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# Ability-Heterogeneity, Entrepreneurship, and Economic Growth

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**Abstract**: This paper develops an endogenous growth model of occupational choice with overlapping generations heterogeneous in entrepreneurial ability. While an increase in the number of entrepreneurs creates a growth-enhancing variety effect, the reduced overall quality of entrepreneurial ability retards growth. As a result, the number of entrepreneurs and output growth need not be positively related, in response to changes in the ability distribution. While cheaper financial operation and higher manufacturing productivity are both growth-enhancing, they have different effects on equilibrium factor prices and equilibrium financial markups. Additionally, the long-run growth consequences of subsidies to entrepreneurship and credit-market imperfections are studied.

JEL Classification: D90, O4.

Keywords: Occupational Choice, Entrepreneurial Ability, Distribution and Growth.



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# 1 Introduction

Entrepreneurship has been widely thought as instrumental in driving modern growth. Recently, Cagetti and De Nardi (2006) document based on the Survey of Consumer Finances that entrepreneurs measured by self-employed business owners account for only 7.6% of the U.S. population but for almost one third of the total net worth. According to Mondragon-Velez (2009), this entrepreneur group receives more than 20% of income of the entire population. Indeed, promoting entrepreneurship has long been a popular idea within policy circles. For example, World Development Report 1991: the Challenge of Development, published by the World Bank, has a whole chapter (Chap. 4: the Climate for Enterprise) devoted to the importance of entrepreneurship in development. In this chapter, it is stated that "...But the key to rapid development is the entrepreneur. Governments need to serve enterprise, large and small, ..." (WDI '91, p.70). The theoretical argument can be traced back to Schumpeter (1911) who emphasized that by allowing for a sufficiently high rent, active entrepreneurship can foster long-run economic growth. Yet, a recent paper by Blanchflower (2000) delivers a striking empirical evidence: across OECD countries, the degree of entrepreneurship (measured by the number of self-employed) need not be positively related to the rate of economic growth. One may therefore wonder whether there is a robust base to hypothesize entrepreneur-driven growth in the process of economic advancement.

To explore this issue, one must understand the major factors determining entrepreneurial choice. In a now-classic paper by Kihlstrom and Laffont (1979), three such factors are highlighted: (i) entrepreneurial ability versus labor skills, (ii) access to capital markets and (iii) individual attitude towards risk. In this paper, we will focus on the first aspect, by developing a dynamic general-equilibrium model with endogenous occupational choice by agents heterogeneous in their entrepreneurial ability. In an extension to our benchmark setup, we will incorporate the considerations allowing individual to have imperfect access to the credit market.[1]

We begin by asking: Just how do entrepreneurial abilities affect occupational choice over time and influence macroeconomic performance in the long run? We then inquire: in response to changes in the ability distribution, do we expect to see a theoretically positive relationship between entrepre-

[1] Although not our main focus, the impact of credit market imperfection on economic growth is also discussed in the present paper (Section 4.3.3). Throughout the paper, we do not model risk attitude. The reader is referred to Kihlstrom and Laffont (1979) and a number of follow-up studies for an analysis of risk attitude and entrepreneurship within the partial equilibrium static framework.

neurship and economic growth highlighted by Schumpeterian theory? If the answer is ambiguous, what are the underlying driving forces that may explain Blanchflower's findings? Moreover, do credit market imperfections necessarily retard entrepreneurial activities and economic growth? By addressing these questions, one gains insights toward understanding the role of entrepreneurial ability and credit market imperfections played in the dynamic process of long-run development.

Specifically, we construct an overlapping-generations model with ability-heterogeneous agents optimally choosing to become workers or entrepreneurs when they are young. We assume that borrowing and lending must all be financially intermediated. Workers work in their youth and deposit the entirety of their wage income in competitive banks. Banks take deposits, employ labor and provide loans to borrowing entrepreneurs. Upon obtaining a loan from a bank, a young entrepreneur of a particular ability type can transform the loan into a capital good. Individual capital goods can then be combined in an imperfectly substitutable manner, together with labor, to produce the final consumption good. With perfect credit markets, as assumed in our benchmark model, agents' differential entrepreneurial ability is the sole force determining the occupational choice between workers and entrepreneurs.

We would like to highlight the key insights generated by our models. Here, occupational choice and the process of transforming loans into capital goods affect capital accumulation and output growth in such a way that the positive relationship between entrepreneurship and growth may be upset. In particular, there are two effects in consideration: (i) a "selectivity effect" as a result of individuals' decision on whether to become an entrepreneur and the overall quality of entrepreneurs as a whole and (ii) a "variety effect" as a result of how effectively the imperfectly substitutable capital goods can be combined into a composite capital for the production of the final good. When the distribution of entrepreneurial ability is fixed, these two effects work in opposite directions: More entrepreneurs are associated with a negative selectivity effect and a positive variety effect on economic growth. Furthermore, there is yet another effect, which we call (iii) a "loanable fund supply effect", further complicating the entrepreneurship-growth relationship. In a general equilibrium setup, the capital forming ability determined by the first two effects must influence workers' productivity, which in turn affects the amount of loanable funds and thus the rate of capital accumulation.

Upon solving the model analytically, we establish the following main findings. First, a uniformly rightward shift in the ability distribution raises the cutoff ability less-than-proportionately and increases the number of entrepreneurs and the equilibrium wage rate. Yet, it creates two conflicting effects on output growth: (i) it makes the production of the composite capital more efficient and (ii) it reduces the size of loanable funds channeled through the banking sector from deposits by manufacturing workers. When the loanable fund supply effect is important, the rate of economic growth need not be higher and hence economic growth and the number of entrepreneurs need not be positively related. Second, a stretch of the distribution in a proportional manner makes the cutoff ability go up proportionately, leaving the fractions of workers and entrepreneurs unchanged. Thus, even if two economies have the same proportion of entrepreneurs, the one with a stochastically dominant entrepreneurial ability distribution features a higher rate of economic growth. Third, while both a reduction in the unit financial operation cost and an improvement in manufacturing productivity are growth enhancing, they have different effects on equilibrium wages, interest rates and financial markups. In particular, a higher manufacturing productivity raises the financial markup whereas a higher banking productivity reduces it. Fourth, policies that aim to encourage entrepreneurship may not enhance long-run growth. Finally, credit market imperfections do lead to fewer entrepreneurs but need not retard economic growth if the selectivity effect is strong compared to the variety effect.

*Related Literature*

Our result that there can be too many entrepreneurs in the competitive equilibrium is related to the literature that questions whether encouraging entrepreneurship is always socially desirable. At least three of such arguments can be found in this literature. First, potential entrepreneurs can be overly optimistic and therefore should not be encouraged. Among several, Astebro (2003) found evidence supporting this view. Second, due to the presence of limited liability and asymmetric information, entrepreneurs with too low an ability can emerge in the competitive equilibrium, as shown in de Meza and Webb (1987). Third, innovators/entrepreneurs may face a higher private return than the social return and as a result, the equilibrium level of innovation/entrepreneurial activity can be too high compared to the socially optimal outcome. Aghion and Howitt (1992) showed such a case in the presence of a strong business-stealing effect when innovators are ex ante competitive and ex post monopolistic. In our paper, however, entrepreneurs are not modeled as ex ante competitive and ex post monopolistic innovators and it is assumed that not only borrowers know their entrepreneurial ability, so do banks. Thus, the argument made in this paper is very different from the three outlined above.

In terms of modeling, this paper can be related to the growing literature on occupational or skill choice, including Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), LLoyd-Ellis (2000), Ghatak and Jiang (2002), and Fender and Wang (2003), to name but a few. Of particular relevance are LLoyd-Ellis and Bernhardt (2000) and Cagetti and De Nardi (2006) in which agents make occupational choice between entrepreneurs and workers. A central theme of most of the papers in this literature is to highlight how the distribution of wealth and the degree of credit constraints influence economic growth and income inequality.

The present paper has some important features that contrast with the setups in the previous literature. The most important feature is our emphasis on heterogeneity in entrepreneurial ability for producing capital goods rather than individual wealth (cf. Aghion and Bolton, Banerjee and Newman, Ghatak and Jiang, LLoyd-Ellis and Bernhardt, and Piketty), or labor skills (cf. Fender and Wang, and LLoyd-Ellis), or setup costs of final goods production (cf. LLoyd-Ellis and Bernhardt), or efficiency in final goods production (cf. Cagetti and De Nardi). This modeling strategy is partly motivated by a recent work of Hurst and Lusardi (2004) where the transition probability into entrepreneurship is inelastic in wealth except for agents at the very top end of the wealth distribution. This feature also enables us to incorporate an interesting variety effect as well as the uncompensated entrepreneurial knowledge spillover; both factors will be shown crucial for driving the long-run relationship between entrepreneurship and economic growth.



Moreover, we explicitly model the process through which entrepreneurs transform loans into productive capitals with and without credit constraints. This enables us to isolate the effects of credit market imperfections from other above-mentioned forces, to better understand the determinants of entrepreneurial choice. Furthermore, we have a banking sector with nontrivial endogenous financial markups due to endogenous labor allocation between manufacturing and banking activities. We also differentiate productivity improvements originated in manufacturing from those in banking, to study their different effects on equilibrium factor prices and financial markups.

## 2 The Basic Environment

There are two sectors in this economy: a final good sector and a banking sector. Time is discrete, indexed by $t$. In addition to an initial old generation at date $t = 0$, the economy consists of an infinite sequence of two-period lived overlapping generations. The population is constant over time, normalized to have a measure of one.

Call one born in period $t$ a $t$-generation agent, who chooses to become a worker or an entrepreneur when young. Each young agent is endowed with one unit of labor and an idea of designing a capital good which can be implemented if he becomes an entrepreneur. All varieties of individual capital goods last only one period (entrepreneur-specific). All agents have identical monotone preferences and value consumption only when they are old and therefore, in the absence of bequest motives, they save the entirety of their income for consumption in the second period of their lifetime.

A young worker can either be employed in the final good sector to manufacture consumables, or in the banking sector to transform deposits into loans. All workers are paid the current competitive wage, and deposit the entire amount into banks for the purpose of future consumption.

A young entrepreneur (i) borrows from a bank and (ii) transforms what he borrowed into a particular capital good which embeds his unique idea; when old, he (iii) sells the capital good to firms in the final good sector and (iv) consumes his entire wealth. The structure of the economy is conveniently depicted in Figure 1 where the timing of events is numerically ordered from 1 to 7.

