort, it enhances the marginal product of labor and raises effective output and consumption. In the benchmark case, the latter effect turns out to be dominant and, as a result, higher wealth mitigates locational segregation.

It is informative to compare our results with those in previous studies, particularly, two closely related papers by Benabou (1996a) and Peng and Wang (2005). Similar to Benabou, local human capital spillovers are crucial for determining whether an economy is stratified. While Benabou models schooling-work trade-off, parents do not make an explicit decision on whether to invest in children's education – parents are taxed and the resulting community tax revenues are in turn used to finance public education. In Benabou's paper, complementarity between individual human capital and the global human capital aggregator in forms of economy-wide knowledge spillovers is the sole underlying driving force of integration. Without economy-wide knowledge spillovers, the equilibrium in this economy is always stratified. In our paper, economy-wide knowledge spillovers are absent, but the economy may still be integrated. Specifically, we consider comparable preferences for the LPG and low interregional commuting costs as key forces for agents of different types to reside with each other in the same region. Moreover, while heterogeneous LPG preferences are incorporated in Peng and Wang, their framework is static, abstracting from any human capital effects. By combining both LPG preference and human capital effects in a dynamic general equilibrium setting, we obtain further insights toward understanding the causes of economic stratification as illustrated above.

5. Concluding remarks

We have examined the conditions under which the balanced growth equilibrium is integrated or stratified. With heterogeneous preferences for the LPG, economic stratification requires that the local income tax rate differential fall in an intermediate range. We also point out that the sustainability of economic stratification (or intergenerational immobility) depends crucially on parental altruism, intergenerational human capital spillovers, the influence of individual schooling effort on human capital accumulation, the peer group effect of schooling effort, and interregional commuting.

Along these lines, there are at least two interesting avenues for future research. One is to characterize the dynamic evolution of the locational equilibrium to analyze the short-run transition of intergenerational mobility. In particular, some determinants mentioned above may have stronger short-run effects while others may be more influential in the long run. Another is to use voting to determine endogenously the local tax rate in each region given the provision of the LPG (as in Konishi, 1996; Nechyba, 1997) or the number of LPG sites given the tax schedules (as in Berliant et al., 2006). An important question to inquire is whether this political equilibrium mechanism may reinforce economic stratification and reduce the degree of intergenerational mobility.

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