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Social Welfare and Income Inequality in a System of Cities*

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This paper develops a general-equilibrium model of a system of core-periphery cities to examine the main determinants of intra- and inter-regional income disparities. The economy is populated by a continuum of (homogeneous) unskilled and (heterogeneous) skilled workers. Unskilled workers, whose wages are determined competitively, specialize in food production in local cities; skilled workers, whose wages are determined according to a Nash bargain, manufacture a high-tech commodity in the metropolis. We establish conditions under which this core-periphery equilibrium spatial configuration emerges. We show that both intra- and inter-regional income disparities are present; the determinants of each type of disparity and the social-welfare implications are fully investigated. Our results suggest that public policy programs that improve income equality may not be necessarily welfare enhancing.

1. INTRODUCTION

What are the underlying forces which have led to widened income disparities not only between core and periphery regions, but also within

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the core, during the post World War II period?[1] Are there any geographical factors in addition to economic changes which play a significant role in explaining this observation? Are public policy programs that reduce income inequalities always welfare enhancing? In order to answer these questions properly, it is necessary to develop a general-equilibrium spatial model that allows for both intra- and inter-regional income heterogeneity. Due to its analytic complexity, however, little has been done in the existing literature and these issues remain open despite their intriguing nature.

Previous studies on urban hierarchial structures have focused primarily on the equilibrium formation of a system of cities.[2] They have generated useful insights in understanding the main determinants of the size and the number of cities in the economy. Nonetheless, their spatial structures are too stylized and fail to explain the unbalanced urban development and widened geographical income inequalities observed in reality.[3] In a companion article, Abdel-Rahman and Wang [2] make a first step toward constructing a general-equilibrium model of a core-periphery system of cities supporting a single metropolis (core) and a number of local cities (periphery). However, for analytical convenience, a symmetric wage bargain rule was adopted in that paper, thus resulting in a uniform income distribution within the core. Moreover, the model's welfare implications remain unexamined.

The present paper builds on our previous work and extends it in three significant aspects. First, we provide an analytic characterization of city size and a two-level hierarchical spatial structure in the presence of *heterogeneous* labor. Second, we allow for a *non-symmetric* Nash wage bargain which permits intra-regional (earned) income inequality. As a consequence, we can examine the main determinants of *both* intra- and inter-regional income disparities in a core-periphery spatial structure. Third, we study the *welfare* implications of changes in geographical and economic factors which lead to widened income inequalities. We accomplish these tasks by constructing a general-equilibrium spatial model in which a system of local cities surround a single metropolis and in which

[1] For example, see the study of Fujita and Ishii [12] for the case of Japan, Thailand, and China where high-tech industries, research and development (R & D) activities, and corporate and financial services are concentrated in a single dominant metropolitan area rather than in the periphery region surrounding the core.

[2] For modeling a system of cities, the reader is referred to Abdel-Rahman [1], Helsley and Strange [13], and Henderson and Abdel-Rahman [14].

[3] Although Helsley and Strange [13] allow for heterogeneous labor, all cities are identical and all workers achieve the same utility level under the symmetric equilibrium considered in their paper. Also Kim [15] applied symmetric Nash wage bargaining to determine the size of an open city where all workers achieve the same utility level in equilibrium. In contrast to previous studies, the present framework allows us to generate unequal utility levels in equilibrium.

labor heterogeneities are present both within the core and between the core and the periphery regions. We specify wages as the outcome of a Nash bargaining process; this greatly simplifies the analysis relative to the conventional competitive or monopolistically competitive framework. Upon establishing the existence of a spatial equilibrium, we examine, in the context of urban economics, two earned income inequality measures: intra- and inter-regional. This income inequality issue has not been addressed in the system of cities literature, but has been discussed under the common labor market and the local public good frameworks.[4] Furthermore, we investigate the social welfare consequences of widened geographical income inequalities and derive useful policy implications accordingly.

To study agglomerative efficiency and income inequality in a two-level hierarchical system, we begin by constructing a two-sector (food and manufacturing) general-equilibrium spatial model in a closed economy consisting of "unskilled" and "skilled" workers.[5] Unskilled workers are perfectly homogeneous and their wages are determined competitively. On the contrary, skilled workers are heterogeneous (in their skill characteristics) and their wages are determined under a non-symmetric Nash bargain. There are two commodities produced in the economy: one is a high-tech good manufactured by the skilled labor and another is a food product produced using the unskilled labor as the sole input. We establish conditions that support a two-level, *completely specialized* hierarchical system in equilibrium: the upper level consists of a single metropolis inhabited by skilled workers who produce the manufacturing good, whereas the lower level has a large number of identical cities populated by unskilled workers who produce the food product.[6]

The spatial tension within each city is specified as consisting of *agglomerative* forces (which lead to concentration of population and economic activities) as well as *dispersive* forces (which results in deconcentration). When these forces are balanced at the margin, the equilibrium size of each city in the core-periphery spatial system is then determined.[7] The only agglomerative force in the periphery is the decreasing average cost of the

[4]See, for example, Wildasin [21] and Thisse and Wildasin [19].

[5]See Christaller [8] for a description of this type of spatial distribution of cities. Notice, however, that the issues concerning agglomeration efficiency and income inequality in this spatial system still remain unexplored.

[6]Although we find conditions for supporting this two-level hierarchical system in equilibrium, we recognize that there are no dynamics regarding the partition of the spatial economy into core and periphery. Moreover, it is not our intention to investigate the spatial distribution of cities. Therefore, following the setup in Fujita [10], among others, we ignore inter-city transport costs. Should such costs be introduced, similar results can still obtain given strong enough scale economies from matching.