[Figure 1 here]

## 2.1 Final Good Sector

This sector features a single final good manufactured by a continuum of perfectly competitive firms of mass one. Firms in this sector combine a variety of capital goods and effective labor to produce the final good $Y_{t+1}$ at time $t+1$ according to the following production technology,

$$Y_{t+1} = A[(\int_0^{N_{t+1}} k_{t+1}(i)^\theta di)^{\frac{1}{\theta}}]^\alpha (h_{t+1}L_{t+1})^{1-\alpha}, \tag{1}$$

where $A$ is a productivity scaling parameter; $k_{t+1}(i)$ denotes the amount of an individual capital good produced by entrepreneur $i$ and there are $N_{t+1}$ entrepreneurs (and hence $N_{t+1}$ types of capital goods) in period $t+1$; $L_{t+1}$ is the amount of labor employed; $h_{t+1}$ measures the level of human capital embodied with each worker; $\alpha \in (0,1)$ is the output elasticity of capital as a whole; and $\theta \in (0,1)$ suggests that individual capitals are less than perfectly substitutable but not as complementary as under the Cobb-Douglas technology. It is convenient to denote $K_{t+1}$ as the composite capital which summarizes the usage of individual capital:

$$K_{t+1} \equiv \left[\int_0^{N_{t+1}} k_{t+1}(i)^\theta di\right]^{\frac{1}{\theta}}. \tag{2}$$

Given the wage rate per effective labor ($\omega_{t+1}$) and rental prices for all individual capitals ($p_{t+1}(i)$), we can solve a representative firm's cost minimization problem and derive the manufacturing demands for effective labor, composite capital, and individual capitals, respectively, as:

$$h_{t+1}L_{t+1} = A^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha}\left(\frac{\omega_{t+1}}{P_{t+1}}\right)^{-\alpha}Y_{t+1} \tag{3}$$

$$K_{t+1} = A^{-1}\alpha^{1-\alpha}(1-\alpha)^{\alpha-1}\left(\frac{\omega_{t+1}}{P_{t+1}}\right)^{1-\alpha}Y_{t+1} \tag{4}$$

$$k_{t+1}(i) = \left[\frac{P_{t+1}}{p_{t+1}(i)}\right]^{\frac{1}{1-\theta}}K_{t+1}, \quad \forall i. \tag{5}$$

Notice that $P_{t+1}$ is the rental price of the composite capital which satisfies:

$$P_{t+1} = \left[\int_0^{N_{t+1}} p_{t+1}(i)^{-\frac{\theta}{1-\theta}}di\right]^{-\frac{1-\theta}{\theta}}. \tag{6}$$

Finally, since the final good sector is perfectly competitive, the competitive profit condition implies that the constant marginal cost must be equal to the price of the final good which we normalize to 1:

$$\frac{P_{t+1}^{\alpha}\omega_{t+1}^{1-\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = 1. \tag{7}$$

## 2.2 Workers, Entrepreneurs and Occupational Choice

Although agents of $t$-generation are born with the same level of human capital $h_t$ they are different in two aspects. First, everyone is endowed with a unique idea of designing a capital good but only those who do become entrepreneurs can implement their ideas.[2] As one can see from the previous section, once those ideas are implemented, they are treated symmetrically in our model. Second, we assume agents are born with different ability $\tau$ on transforming the final good into the capital good.

More specifically, if entrepreneur $i$ is endowed with ability $\tau$ and borrows $x_t$ units of final goods from the bank, he is able to convert them into $k_{t+1}(\tau)$ units of the type-$i$ capital good, where

$$k_{t+1}(\tau) = \tau\frac{M_{t+1}}{\Omega_{t+1}^{\zeta}}x_t. \tag{8}$$

where we have dropped the index $i$ to minimize the usage of the notation.[3] Thus, a more able entrepreneur with higher $\tau$ can convert more effectively the loan obtained from the bank into a

[2] In a pivotal study, Romer (1993) illustrates the importance of ideas for fostering economic growth.

[3] The reader, however, should keep in mind that all entrepreneurs, even those with same ability $\tau$, are producing different types of capital goods.

variety specific capital good. Here, $M_{t+1}$ and $\Omega_{t+1}$ are, respectively, an overall quality index and a variety adjustment index of entrepreneurs at date $t+1$, whose functional forms will be specified toward the end of this subsection. The higher the overall quality of entrepreneurs is, the greater the positive spillovers each entrepreneur can benefit from and hence the more productive his capital transformation will be. The variety-embedded production technology (1) naturally implies a positive variety-induced scale effect in which output and marginal product of each capital good increase in the measure of entrepreneurs. The introduction of a general variety adjustment index is therefore to isolate this variety-induced scale effect: the scale effect is fully accounted for with $\zeta = 0$, completely eliminated with $\zeta = 1$, and discounted with $\zeta \in (0, 1)$.

Notably, our setup captures the spirit of Lucas (1978) that the more able can produce more than the less able given the same amount of inputs, though we introduce a direct Marshallian externality and a variety adjustment factor into the entrepreneur's production function. The externality effect captures positive knowledge spillovers emphasized by Romer (1986) and Lucas (1988), as well as average pricing facing ability-heterogeneous entrepreneurs highlighted by Ghatak, Morelli and Sjöström (2006).[4] The variety adjustment follows Young (1998), motivated by the arguments by Jones (1995) that the scale effect obtained in R&D-based variety models is not supported empirically.



Let $\delta_{t+1}$ be the (gross) loan rate prevailed from $t$ to $t+1$. An entrepreneur with ability $\tau$ will then face a marginal cost,

$$MC_t(\tau) = \frac{\delta_{t+1}}{\tau M_{t+1}/\Omega_{t+1}^{\zeta}},$$

of producing a capital good. Since the derived demand for an individual capital good has a constant price elasticity $-\frac{1}{1-\theta}$ (see (5)), the profit-maximizing price must be a constant mark-up over the marginal cost. That is, an entrepreneur with ability $\tau$ must set the price of his capital good as:

$$p_{t+1}(\tau) = \frac{1}{\theta}\frac{\delta_{t+1}}{\tau M_{t+1}/\Omega_{t+1}^{\zeta}}. \tag{9}$$

From (5) and (8), his demand for loans and the resulting entrepreneurial profit will be:

$$x_t(\tau) = \frac{1}{\tau M_{t+1}/\Omega_{t+1}^{\zeta}} \left(\frac{P_{t+1}}{p_{t+1}(\tau)}\right)^{\frac{1}{1-\theta}} K_{t+1}, \tag{10}$$

[4] In an asymmetric information environment where entrepreneurial ability is unknown, the product price and the loan rate both depend on the average quality of entrepreneurs, thereby creating an external effect. This external effect in our perfect information world is in the Marshallian form.

$$\pi_{t+1}(\tau) = (1-\theta)\left(\frac{P_{t+1}}{p_{t+1}(\tau)}\right)^{\frac{\theta}{1-\theta}} P_{t+1}K_{t+1}. \tag{11}$$

Because all agents have identical monotone preference, inelastic labor supply, and consume only when they are old, their intertemporal utility maximization problem reduces to the maximization of lifetime income, which is simply governed by the occupational choice decision. Notice that since all workers earn the same wage income and $\frac{\partial \pi_{t+1}(\tau)}{\partial \tau} > 0$, there must exist a unique cutoff ability $\tau_t^*$ that satisfies

$$\pi_{t+1}(\tau_t^*) = r_{t+1}\omega_t h_t, \tag{12}$$

so that agents with higher ability strictly prefer to become entrepreneurs (see Figure 2). This determines the *occupational choice* of young agents. As a result, the mass of entrepreneurs (in period $t+1$) is:

$$N_{t+1} = \int_{\tau_t^*}^{\tau^H} dF, \tag{13}$$

where $F$ is the distribution function of entrepreneurial ability $\tau$.

Thus, the overall quality index of entrepreneurs ($M_{t+1}$) is specified as:

$$M_{t+1} = \frac{\int_{\tau_t^*}^{\tau^H} \tau dF}{\left(\int_{\tau_t^*}^{\tau^H} dF\right)^{1-\eta}}, \tag{14}$$

where $\eta \in [0,1]$. When $\eta = 1$, the positive spillover of entrepreneurship is the strongest where the overall quality is measured by the aggregate ability of all entrepreneurs. When $\eta = 0$, the overall quality is measured by the average ability of entrepreneurs. This latter case resembles the human capital spillover setup in Lucas (1988) and is regarded as the benchmark specification. Our results will remain qualitatively unchanged for the case of $\eta > 0$, provided that $\eta$ is not too large.

Moreover, the variety adjustment index is given by,

$$\Omega_{t+1} = \left(\int_{\tau_t^*}^{\tau^H} \tau^{\frac{\theta}{1-\theta}} dF\right)^{\frac{1-\theta}{\theta}}. \tag{15}$$

Intuitively, this is weighted entrepreneurial quality index where entrepreneurial ability is weighted by the same factor as the relative price of the entrepreneurial capital good in the profit function as shown in (11). One can now see this is a factor capturing the variety effect: when varieties are perfect substitutes ($\theta = 1$), the variety effect vanishes ($\Omega_{t+1} = 1$).

[Figure 2 here]

Also notice that comparing to the decision made by the marginal entrepreneur, those with a higher ability $\tau$ will set a lower price, produce more individual capital, borrow a greater amount of loans, and earn more profits. Specifically, for any $\tau \geq \tau_t^*$,

$$\begin{aligned} \frac{p_{t+1}(\tau)}{p_{t+1}(\tau_t^*)} &= \frac{MC(\tau)}{MC(\tau_t^*)} = \left(\frac{\tau}{\tau_t^*}\right)^{-1} \\ \frac{k_{t+1}(\tau)}{k_{t+1}(\tau_t^*)} &= \left(\frac{\tau}{\tau_t^*}\right)^{\frac{1}{1-\theta}} \\ \frac{\pi_{t+1}(\tau)}{\pi_{t+1}(\tau_t^*)} &= \frac{x_t(\tau)}{x_t(\tau_t^*)} = \left(\frac{\tau}{\tau_t^*}\right)^{\frac{\theta}{1-\theta}}. \end{aligned} \tag{16}$$

These relationships will be used throughout to simplify various integral expressions.

## 2.3 Banking Sector

We now turn to describing the perfectly competitive banking sector, where banks accept deposits from young workers and lend them to young entrepreneurs. At any time $t$, any worker can elect to form (and own) a bank, operating under a market (gross) deposit rate $r_{t+1}$ prevailing from $t$ to $t+1$ to absorb loanable funds from young workers and a market loan rate $\delta_{t+1}$ to provide loanable funds to potential young entrepreneurs.[5] At time $t+1$, the bank receives the repayment from its borrowers and pays back to depositors. For simplicity, assume such a transformation from deposits into loans is undertaken purely by the bank employees, requiring no capital inputs.

The technology of transforming deposits into loans is Ricardian: in order to convert $X$ units of deposits into $X$ units of loans, a bank must employ $X/\phi$ units of effective labor. That is, we assume zero reserve requirement (or, total deposits equal total loans) and a unit labor requirement $1/\phi$ for bank operation. Parameter $\phi$ serves as a measure of banking productivity. Under perfect competition, a bank must end up with zero profit, which implies the following equality must hold:

$$\delta_{t+1} = \left(1 + \frac{\omega_t}{\phi}\right) r_{t+1}. \tag{17}$$

The LHS is the repayment that a bank receives at date $t+1$ for one unit of loan. The RHS is the amount it has to pay which can have two interpretations depending on the contract between the bank and its workers. One interpretation is that the bank uses the deposit to pay its employees and upon receiving their wage income, workers re-deposit back into the bank. Alternatively, one may imagine that the bank promises its own employees that they will be paid $r_{t+1}\omega_t$ per unit of

[5] We index the interest rates by the payment rather than the contract dates.

effective labor when they become old via an implicit contract. While both yield the same zero-profit condition (17), we stick to the latter interpretation throughout the paper. That is, only workers at the final goods sector are depositors, under which it is much easier to maintain not only the balance sheet accounting but the equilibrium flow of funds.[6]

Under this setup, it is obvious that there must be a gap between deposit and loan rates to guarantee nonnegative profit for banks (i.e., a positive loan-deposit interest rate spread). Throughout the paper, we measure the *financial markup* by $S_{t+1} \equiv \frac{\delta_{t+1} - r_{t+1}}{r_{t+1}}$. From (17), we thus obtain:

$$S_{t+1} = \frac{\omega_t}{\phi}.$$

This suggests that without accounting for the wage adjustment, the direct effect of a reduction in the unit banking operation cost (higher $\phi$) is to lower the financial markup.

### 2.4 Human Capital Formation

To close the model, we must specify the formation of human capital. Since we want to focus on heterogeneity in entrepreneurial ability and idea variety, we simplify the human capital evolution process as:

$$h_{t+1} = K_t^{\beta} h_t^{1-\beta}, \tag{18}$$

where a scaling constant is omitted as it plays no additional role than $A$ under our setting. This equation says that the (gross) rate of human capital accumulation ($h_{t+1}/h_t$) is driven by the physical-human capital ratio ($K_t/h_t$), which is in turn driven by the entrepreneurial productivity as a whole. Such an evolution pattern is justified if the economy exhibits capital-skill complementarity (cf. Griliches 1969) – a larger value of $\beta$ means a greater degree of capital-skill complementarity. Throughout the paper, we assume that the production of human capital is relatively human-capital intensive, thereby requiring $\alpha > \beta$. If there is a positive relationship between individual's effort devoted to human capital accumulation and the (average) physical capital-human capital ratio in the spirit following Azariadis and Drazen (1990), the human capital evolution process essentially captures that in Lucas (1988).

[6] Otherwise, a condition is needed to ensure that each bank would have enough funds to prepay wages to its own employees, which would create unnecessary complexity without providing any additional insights toward understanding the issues examined in this paper.

# 3 Equilibrium

We are now ready to study the equilibrium of this economy. To begin, we define the concept of equilibrium.

**Definition 1**: *Given $h_0$, an initial distribution of individual entrepreneur's capital $\widetilde{k}_0(\tau)$, and an entrepreneurial ability distribution $F(\tau)$, a* ***dynamic equilibrium*** *is a collection of quantity sequences $\{Y_{t+1},\ L_{t+1},\ K_{t+1},\ k_{t+1}(\tau),\ x_t(\tau),\ h_{t+1},\ \overline{\tau}_t,\ N_{t+1},\ M_{t+1}\}$ together with a collection of price sequences $\{\omega_{t+1},\ P_{t+1},\ p_{t+1}(\tau),\ \delta_{t+1},\ r_{t+1}\}$ such that:*

**(i)** *a t-generation agent with ability $\tau$ becomes an entrepreneur if and only if $\tau \geq \tau_t^*$, where $\tau_t^*$ is determined by (12);*

**(ii)** *the number of entrepreneurs and their overall quality index are given, respectively, by (13) and (14);*

**(iii)** *given $\omega_{t+1}$ and $p_{t+1}(\tau)$, firms in the final good sector adopt the optimal manufacturing plan so that (3), (4), (5) hold, where the price of the composite capital $P_{t+1}$ satisfies (6);*

**(iv)** *given $\delta_t, M_{t+1,} P_{t+1}$, and $K_{t+1}$, entrepreneurs adopt the optimal investment and pricing plans so that (10) and (9) hold;*

**(v)** *human capital evolves according to (18);*

**(vi)** *the final good competitive profit condition (7) holds;*

**(vii)** *banks earn zero profit so that (17) holds;*

**(viii)** *loan, labor and goods markets all clear every period.*



## 3.1 Market Clearing Conditions

In this subsection, we derive conditions that characterize loan and labor market clearing. One can easily show that the goods market then clears automatically (i.e., the Walras' law holds true).

The total demand for loan is simply the summation of individual entrepreneur's loan demands while the total supply of loan is more subtle. In particular, the total supply of loan is the aggregate wage income of, not all workers, but only those who work in the final good sector. This can be best illustrated by regarding the payment arrangement between banks and their employees as a promised contract so that only manufacturing workers actually receive wage payments immediately. Furthermore, for one unit of effective labor hired by the manufacturing sector at time $t$, it costs $\frac{\omega_t}{\phi}$ to pay for the required labor in the banking sector to transform deposits into loanable funds. Hence, the fraction of manufacturing workers to all workers is $(1+\frac{\omega_t}{\phi})^{-1}$, and the loan market clearing condition can be written as:

$$\int_{\tau_t^*}^{\tau^H} x_t(\tau)dF = \omega_t h_t \left[\left(1+\frac{\omega_t}{\phi}\right)^{-1}\int_0^{\tau_t^*} dF\right], \quad \forall t \geq 0. \tag{19}$$

Here, the term in the bracket on the RHS is the total number of manufacturing workers at time $t$.

We can specify the labor market clearing condition in a similar fashion,

$$L_{t+1} = \left(1+\frac{\omega_{t+1}}{\phi}\right)^{-1}\int_0^{\tau_{t+1}^*} dF, \quad \forall t \geq 0. \tag{20}$$

Under equation (20), labor market clears every period starting period 1, and it is the initial period wage rate $\omega_0$ that clears the labor market in period 0. More specifically, $\omega_0$ must equate the labor supply in the manufacturing sector $((1+\frac{\omega_0}{\phi})^{-1}\int_0^{\overline{\tau}_0} dF)$ and the labor demand which depends on the initial distribution of individual entrepreneur's capital $(\tilde{k}_0)$. It turns out that the wage dynamics in this model converges to a unique fixed point (to be shown formally in Section 3.4 below). Without loss of generality, we assume $\omega_0$ as the given initial condition instead of $\tilde{k}_0$.

## 3.2 Determination of the Cutoff Ability

In this subsection, we solve the cutoff ability $\tau_t^*$ which turns out to be time-invariant. To establish this result, let us start working on the LHS of the occupational choice condition (12) – a marginal entrepreneur's profit. Integrating both sides of (11) and applying (6), one obtains:

$$\int_{\tau_t^*}^{\tau^H} \pi_{t+1}(\tau)dF = (1-\theta)P_{t+1}K_{t+1}. \tag{21}$$

This equation says entrepreneurs' profits as a whole are a constant fraction $(1-\theta)$ of the aggregate capital rentals $P_{t+1}K_{t+1}$. By applying (16), the above equation implies:

$$\pi_{t+1}(\tau_t^*) = (1-\theta)\left[\int_{\tau_t^*}^{\tau^H}\left(\frac{\tau}{\tau_t^*}\right)^{\frac{\theta}{1-\theta}} dF\right]^{-1} P_{t+1}K_{t+1}.$$

Thus, the profit of a marginal entrepreneur is a fraction $\nu(\tau_t^*) \equiv \frac{1}{\int_{\tau_t^*}^{\tau^H}(\frac{\tau}{\tau_t^*})^{\frac{\theta}{1-\theta}}dF}$ of the total entrepreneurial profits. To solve $P_{t+1}K_{t+1}$, we first combine (6) and (9) to get,

$$P_{t+1} = \frac{\delta_{t+1}\Omega_{t+1}^{\zeta-1}}{\theta M_{t+1}}.$$

Then, from (2) and (8), we can derive how the amount of the composite capital is related to the loan size:

$$K_{t+1} = M_{t+1}\Omega_{t+1}^{1-\zeta}\left[\int_{\tau_t^*}^{\tau^H} x_t(\tau)dF\right],$$

which together with (19) yields:

$$K_{t+1} = M(\tau^*)\Omega(\tau^*)^{1-\zeta}\left[\omega_t h_t(1+\frac{\omega_t}{\phi})^{-1}\int_0^{\tau^*} dF\right]. \tag{22}$$

By substituting in the expressions for $P_{t+1}$ and $K_{t+1}$, the profit of a marginal entrepreneur can be written as:

$$\pi_{t+1}(\tau_t^*) = \frac{1-\theta}{\theta}\left[\int_{\tau_t^*}^{\tau^H}(\frac{\tau}{\tau_t^*})^{\frac{\theta}{1-\theta}}dF\right]^{-1}\delta_{t+1}\omega_t h_t(1+\frac{\omega_t}{\phi})^{-1}\int_0^{\tau_t^*}dF,$$

which can be combined with (17) to simplify the occupational choice condition (12) to:

$$\frac{\int_{\tau_t^*}^{\tau^H}(\frac{\tau}{\tau_t^*})^{\frac{\theta}{1-\theta}}dF}{\int_0^{\tau_t^*} dF} = \frac{1-\theta}{\theta}. \tag{23}$$

The intuition is simple: since entrepreneurs as a whole earn a constant fraction of the capital payment ($(1-\theta)P_{t+1}K_{t+1}$), the rest ($\theta P_{t+1}K_{t+1}$) must go to $t$-generation workers whose mass is $\int_0^{\tau_t^*} dF$. Therefore, at the margin, $\frac{(1-\theta)P_{t+1}K_{t+1}}{\int_{\tau_t^*}^{\tau^H}(\frac{\tau}{\tau_t^*})^{\frac{\theta}{1-\theta}}dF} = \frac{\theta P_{t+1}K_{t+1}}{\int_0^{\tau_t^*} dF}$ must hold, which yields the condition in (23).

**Proposition 1**: (Cutoff Ability) *The cutoff ability $\tau_t^*$ is uniquely determined and is time-invariant.*

*Proof.* For the first part, notice that as $\tau^*$ increases from 0 to $\tau^H$, the LHS of (23) strictly decreases from infinity to zero. So $\tau^*$ exists and is unique. The second part follows directly from (23) which involves no time-varying variables. □

Proposition 1 greatly simplifies our equilibrium analysis because one state variable $\tau_t^*$ remains constant at all times.[7] As a result, $K_t$ and $h_t$ serve as the only state variables in our model economy.[8]

[7] We would like to point out that the result on time-invariance cutoff ability is not robust to more general socially constant-return-to-scale production functions. Since factor shares are constant under Cobb-Douglas technology, the comparison between being an entrepreneur and being a worker is independent of tomorrow's states. This can be seen from the derivation of (23) where all other time-dependent variables cancel out.

[8] Instead of $K_t$, it is more straightfroward to perceive $\tilde{k}_t$, the whole distribution of individual capital owned by $(t-1)$-generation entrepreneurs, as the state variable. However, since we know how $\tau$-entrepreneur's capital stock

### 3.3 Capital, Wage and Rental Dynamics

In this subsection, we study the equilibrium by first determining how the equilibrium wage rate depends on the state variables and then studying the dynamics of this economy. From (3), (4), and (7), we can write the demand for labor as a function of physical-human capital ratio and the wage rate only. That is,

$$L_{t+1} = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}\omega_{t+1}^{-\frac{1}{\alpha}}\frac{K_{t+1}}{h_{t+1}}.$$

Substituting this expression into the labor market clearing condition (20) gives:

$$\omega_{t+1}^{\frac{1}{\alpha}}\left(1+\frac{\omega_{t+1}}{\phi}\right)^{-1} = [A(1-\alpha)]^{\frac{1}{\alpha}}\left(\int_0^{\tau^*} dF\right)^{-1}\frac{K_{t+1}}{h_{t+1}}. \tag{24}$$

Therefore, (24) tells us how the equilibrium wage rate is determined once the ratio of the two state variables, $K_{t+1}$ and $h_{t+1}$, is given.

Next, we study the dynamics of this economy. Since the evolution of human capital is simply captured by its law of motion (18), we focus primarily on the dynamics of physical capital. Combining (22) with (18) and (24), we can obtain the evolution of the transformed state variable:

$$\frac{K_{t+1}}{h_{t+1}} = [A(1-\alpha)]^{\frac{1}{\alpha}}M(\tau^*)\Omega(\tau^*)^{1-\zeta}\omega_t^{-\frac{1-\alpha}{\alpha}}\left(\frac{K_t}{h_t}\right)^{1-\beta}. \tag{25}$$

Since from (24) $\frac{K_t}{h_t}$ can be written as a function of $\omega_t$ explicitly but not vice versa, it is more convenient to reduce the $2\times 2$ dynamical system in $(\omega_t, \frac{K_t}{h_t})$ into one in terms of $\omega_t$ alone,

$$\omega_{t+1}^{\frac{1}{\alpha}}\left(1+\frac{\omega_{t+1}}{\phi}\right)^{-1} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}B(\tau^*)\omega_t^{\frac{\alpha-\beta}{\alpha}}\left(1+\frac{\omega_t}{\phi}\right)^{-(1-\beta)}, \tag{26}$$

where $B(\tau^*) \equiv M(\tau^*)\Omega(\tau^*)^{1-\zeta}\left(\int_0^{\tau^*} dF\right)^{-\beta}$ summarizes all the terms directly related to the cutoff ability. Since for any given $\omega_t$, $\omega_{t+1}$ is uniquely determined from (26), the entire dynamic path of the effective wage rate $\{\omega_t\}_{t=1}^{\infty}$ is determined once $\omega_0$ is given. Furthermore, the wage rate evolution actually implies the dynamics of the whole price system. To see this, note that $P_{t+1}$ can be solved from (7),

$$P_{t+1} = A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\omega_{t+1}^{\frac{\alpha-1}{\alpha}}, \tag{27}$$

is related to that of the marginal entrepreneur (namely, $k_t(\tau) = (\frac{\tau}{\tau_{t-1}^*})^{\frac{1}{1-\theta}}k_t(\tau_{t-1}^*)$) and how composite capital is produced from individual capital (equation (2)), a constant $\tau^*$ implies $K_t$ becomes a sufficient statistic of $\tilde{k}_t$ and the pair $(K_t, h_t)$ fully describes the state at time $t$.

and from (6), (9), (16) and (17), $p_{t+1}(\tau)$, $\delta_{t+1}$ and $r_{t+1}$ are also functions of $\omega_{t+1}$ alone,

$$
\begin{aligned}
p_{t+1}(\tau) &= A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tau^{-1}\Omega(\tau^*)\omega_{t+1}^{\frac{\alpha-1}{\alpha}} && (28)\\
\delta_{t+1} &= A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\theta M(\tau^*)\Omega(\tau^*)^{1-\zeta}\omega_{t+1}^{\frac{\alpha-1}{\alpha}} && (29)\\
r_{t+1} &= A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\theta M(\tau^*)\Omega(\tau^*)^{1-\zeta}\frac{\omega_{t+1}^{\frac{\alpha-1}{\alpha}}}{1+\frac{\omega_{t+1}}{\phi}}. && (30)
\end{aligned}
$$

We summarize these results as follows.

**Proposition 2**: (Equilibrium Dynamics) *Given $\omega_0 > 0$, the equilibrium dynamics of this economy $\{\omega_{t+1},\, P_{t+1},\, p_{t+1},\, \delta_{t+1},\, r_{t+1},\, \frac{K_{t+1}}{h_{t+1}}\}_{t=0}^{\infty}$ are fully captured by the dynamical system (26), (27), (28), (29), (30) and (24).*

### 3.4 Balanced Growth Path

Next, we consider,

**Definition 2**: *Given $h_0$ and $\omega_0$, a* ***balanced growth equilibrium*** *is a dynamic equilibrium where* $\{Y_{t+1},\, K_{t+1},\, k_{t+1}(\tau),\, x_t(\tau),\, h_{t+1}\}_{t=0}^{\infty}$ all grow at constant rates and $\{\tau_t^*,\, N_{t+1},\, M_{t+1},\, L_{t+1},\, \omega_{t+1},$ $P_{t+1},\, p_{t+1}(\tau),\, \delta_t,\, r_t\}_{t=0}^{\infty}$ *are all constant.*



Our goal here is to show that the dynamic equilibrium converges to a *balanced growth path* along which the stocks of physical and human capital grow at a common rate $g$. Since the dynamics of the whole system can be fully described by that of the wage rate, it suffices to show that the wage rate converges to a unique fixed point under any initial $\omega_0 > 0$. For illustrative convenience, let us express the first-order difference equation (26) as:

$$\omega_{t+1} = \Psi(\omega_t), \tag{31}$$

where $\Psi : \mathbf{R}_+ \to \mathbf{R}_+$ is a continuous function. Our first job is to show that $\Psi$ has a unique non-zero fixed point $\omega$ and this is done in Lemma 1 below. It is also easy to verify that function $\Psi$ is hump-shaped with $\Psi(0) = \lim_{\omega_t\to\infty}\Psi(\omega_t) = 0$, $\Psi'(0) = \infty$ and has a unique local maximum at $\omega_t = \frac{\alpha-\beta}{\beta(1-\alpha)}\phi$, as shown in Figures 3a and 3b. The difference between these two figures is whether $\Psi$ is increasing or decreasing at the fixed point. In the first case (Figure 3a), it is clear that the wage rates converge to $\omega$ monotonically, with a possible exception of the initial period. In the second case

(Figure 3b), however, we must first rule out the possibility of cycles in order to establish oscillating convergence. This is further proved in Lemma 2.

**Lemma 1:** $\Psi$ *has a unique, locally stable fixed point* $\omega$.

*Proof.* See Appendix. □

[Figures 3a and 3b here]

Although $\omega$ is the only locally stable fixed point, $\{\omega_t\}_{t=1}^{\infty}$ may not converge to $\omega$ in the presence of cycles. The next lemma precludes such a possibility by applying the Sarkovskii Theorem.[9]

**Lemma 2:** $\Psi$ *has no periodic point.*

*Proof.* See Appendix. □

By substituting $\omega_{t+1} = \omega_t = \omega$ into (26), the equilibrium wage rate along the balanced growth path satisfies:

$$\omega^{\frac{1-\alpha+\beta}{\alpha}}(1+\frac{\omega}{\phi})^{-\beta} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}} B(\tau^*). \tag{32}$$

Utilizing Lemmas 1 and 2, we can draw the main conclusion regarding the balance growth path in the following Proposition.

**Proposition 3**: (Balanced Growth Equilibrium) *Given* $\omega_0 > 0$, *the economy converges to a balanced growth path where all prices remain constant and where capitals and goods output all grow at the same constant rate,*

$$g = [A(1-\alpha)]^{\frac{1}{\alpha}} M(\tau^*)\Omega(\tau^*)^{1-\zeta}\omega^{-\frac{1-\alpha}{\alpha}} - 1, \tag{33}$$

*where* $\tau^*$ *and* $\omega$ *satisfies (23) and (32), respectively.*

*Proof.* The result in (33) follows immediately from manipulating (18) and (25) by imposing balanced growth. □

We have solved the balanced growth equilibrium and are prepared to characterize the equilibrium properties to which we now turn.

[9] See Sharkovskii (1964).

## 4 Characterization of Balanced Growth Equilibrium

Defining $Z(\tau^*) \equiv \int_0^{\tau^*} dF$, we can then rewrite (32) and (33) as:

$$\omega^{\frac{1-\alpha+\beta}{\alpha}}(1+\frac{\omega}{\phi})^{-\beta} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}} M(\tau^*)\Omega(\tau^*)^{1-\zeta} Z(\tau^*)^{-\beta} \tag{34}$$

$$g = [A(1-\alpha)]^{\frac{1}{\alpha}} M(\tau^*)\Omega(\tau^*)^{1-\zeta}\omega(\tau^*)^{-\frac{1-\alpha}{\alpha}} - 1. \tag{35}$$

To see how changes in $M$, $\Omega$, and $Z$ affect $\omega$ and $g$, we totally differentiate the above expressions:

$$\frac{d\omega}{\omega} = \frac{\alpha}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})}\left(\frac{dM}{M}+(1-\zeta)\frac{d\Omega}{\Omega}-\beta\frac{dZ}{Z}\right) \tag{36}$$

$$\frac{dg}{1+g} = \frac{\beta}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})}\left[(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})(\frac{dM}{M}+(1-\zeta)\frac{d\Omega}{\Omega})+(1-\alpha)\frac{dZ}{Z}\right]. \tag{37}$$

Recall that $M$ measures the overall quality of entrepreneurs which directly affects each entrepreneur's efficiency in producing individual capital good. As a result, a change of $M$ will generate a long-run effect on the aggregate performance of the economy. We refer to this as a *selectivity effect* because $M$ is directly related to the question of who, with what ability, chooses to become an entrepreneur. Moving beyond those imperfectly substitutable individual capital goods, how effectively they are combined into the composite capital good, captured by the variable $\Omega$, will also affect aggregate performance except for the limiting case where $\zeta = 1$. This latter channel is referred to as a *variety effect*. These two effects govern the effectiveness of transforming loanable funds into productive composite capital. Additionally, there is yet a third channel via changes in the supply of loanable funds under our general equilibrium setup. Specifically, the loan supply in our economy comes from the wage earnings of final-good sector workers and hence is related to the total mass of workers, denoted by $Z$. When $Z$ changes, the amount of loan supply will change in response, which can be referred to as a *loanable fund supply effect*.[10] Equations (36) and (37) show what the impacts of these three effects are on the steady-state wage rate and the balanced growth rate.

The intuition behind these two equations is as follows. First, fix $Z$ ($dZ = 0$). A higher $M$ or $\Omega$ implies greater efficiency in producing the composite capital given the same amount of loanable funds. This in turn raises the capital-labor ratio and the rate of balanced growth. Since capital

[10] One notable feature of this two-period overlapping generation model is that the loanable fund supply effect and the variety effect always go in opposite directions. This is because only wage earnings contribute to the loanable fund and more workers must lead to more loanable fund and less capital variety. This relationship could be upset, for example, in a three-period setup where entrepreneurs' profits may also contribute to the loanable fund.

and labor are Pareto-Edgeworth complements, workers are also benefitted from a larger amount of capital and hence, their (effective) wage rate goes up. Next, let us fix $M$ and $\Omega$, and let the number of workers increase ($dZ > 0$). As labor supply increases, the wage rate must go down. However, the size of total loanable funds should go up which results in a higher balanced growth rate.

It may be noted that when the cutoff ability $\tau^*$ determined by occupational choice is low, there is a large number of entrepreneurs with complementary capital projects and a small number of workers available for goods production. As a consequence, the effect via $\Omega$ is strong whereas that via $Z$ is weak (to be shown formally in the next subsection). One can therefore see the crucial role played by occupational choice in influencing the balanced growth rate of the economy ($g$).

## 4.1 Distribution, Entrepreneurship and Growth

In the following two exercises, we will study how changes of the underlying ability distribution, a shift and a stretch, will affect entrepreneurship, the equilibrium wage rate and the balanced growth rate. We restrict our attention to the case where the entrepreneurial ability distribution has a compact support $[0, \tau^H]$ with $\tau^H < \infty$. The main results are summarized in Table 1.

First, consider a uniformly rightward shift of the ability distribution function from a compact support $[0, \tau^H]$ to $[\lambda, \tau^H + \lambda]$, where $\lambda > 0$. We then have:

**Proposition 4:** (Shift in Ability Distribution) *A uniformly rightward shift in the ability distribution by* $\lambda > 0$ *raises the cutoff ability* $\tau^*$ *less-than-proportionately, increases the number of entrepreneurs and the equilibrium wage rate, but has an ambiguous effect on the balanced growth rate.*

*Proof.* First, we show that the new cutoff ability $\tau_\lambda \in (\tau^*, \tau^* + \lambda)$ and the number of entrepreneurs rises. By occupational choice, we have:

$$\int_{\tau_\lambda}^{\tau^H+\lambda} \left(\frac{\tau}{\tau_\lambda}\right)^{\frac{\theta}{1-\theta}} dF(\tau - \lambda) = \frac{1-\theta}{\theta} \int_{\lambda}^{\tau_\lambda} dF(\tau - \lambda).$$

By changing variables, we can rewrite the above equation as:

$$\Lambda(\tau_\lambda) \equiv \int_{\tau_\lambda - \lambda}^{\tau^H} \left(\frac{\tau + \lambda}{\tau_\lambda}\right)^{\frac{\theta}{1-\theta}} dF(\tau) - \frac{1-\theta}{\theta} \int_0^{\tau_\lambda - \lambda} dF(\tau) = 0.$$

Notice that $\Lambda(\tau_\lambda)$ is strictly decreasing in $\tau_\lambda$, satisfying $\Lambda(\tau^*) > 0$ and $\Lambda(\tau^* + \lambda) < 0$. Thus, by the Mean Value Theorem, $\tau_\lambda \in (\tau^*, \tau^* + \lambda)$. The number of entrepreneurs must rise because,

$$N_\lambda = \int_{\tau_\lambda}^{\tau^H+\lambda} dF(\tau - \lambda) = \int_{\tau_\lambda - \lambda}^{\tau^H} dF(\tau) > \int_{\tau_\lambda}^{\tau^H} dF(\tau) = N.$$

Next we show that the effect on $\omega$ is positive but that on $g$ is ambiguous. It is clear that $M$ goes up and the number of workers $Z$ goes down ($Z = 1 - N$). Moreover, $\Omega$ must go up because,

$$\Omega(\tau_\lambda)^{\frac{\theta}{1-\theta}} = \int_{\tau_\lambda}^{\tau^H+\lambda} \tau^{\frac{\theta}{1-\theta}} dF(\tau-\lambda) = \int_{\tau_\lambda-\lambda}^{\tau^H} (\tau+\lambda)^{\frac{\theta}{1-\theta}} dF(\tau) > \int_{\tau^*}^{\tau^H} \tau^{\frac{\theta}{1-\theta}} dF(\tau) = \Omega(\tau)^{\frac{\theta}{1-\theta}}.$$

From (36), $\omega$ must go up, but from (37) $g$ need not rise if the effect via $Z$ is strong. $\square$

Intuitively, an uniformly rightward shift in the ability distribution creates two conflicting effects. On the one hand, it makes the production of the composite capital more efficient (higher $M$ and $\Omega$). On the other hand, it reduces the size of loanable funds channeled through the banking sector from deposits by manufacturing workers. When the *loanable fund supply effect* is important, the rate of economic growth need not be higher. This result implies economic growth and the number of entrepreneurs may not be positively related in response to such a shift in entrepreneurial ability.

Next, consider a proportional stretch of the ability distribution from $F$ to $G$ such that $F(\tau) = G(a\tau)$ where $a > 1$. The following property can be established:

**Proposition 5:** (Stretch of Ability Distribution) *A proportional stretch of the ability distribution from $F$ to $G$ such that $F(\tau) = G(a\tau)$ with $a > 1$ raises the cutoff ability $\tau^*$ proportionately ($\tau_a = a\tau^*$), the equilibrium wage rate and the balanced growth rate unambiguously, but has no effect on the number of entrepreneurs.*

*Proof.* By occupational choice, we have:

$$\int_{\tau_a}^{a\tau^H} (\frac{\tau}{\tau_a})^{\frac{\theta}{1-\theta}} dG = \frac{1-\theta}{\theta} \int_0^{\tau_a} dG.$$

By changing variables, we obtain:

$$\int_{\tau_a/a}^{\tau^H} (\frac{a\tau}{\tau_a})^{\frac{\theta}{1-\theta}} dF = \frac{1-\theta}{\theta} \int_0^{\tau_a/a} dF.$$

Plugging $\tau_a = a\tau^*$ into the above equation yields (23), which implies $\tau_a = a\tau^*$ is the new cutoff ability. It is immediate that the number of entrepreneurs remains unchanged, as $N_a = \int_{\tau_a/a}^{\tau^H} dF = \int_{\tau^*}^{\tau^H} dF = N$. Since both $M$ and $\Omega$ go up (by a multiplicative factor $a > 1$) and $Z$ remains the same, both $\omega$ and $g$ must go up as suggested by (36) and (37). $\square$

Intuitively, a stretch of the distribution in a proportional manner makes the cutoff ability go up proportionately, so the fractions of workers and entrepreneurs remain the same. In this case, we have both a positive selectivity effect and a positive variety effect, without any offsetting loanable fund supply effect. As a result, the growth rate increases unambiguously. This proposition suggests that even if two economies have the same number (proportion) of entrepreneurs, the one with a stochastically dominant entrepreneurial ability distribution captured by a proportional stretch features a higher rate of economic growth.

One should note that the lack of a robust positive relationship between the number of entrepreneurs and economic growth, as suggested in Propositions 4 and 5, does not rely on the existence of entrepreneurial externality, captured by $M$.[11] Our result therefore lends a plausible theoretical support to the somewhat surprising empirical finding by Blanchflower (2000) who finds no evidence that a higher self-employed to total employment ratio is associated with a higher growth rate across OECD countries.

One may now check in general how an *exogenous* shift in $\tau^*$ (while holding the underlying ability distribution *unchanged*) affects the balanced growth rate. Straightforward calculation gives,

$$\begin{aligned}
\frac{1}{\Omega}\frac{d\Omega}{d\tau^*} &= -\frac{1-\theta}{\theta}f(\tau^*)\left[\int_{\tau^*}^{\tau^H}(\frac{\tau}{\tau^*})^{\frac{\theta}{1-\theta}}dF\right]^{-1} < 0 \\
\frac{1}{Z}\frac{dZ}{d\tau^*} &= \frac{f(\tau^*)}{\int_0^{\tau^*}dF} > 0 \\
\frac{1}{M}\frac{dM}{d\tau^*} &= \frac{f(\tau^*)\int_{\tau^*}^{\tau^H}\left[(1-\eta)\,\tau-\tau^*\right]dF}{(\int_{\tau^*}^{\tau^H}\tau dF)(\int_{\tau^*}^{\tau^H}dF)},
\end{aligned}$$

where $\frac{1}{M}\frac{dM}{d\tau^*} > 0$ as long as $\eta$ is not too large. Notice that under (23), $\frac{1}{Z}\frac{dZ}{d\tau^*} = -\frac{1}{\Omega}\frac{d\Omega}{d\tau^*}$. Utilizing this property and (37), we obtain:

$$\begin{aligned}
\frac{1}{1+g}\frac{dg}{d\tau^*} &= \frac{\beta}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})}\cdot \\
&\quad \left[(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})\frac{1}{M}\frac{dM}{d\tau^*}+\left(\frac{\alpha}{1+\omega/\phi}-\zeta(1-\frac{\alpha\omega/\phi}{1+\omega/\phi})\right)\frac{1}{\Omega}\frac{d\Omega}{d\tau^*}\right] \qquad (38)
\end{aligned}$$

Therefore, raising $\tau^*$ determined in the decentralized economy may raise the balanced growth rate (since $M$ goes up) or lower it if $\zeta$ is not too large (since $\Omega$ goes down). This ambiguity, once again, is due to the presence of the two conflicting effects. When the selectivity effect is stronger (weaker) than the variety effect, a higher cutoff ability or a lower proportion of entrepreneurs is associated with higher (lower) economic growth. In the case where $\zeta$ is not too large, a reduction in $\tau^*$ determined in the decentralized economy raises the number of entrepreneurs as well as the

[11] Even by setting $M = 1$, Propositions 4 and 5 still hold true.

endogenous growth rate, which restores the precept in the conventional R&D-based literature in the presence of a positive scale effect.

[Table 1 here]

### 4.2 Productivity, Growth and Financial Markups

We next turn to characterizing the different effects of improvements in production technology versus in banking efficiency (i.e., increases in $A$ versus in $\phi$). We summarize the results in Table 2 and relegate the detailed proof to the Appendix. Briefly, the effective wage rate ($\omega$) and the financial markup ($S \equiv \frac{\delta - r}{r} = \frac{\omega}{\phi}$) are decreasing in banking productivity but increasing in manufacturing productivity. The negative relationship between banking productivity and the financial markup corroborates with empirical findings in Lehr and Wang (2000). Also, while higher banking productivity raises both loan and deposit rates ($\delta$ and $r$), higher manufacturing productivity increases the loan rate but its effect on the deposit rate is generally ambiguous. Moreover, since both manufacturing and banking productivity shifts change labor wages and entrepreneurial returns equi-proportionately, neither productivity shift alters occupational choice, so the number of entrepreneurs remains unchanged. Finally, while both banking and manufacturing productivity improvements are growth-enhancing, their effects on the financial markup are entirely opposite. Therefore, whether economic development is accompanied by a lower or higher financial markup depends crucially on the origin of the productivity increase, which provides a plausible explanation for the ambiguity of this relationship established in the cross-country, cross-industry study of Cetorelli and Gambera (2001).



[Table 2 here]

### 4.3 Further Discussion

This subsection provides further analysis under a uniform ability distribution in a simplified environment, as well as two extensions concerning entrepreneur subsidies and credit-market imperfections.

### 4.3.1 Special Case: Uniform Ability Distribution

We now consider a useful benchmark with a uniform ability distribution to analyze the growth effect of a mean-preserving concentration where the support of the ability distribution shrinks from $\left[0, \tau^H\right]$ to $\left[\varepsilon, \tau^H - \varepsilon\right]$ with $\varepsilon > 0$. In order to obtain analytic results, we further impose three simplifying assumptions: $\eta = 0$ (i.e., only average quality of entrepreneurs enters the Marshallian externality term), $\zeta = 1$ (i.e., there is no variety effect), and $\phi \to \infty$ (i.e., the banking sector bears no operation cost).

In this case, the occupational choice equation becomes:

$$\frac{\int_{\tau_\varepsilon}^{\tau^H - \varepsilon} \left(\frac{\tau}{\tau_\varepsilon}\right)^{\frac{\theta}{1-\theta}} \frac{1}{\tau^H - 2\varepsilon} d\tau}{\int_{\varepsilon}^{\tau_\varepsilon} \frac{1}{\tau^H - 2\varepsilon} d\tau} = \frac{1-\theta}{\theta}.$$

Thus, the unique time-invariant cutoff ability, $\tau_\varepsilon$, is determined by,

$$\theta \left[ \left( \frac{\tau^H - \varepsilon}{\tau_\varepsilon} \right)^{\frac{1}{1-\theta}} - 1 \right] = \frac{\tau_\varepsilon - \varepsilon}{\tau_\varepsilon}, \tag{39}$$

where $\tau_\varepsilon \to \tau^* = \tau^H \left(\frac{\theta}{1+\theta}\right)^{1-\theta}$ as $\varepsilon \to 0$. Straightforward differentiation yields,

$$\begin{aligned}
\left.\frac{d\tau_\varepsilon}{d\varepsilon}\right|_{\varepsilon \to 0} &= \frac{1-\theta}{1+\theta} - \left(\frac{\theta}{1+\theta}\right)^{1-\theta}, \\
\left.\frac{d\left(\frac{\tau^H - \varepsilon - \tau_\varepsilon}{\tau^H - 2\varepsilon}\right)}{d\varepsilon}\right|_{\varepsilon \to 0} &= \frac{1}{\tau^H}\left[\frac{2\theta}{1+\theta} - \left(\frac{\theta}{1+\theta}\right)^{1-\theta}\right] > 0.
\end{aligned}$$

as $\frac{2\theta}{1+\theta} > \left(\frac{\theta}{1+\theta}\right)^{1-\theta}$ for all $\theta \in (0,1)$. This implies more entrepreneurship in response to a mean-preserving concentration.

We can also solve:

$$M = \frac{1}{2}\left(\tau^H - \varepsilon + \tau_\varepsilon\right), Z = \frac{\tau_\varepsilon - \varepsilon}{\tau^H - 2\varepsilon},$$

and

$$\begin{aligned}
\frac{1}{1+g}\frac{dg}{d\varepsilon} &= \frac{\beta}{(1-\alpha)+\beta}\left[\frac{1}{M}\frac{dM}{d\varepsilon} + (1-\alpha)\frac{1}{Z}\frac{dZ}{d\varepsilon}\right] \\
&\propto -\left\{ \left(\frac{\theta}{1+\theta}\right)^{1-\theta} \left[\frac{2\theta}{1+\theta} + \left(\frac{\theta}{1+\theta}\right)^{1-\theta}\right] \right. \\
&\quad \left. + (1-\alpha)\left[1 + \left(\frac{\theta}{1+\theta}\right)^{1-\theta}\right]\left[\frac{2\theta}{1+\theta} - \left(\frac{\theta}{1+\theta}\right)^{1-\theta}\right] \right\} \\
&< 0
\end{aligned}$$

as $\frac{2\theta}{1+\theta} > \left(\frac{\theta}{1+\theta}\right)^{1-\theta}$ for all $\theta \in (0,1)$. Thus, economic growth retards in response to a mean-preserving concentration.

This result is interesting. In the absence of tax distortions or market imperfections, larger dispersion in ability is usually growth-retarding in human capital-based balanced growth models. Our finding suggests that a more concentrated ability distribution encourages entrepreneurship but lowers the long-run rate of growth in our economy where the selectivity effect plays an important role. In summary, we have:

**Proposition 6:** (Mean-Preserving Concentration of a Uniform Ability Distribution) *Under $\eta = 0$, $\zeta = 1$, and $\phi \to \infty$, a mean-preserving concentration of the uniform ability distribution by shrinking the support by $2\varepsilon$ raises the equilibrium number of entrepreneurs but reduces the balanced growth rate.*

Our result can be applied to the analysis of the relationship between the firm size distribution and economic growth. In our economy, the size of intermediate producers can be measured by their values $\pi(\tau)$. From (16), we can see that this firm size distribution is entirely captured by the distribution of the entrepreneurial ability $\tau$. It is immediate that a more dispersed firm size distribution is associated with higher growth in our economy.



### 4.3.2 Extension 1: Subsidizing Entrepreneurship

Understanding the relationship between entrepreneurial ability and long-run growth, we now extend the benchmark model by incorporating a particularly relevant policy instrument, a proportional subsidy to entrepreneurship.

Specifically, we suppose that, the government subsidizes each entrepreneur based upon his profit (call the constant rate of subsidy $\gamma$) and finances the entire subsidy amount by a uniform per capita tax (denoted $T_t$) levied on every agent alive. Then, an entrepreneur of ability $\tau$ receives a gross profit $(1+\gamma)\pi_{t+1}(\tau_t)$ and the cutoff ability (denoted $\tau_t^E$) must now satisfy:

$$(1+\gamma)\pi_{t+1}(\tau_t^E) - T_t = r_{t+1}\omega_t h_t - T_t,$$

where as $\gamma \to 0$, it reduces to the benchmark counterpart (12).[12] By repeating the same exercises

[12] One may reinterpret $\gamma$ as a percentage non-pecuniary gain from being an entrepreneur, following the spirit of Blanchflower and Oswald (1998). In this case, there is no need for considering tax financing.

as in the previous sections, the occupational choice relationship becomes:

$$\frac{\int_{\tau^E}^{\tau^H} (\frac{\tau}{\tau^E})^{\frac{\theta}{1-\theta}} dF}{\int_0^{\tau^E} dF} = (1+\gamma)\frac{1-\theta}{\theta}, \tag{40}$$

which determines a unique time-invariant cutoff ability, $\tau^E$. By comparing (40) with (23), it is easily seen that $\tau^E < \tau^*$. That is, those less-able agents may choose to be entrepreneurs due to the government subsidy, even when they earn pre-subsidy entrepreneurial profits less than the alternative wage income. Thus, the economy is populated with a greater number of entrepreneurs with a lower average entrepreneurial quality. It is immediate that one can apply the result established in (38) to conclude that the provision of a proportional subsidy to entrepreneurs may be harmful for growth if the selectivity effect dominates the variety effect.

#### 4.3.3 Extension 2: Imperfect Credit Market

Into our benchmark model, we now introduce credit market imperfection with moral hazard problem and nontrivial default costs, following the spirit of Evans and Jovanovic (1989).[13] Specifically, we extend the take-the-money-and-run (absconding) setup of Sappington (1993), Banerjee and Newman (1993) and Fender and Wang (2003) to suit the purpose of our paper. We suppose that although banks can observe borrowers' ability $\tau$ and can ensure that the money lent is invested, they cannot secure the return as borrowers might choose to escape upon receiving revenues, denoted by $R$. Should the borrower abscond from repayments, the bank can either (i) track the defaulter down, in which case the bank appropriates the borrower's total assets ($R$) with probability one where the unit labor requirement for this task is $q > 0$, or (ii) just let the borrower escape. Given this simple tracking technology, banks would never bother to track down absconding borrowers with small loans and hence would never approve such loan applications in the first place. Formally, banks at time $t$ would lend money to an entrepreneur with ability $\tau$ only if the following incentive compatibility constraint holds:

$$R_{t+1}(\tau) \geq q\omega_{t+1}h_{t+1}. \tag{41}$$

Since $R_{t+1}(\tau) = \frac{\pi_{t+1}(\tau)}{1-\theta}$ is strictly increasing in $\tau$, there must be a unique $\tau$, denoted by $\hat{\tau}_t$, such that the equality in (41) holds. That is, any $t$-generation agent whose entrepreneurial ability is

[13] Following Fazzari, Hubbard and Petersen, (1988) and Evans and Jovanovic (1989), there has been a large volume of empirical studies documenting how credit constraints as a result of imperfect credit markets limit entrepreneurial practice. For example, see Evans and Leighton (1989), Gertler and Gilchrist (1994), and Holtz, Eakin, Joulfaian and Rosen (1994), to name but a few.

lower than $\hat{\tau}_t$ cannot get the loan from the credit market and therefore cannot become an entrepreneur. Together with the analysis of agents' occupational choice in Section 2.2, a $t$-generation agent actually becomes an entrepreneur only if his ability $\tau \geq \overline{\tau}_t$ where $\overline{\tau}_t = \max\{\tau_t^*, \hat{\tau}_t\}$. In contrast with the conventional literature cited in the introduction, wealth plays no role in determining the entrepreneur ability cutoff under our setup.[14]

How would this modification complicate our previous dynamic equilibrium analysis? One can immediately notice that since the tracking cost varies during transition, $\hat{\tau}_t$ and consequently $\overline{\tau}_t$ will not remain constant. As a result, deriving the whole equilibrium dynamics would be difficult, if not impossible. Nevertheless, it would be interesting to know how a balanced growth path with credit constraint compares to our benchmark case. Specifically, we ask the question: can the credit market imperfection lead to a higher balanced growth rate?

To characterize the credit-constrained balanced growth equilibrium, we need to solve the steady-state values of the cutoff ability ($\hat{\tau}$), the wage rate ($\hat{\omega}$), and the physical-human capital ratio $(\widehat{\frac{K}{h}})$. The procedure of solving them is similar to that of solving the unconstrained equilibrium except that the occupational choice condition (12) should now be replaced by the incentive compatibility condition (41). Substituting (6), (7) and (16) into (41) and replacing variables by their steady-state values, (41) can be written as

$$[\hat{\tau}\Omega(\hat{\tau})^{-1}]^{\frac{\theta}{1-\theta}} = qA^{-\frac{1}{\alpha}}\alpha^{-1}(1-\alpha)^{-\frac{1-\alpha}{\alpha}}\hat{\omega}^{\frac{1}{\alpha}}\widehat{(\frac{K}{h})}^{-1}. \tag{42}$$

The loan and labor market clearing conditions, (19) and (20), remain virtually the same.[15] These can be combined to eliminate $(\widehat{\frac{K}{h}})$ and yield the following market equilibrium condition (referred to as the $MM$ locus):

$$\hat{\omega}^{\frac{1-\alpha+\beta}{\alpha}}(1+\frac{\hat{\omega}}{\phi})^{-\beta} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}M(\hat{\tau})\Omega(\hat{\tau})^{1-\zeta}Z(\hat{\tau})^{-\beta}, \tag{43}$$

which resembles (34). Moreover, by utilizing the labor market clearing condition, one can substitute

[14] One may however consider a more general framework where either collaterals are required for obtaining a loan or part of the wealth is immobile and hence lost in absconding. In either case, the wealthier are less likely to be credit constrained.

[15] These equilibrium conditions, respectively, become:

$$\hat{\omega}^{\frac{1}{\alpha}}(1+\frac{\hat{\omega}}{\phi})^{-1} = [A(1-\alpha)]^{\frac{1}{\alpha}}Z(\hat{\tau})^{-1}\widehat{(\frac{K}{h})} \quad \text{and} \quad \widehat{(\frac{K}{h})}^{1+\beta} = M(\hat{\tau})\Omega(\hat{\tau})Z(\hat{\tau})\hat{\omega}(1+\frac{\hat{\omega}}{\phi})^{-1}.$$

$\widehat{(\frac{K}{h})}$ in (42) away to obtain the following steady-state credit constraint (referred to as the $CC$ locus):

$$q\frac{1-\alpha}{\alpha}(1+\frac{\hat{\omega}}{\phi}) = [\hat{\tau}\Omega(\hat{\tau})^{-1}]^{\frac{\theta}{1-\theta}} Z(\hat{\tau}). \tag{44}$$

This latter relationship implies that a less efficient tracking technology (a larger $q$) leads to, holding $\hat{\omega}$ constant, a higher cutoff ability $\hat{\tau}$ and, as a result, a larger number of potential entrepreneurs (measured by $F(\hat{\tau}) - F(\tau^*)$) are credit constrained and forced to become workers.

In Figure 4, we plot the $MM$ and $CC$ loci and, as it suggests, the solution may not be unique. To be more specific, while the $CC$ locus is upward-sloping in $(\hat{\tau}, \hat{\omega})$ space, the $MM$ locus is generally non-monotone. From (43), the LHS is increasing in $\hat{\omega}$; the RHS is decreasing in $\hat{\tau}$ if and only if the selectivity effect (via $M$) is dominated by the variety and the loanable fund supply effects (via $\Omega$ and $Z$, respectively), which is more likely when the cutoff ability is extremely low (strong variety effect) or extremely high (strong loanable fund supply effect). Figure 4 is drawn such that both $\hat{\tau}_1$ and $\hat{\tau}_2$ are greater than $\tau^*$; thus, they are both possible equilibrium cutoff points in a credit-constrained balanced growth economy.

[Figure 4 here]

To understand this global indeterminacy result, imagine two economies, 1 and 2, ending up with different balanced growth paths associated with $\hat{\tau}_1$ and $\hat{\tau}_2$. Comparing to economy 1, the steady-state wage rate in economy 2 is higher ($\hat{\omega}_2 > \hat{\omega}_1$), implying that the tracking cost is higher and hence the credit market imperfection is more severe ($\hat{\tau}_2 > \hat{\tau}_1$). Although a larger number of potential entrepreneurs are credit-constrained in economy 2 and thus a weaker variety effect, those who do become entrepreneurs are more productive due to a stronger selectivity effect. When this positive selectivity effect dominates the negative variety and loanable fund supply effects, a higher steady-state wage rate can be supported. Similar arguments apply to the constrained equilibrium in economy 1. In summary, both high and low combinations of the effective wage rate and the cutoff ability can be equilibrium phenomena and the equilibrium selection is due entirely to self-fulfilling prophecies.

Compared to the unconstrained balanced growth equilibrium, the steady-state wage rate $\hat{\omega}$ may not be lower since the RHS of (43) is not monotonic in $\tau$. This is counter-intuitive from the static point of view since as the credit constraint tightens, labor supply goes up which would drive the contemporary wage rate down. However, since the loanable fund supply goes up, such an economy may experience faster growth which leads to a higher wage rate in the long run. Formally, the balanced growth rate of the credit-constrained economy can be derived as:

$$\hat{g} = \widehat{\left(\frac{K}{h}\right)}^{\beta} - 1 = [A(1-\alpha)]^{\frac{1}{\alpha}} M(\hat{\tau})\Omega(\hat{\tau})^{1-\zeta}\hat{\omega}^{-\frac{1-\alpha}{\alpha}} - 1.$$

Comparing to the unconstrained balanced growth rate $g$ in (35), credit market imperfections result in a stronger selectivity effect ($M(\hat{\tau}) > M(\tau^*)$), a weaker variety effect ($\Omega(\hat{\tau}) < \Omega(\tau^*)$) and an ambiguous effect on the steady state wage rate, and its overall effect on balanced growth rate is ambiguous ($\hat{g} \gtrless g$).

Thus, we obtain a striking result: should credit constraints serve as an effective filtering device improving the overall quality of entrepreneurs, the associated rate of economic growth may be *higher* than one in the unconstrained equilibrium. In this case, any nonselective policies which help alleviate credit constraints would lower overall entrepreneurial quality and may be growth-retarding.

# 5 Concluding Remarks

We have developed a dynamic general-equilibrium model of endogenous growth with entrepreneurial ability heterogeneity. Rather than being employed as a worker, an agent with higher entrepreneurial ability may choose to become an entrepreneur, transforming loans into capital goods. Competitive final good firms undertake production by employing young workers and imperfectly substitutable individual capitals. Competitive banks earn zero profit by hiring workers to transform deposits into loans. We have solved the closed-form solution of this model and shown that for a given ability distribution, a decrease in the number of entrepreneurs may create a selectivity effect, raising the overall quality of entrepreneurs and enhancing the rate of growth of the economy. Moreover, for a given family of distributions, a change of distribution parameters may generate a permanent growth effect. Due to the presence of a selectivity effect and a variety effect (that affect the effectiveness of combining individual capitals into a composite capital) and the presence of a loanable fund supply effect, the number of entrepreneurs and income growth need not be positively related. Furthermore, production and banking technologies may have very different long-run implications for factor returns and financial markups. Additionally, we have shown that government subsidy to entrepreneurship need not enhance growth and that the presence of credit constraints as a result of credit market imperfections hinders entrepreneurship but need not retard growth. Our paper has therefore promoted better understanding about the workings of entrepreneurship-driven growth in the process of economic development.

Our model predicts that a rightward shift of the entrepreneurial ability distribution may lead to a negative relationship between the number of entrepreneurs and economic growth, particularly when the loanable fund supply effect is strong as compared to the effects via occupational choice. This may be tested empirically using cross-country, cross-industry data. Furthermore, our model also suggests that an improvement in financial efficiency and an advancement in production technology can induce different relationships between financial markups and economic growth. It may therefore be interesting to separate these two types of productivity enhancement empirically to examine the relative magnitude of their effects on the financial markup-output growth relationship across countries and industries.

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# Appendix

*Proof of Lemma 1.* Let $\bar{\omega}$ be a fixed point of $\Psi$, then $\bar{\omega}$ must satisfy $\bar{\omega} = \Psi(\bar{\omega})$. Replacing $\omega_{t+1}$ and $\omega_t$ in equation (26) with $\bar{\omega}$ yields,

$$\bar{\omega}^{\frac{1}{\alpha}}(1+\frac{\bar{\omega}}{\phi})^{-1} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}B(\tau^*)[\bar{\omega}^{\frac{\alpha-\beta}{\alpha}}(1+\frac{\bar{\omega}}{\phi})^{-(1-\beta)}]. \tag{A1}$$

Obviously, $\bar{\omega} = 0$ is a fixed point of (A1). Assuming $\bar{\omega} = \omega > 0$, (A1) implies:

$$\omega^{\frac{1-\alpha+\beta}{\alpha}}(1+\frac{\omega}{\phi})^{-\beta} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}B(\tau^*). \tag{A2}$$

Since the RHS of (A2) is a constant and the LHS strictly increases in $\omega$ from 0 to infinity, $\omega$ ($> 0$) exists and is unique.

To prove that $\omega$ is the only locally stable fixed point, it suffices to show that $|\Psi'(\omega)| < 1$ and $|\Psi'(0)| > 1$. From equation (26),

$$\Psi'(\omega_t) = \frac{d\omega_{t+1}}{d\omega_t} = \frac{[(\alpha-\beta)\phi - \beta(1-\alpha)\omega_t][\omega_{t+1}(1+\frac{\omega_{t+1}}{\phi})]}{[\phi+(1-\alpha)\omega_{t+1}][\omega_t(1+\frac{\omega_t}{\phi})]}. \tag{A3}$$

Therefore,

$$\Psi'(\omega) = \frac{(\alpha-\beta)\phi - \beta(1-\alpha)\omega}{\phi+(1-\alpha)\omega},$$

and it is easy to verify that $|\Psi'(\omega)| < 1$.

To show that $\bar{\omega} = 0$ is an unstable fixed point, we utilize equation (26) again and rewrite equation (A3) as:

$$\Psi'(\omega_t) = \frac{[(\alpha-\beta)\phi - \beta(1-\alpha)\omega_t]\left\{[A(1-\alpha)]^{1+\beta}B(\tau^*)^{\alpha}(1+\frac{\omega_{t+1}}{\phi})^{1+\alpha}\right\}}{[\phi+(1-\alpha)\omega_{t+1}][\omega_t^{1-\alpha+\beta}(1+\frac{\omega_t}{\phi})^{1+\alpha(1-\beta)}]}.$$

Since $\omega_{t+1} = 0$ when $\omega_t = 0$, $\Psi'(0) = \infty$. $\square$

*Proof of Lemma 2:* Since $\Psi$ is continuous, the Sarkovskii Theorem applies. Therefore, if we can show that $\Psi$ has no two-period cycle, then the proof is completed.

Suppose $\Psi$ has a periodic point $\omega_p (> 0)$ with prime period two. That is, $\Psi(\omega_p) = b\omega_p$ and $\Psi(b\omega_p) = \omega_p$ for some positive $b$ and $b \neq 1$. From equation (26), this implies:

$$(b\omega_p)^{\frac{1}{\alpha}}(1+\frac{b\omega_p}{\phi})^{-1} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}B(\tau^*)(\omega_p)^{\frac{\alpha-\beta}{\alpha}}(1+\frac{\omega_p}{\phi})^{-(1-\beta)} \tag{A4}$$

$$(\omega_p)^{\frac{1}{\alpha}}(1+\frac{\omega_p}{\phi})^{-1} = [A(1-\alpha)]^{\frac{1+\beta}{\alpha}}B(\tau^*)(b\omega_p)^{\frac{\alpha-\beta}{\alpha}}(1+\frac{b\omega_p}{\phi})^{-(1-\beta)}. \tag{A5}$$

From (A4) and (A5), we get:

$$b^{\frac{1+\alpha-\beta}{\alpha}} = (\frac{1+\frac{b\omega_p}{\phi}}{1+\frac{\omega_p}{\phi}})^{2-\beta},$$

which yields,

$$(1 - b^{-\frac{(1-\alpha)(1-\beta)}{\alpha(2-\beta)}})\omega_p = (b^{-\frac{1+\alpha-\beta}{\alpha(2-\beta)}} - 1)\phi. \tag{A6}$$

Since $\text{sign}(1 - b^{-\frac{(1-\alpha)(1-\beta)}{\alpha(2-\beta)}}) = -\text{sign}(b^{-\frac{1+\alpha-\beta}{\alpha(2-\beta)}} - 1)$ for any positive $b \neq 1$, $\omega_p < 0$ which contradicts our assumption. Therefore, $\Psi$ has no two-period cycle and by the Sarkovskii Theorem, $\Psi$ has no cycle of any period. $\square$

*Proof of Comparative Statics in Section 4.2:* From (32) and (33), straightforward differentiation with respect to $A$ and $\phi$ yields:

$$\frac{\phi}{\omega}\frac{\partial\omega}{\partial\phi} = -\frac{\alpha\beta\frac{\omega/\phi}{1+\omega/\phi}}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})} < 0.$$

$$\frac{A}{\omega}\frac{\partial\omega}{\partial A} = \frac{1+\beta}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})} > 0$$

$$\begin{aligned}\frac{\phi}{1+g}\frac{\partial g}{\partial\phi} &= \frac{1}{\alpha}\left[1-(1-\alpha)\left(\frac{\phi}{\omega}\frac{\partial\omega}{\partial\phi}\right)\right] \\ &= \frac{1}{\alpha}\frac{(1-\alpha)+\beta}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})} > 0\end{aligned}$$

$$\begin{aligned}\frac{A}{1+g}\frac{\partial g}{\partial A} &= \frac{1}{\alpha}\left[1-(1-\alpha)\left(\frac{A}{\omega}\frac{\partial\omega}{\partial A}\right)\right] \\ &= \frac{\frac{1}{1+\omega/\phi}}{(1-\alpha)+\beta(1-\alpha\frac{\omega/\phi}{1+\omega/\phi})} > 0.\end{aligned}$$

By definition, the financial markup along the balanced growth path is $S = \frac{\omega}{\phi}$.Thus, it is immediate that

$$\begin{aligned}\frac{\phi}{S}\frac{\partial S}{\partial\phi} &= \frac{\phi}{\omega}\frac{\partial\omega}{\partial\phi} - 1 < 0 \\ \frac{A}{S}\frac{\partial S}{\partial A} &= \frac{A}{\omega}\frac{\partial\omega}{\partial A} > 0,\end{aligned}$$

thereby yielding the claimed properties. $\square$

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Figure 1: The Structure of the Economy

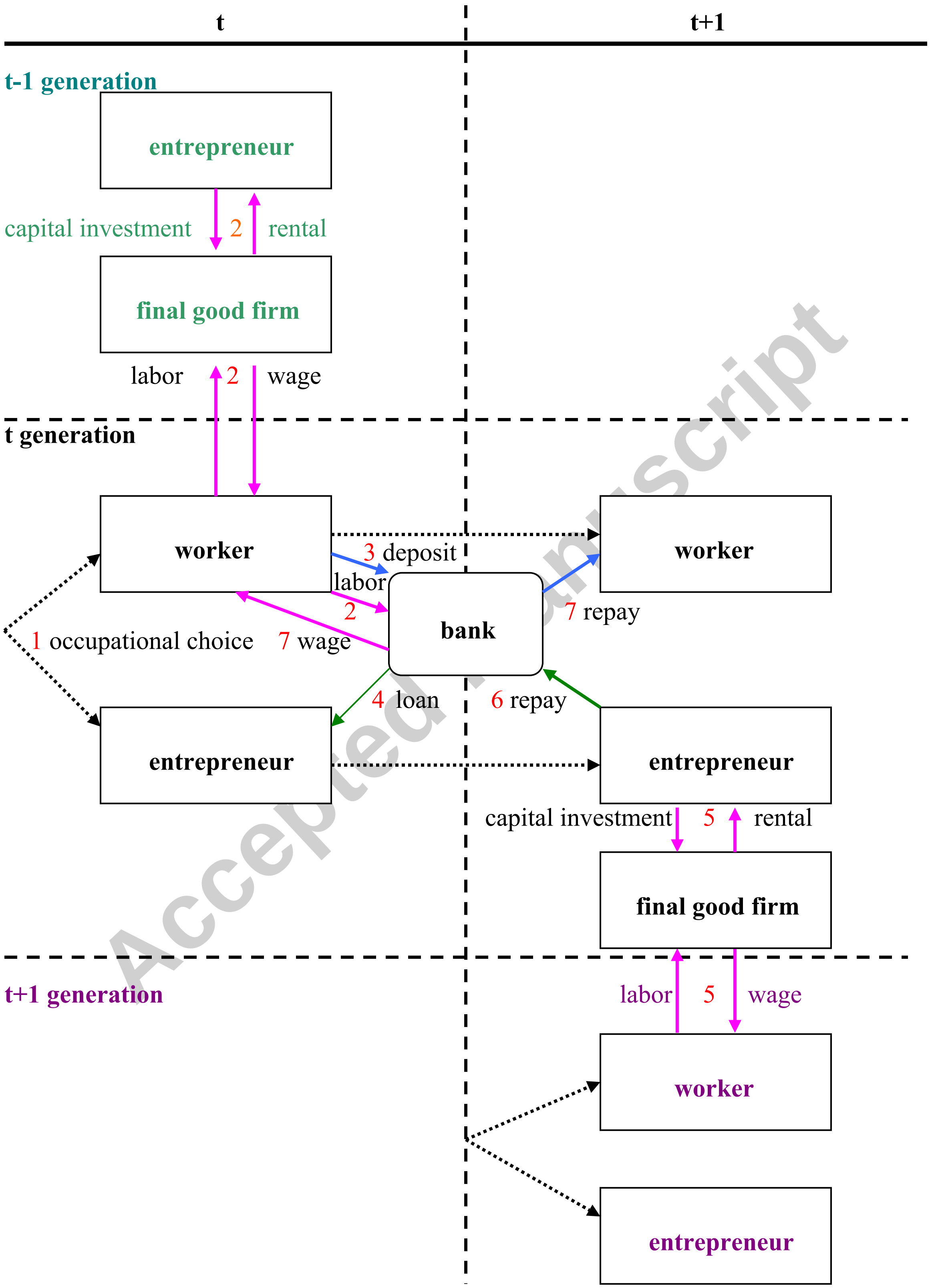

Figure 2: Occupational Choice

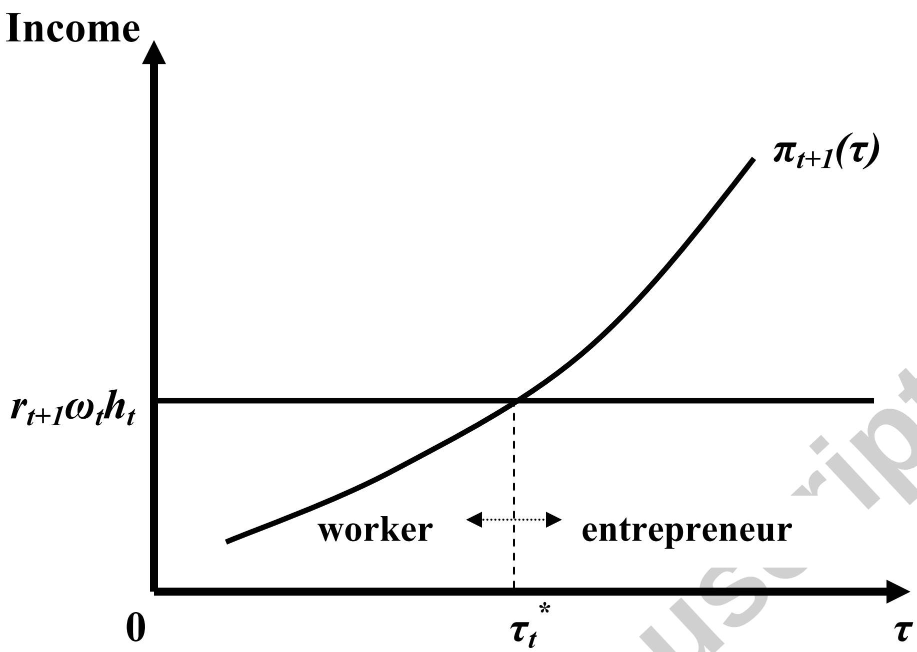


Figure 3a: Wage Dynamics (Monotonic Convergence)

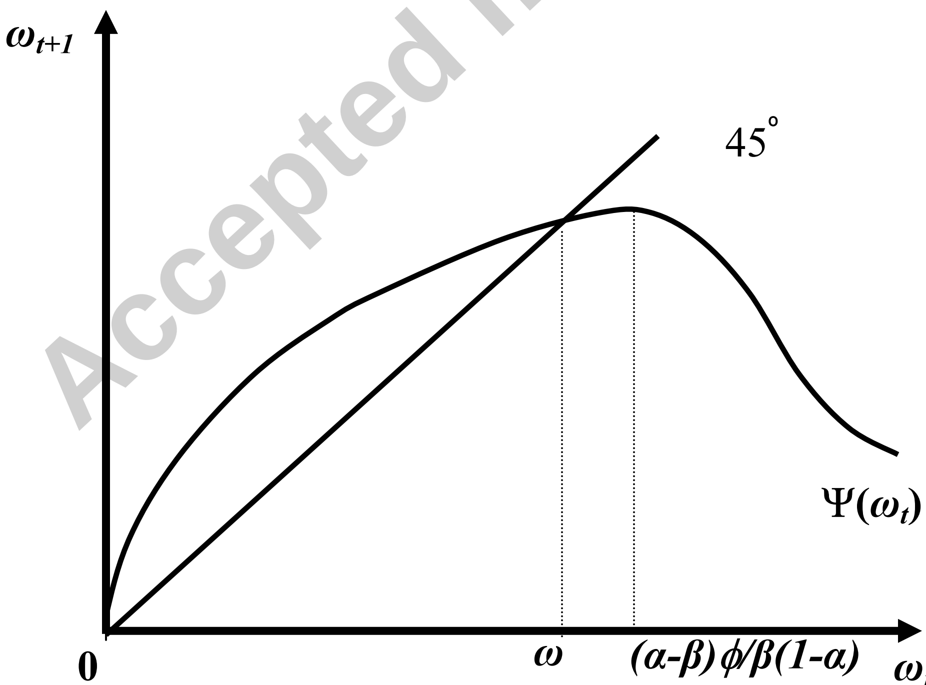

Figure 3b: Wage Dynamics (Oscillating Convergence)

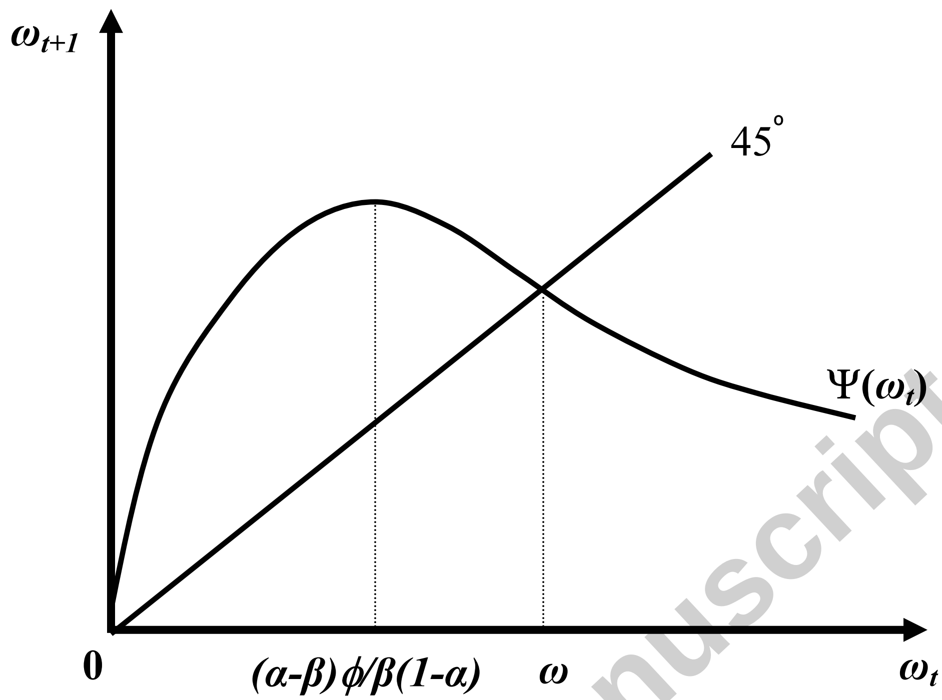


Figure 4: Multiple Credit-Constrained Balanced Growth Equilibria

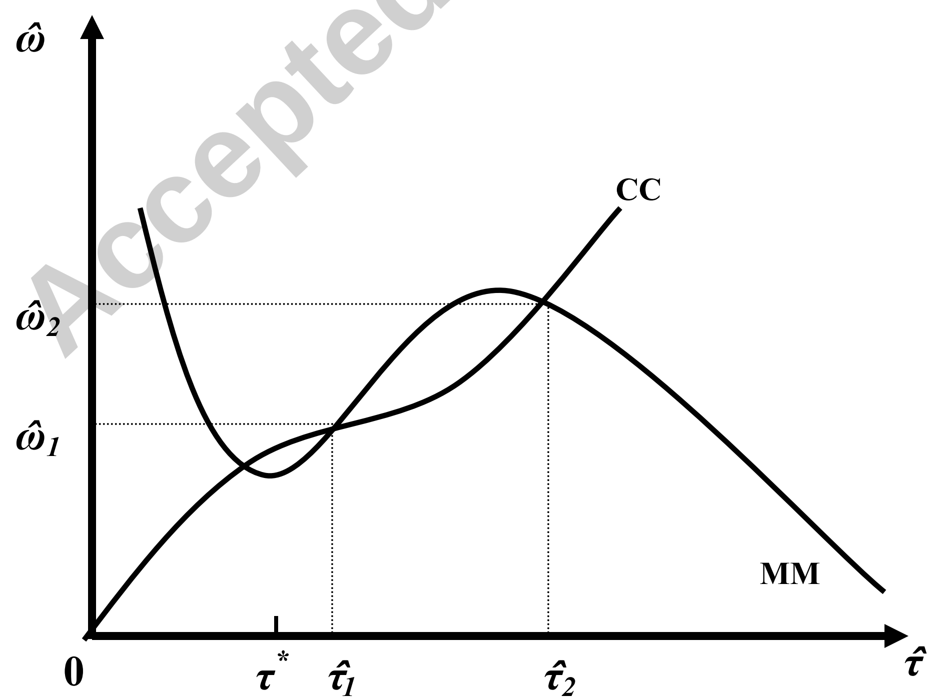

Table 1: Distribution, Entrepreneurship and Growth

| | Uniformly Rightward Shift in Ability Distribution (by $\lambda > 0$) | Proportional Stretch of Ability Distribution (with $F(\tau) = G(a\tau)$, $a>1$) |
|---|---|---|
| Wage Rate ($\omega$) | + | + |
| Cutoff Ability ($\tau^*$) | + | + |
| Mass of Entrepreneurs (N) | + | 0 |
| Growth Rate ($g$) | ? | + |

Table 2: Productivity, Financial Markup and Growth

| | Improvement in Financial Efficiency (higher $\phi$) | Advancement in Production Technology (higher $A$) |
|---|---|---|
| Wage Rate ($\omega$) | - | + |
| Loan Rate ($\delta$) | + | + |
| Deposit Rate ($r$) | + | ? |
| Financial Markup ($S = \delta/r - 1$) | - | + |
| Mass of Entrepreneurs (N) | 0 | 0 |
| Growth Rate ($g$) | + | + |