[7]See Fujita [10, 11] for a discussion of spatial agglomeration and Berliant and Fujita [4] for an elaboration of the general equilibrium properties of a prototypical urban economic model.

infrastructure required for an intracity transportation system (which can be regarded as increasing-returns-to-scale in the form of setup costs). In the metropolis, however, there is an additional agglomerative force. As the mass of skilled workers increases, better matches emerge for specialized skilled labor (due to a positive aggregation externality from matching to be discussed in Section 2 below). As a consequence, productivity of each manufacturing firm-skilled worker pair increases, giving an additional benefit to agglomeration. In both the core and the periphery regions, the only dispersive force considered is the increased commuting costs resulting from the expansion of the physical size of a particular city.

The organization of the remainder of the paper is as follows. In Section 2, the basic environment and assumptions are outlined. Section 3 formally develops a general-equilibrium model of a core-periphery system of cities and Section 4 establishes the existence of spatial equilibrium under a non-symmetric Nash wage bargain. In Section 5, we investigate the determinants of within-the-core and inter-regional income inequalities as well as the determinants of aggregate social welfare, and address the model's policy implications. Finally, Section 6 concludes the paper.

2. BASIC ENVIRONMENT AND ASSUMPTIONS

It is noteworthy that our ultimate goals are (i) to derive formally the conditions for supporting a core-periphery hierarchical system with earned income inequalities both within the core and between the core and the periphery regions, and (ii) to characterize the determinants of intra- and inter-regional income disparities and the level of aggregate social welfare achieved in equilibrium. This requires a model with specific functional forms for preferences as well as for production, transportation, and public infrastructure technologies. Although the use of specific functional forms will produce tedious expressions, it will also enable us to solve for the equilibrium allocations, the equilibrium welfare level and the earned income inequality measure analytically.

We consider an economy in which a system of cities are spread over a flat featureless plain. There are two final goods (denoted by x and z) which are produced and traded at no transportation cost within the spatial system.[8] Both goods are assumed to be homogeneous: x is a food product made by (a continuum of) unskilled workers (of mass N) and z is a high-tech product manufactured by (a continuum of) skilled workers (of mass M). For convenience, we treat z as the numeraire. Unskilled workers are assumed to be perfectly homogeneous. Skilled workers, however, are

[8] If transporting and trading goods are costly, we can still rule out autarky by assuming that the utility gains from trade are sufficient to dominate these costs. See, for example, the assumption imposed in Berliant and Wang [6].

heterogeneous in their *skill characteristics*. Their types (of T) can be thought of as uniformly distributed on a circle with unit circumference such that $M(T) = M_0$ for all $T \in [0, 1]$. Since the mass of skilled workers can be written as $M = \int_0^1 M(T)\, dT = M_0$, it implies $M(T) = M$ for all $T \in [0, 1]$. The wages of unskilled workers (W_L) are determined competitively, whereas those of skilled workers (W_M) are determined via a Nash bargain with high-tech firms.

While the food production sector is entirely conventional, we should elaborate on the distinctive features of the production process for the high-tech good. Figure 1 provides a graphical description for the productivity profile over the ‘‘skill-characteristic’’ space (rather than the commonly used space of locations). Notably, each individual firm of type $S \in [0, 1]$ in the high-tech industry has a different skill requirement. In particular, it would be ideal for a firm of type S to employ a worker of skill-type $T = S$ since a perfect match generates maximal output per worker (i.e., maximal productivity), denoted by $a > 0$. Skill matching is generally imperfect: a high-tech firm of type S may need to employ a skilled worker of type T with distance $\delta \equiv |T - S|$ away from the ideal skill match. In this latter

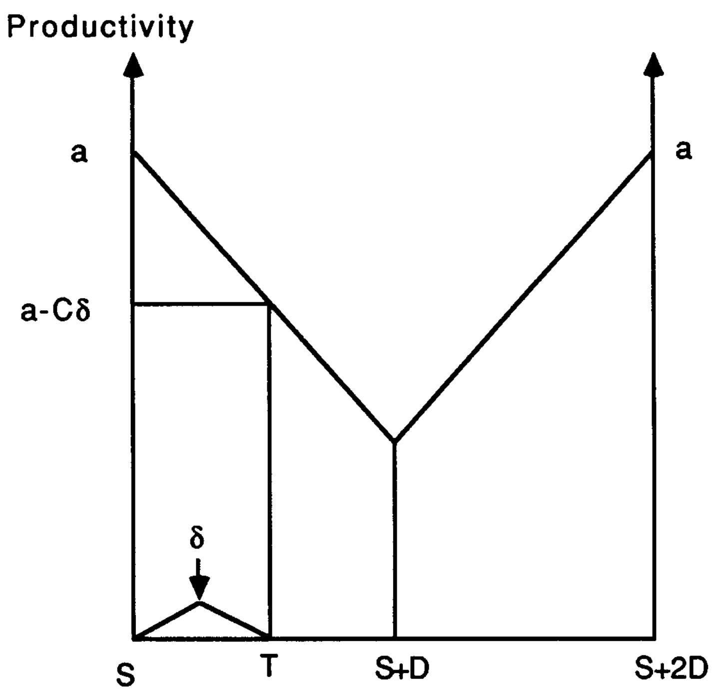

FIG. 1. Skill characteristics and productivity.

case, the correspondent productivity is given by $a - C\delta$, where $C > 0$ measures the productivity loss from "mismatch." After paying a fixed entry cost (K), a matched high-tech firm can bargain with a skilled worker of a particular type. We will show in Section 3.E below that under a non-symmetric Nash bargain, the wage of a skilled worker of type T employed by a high-tech firm of type S will depend on the distance of mismatch, δ.

Evidently, skilled workers are capable of producing the food product as such activities require no skills. In contrast, due to particular skill requirements, the high-tech commodity cannot be manufactured by unskilled workers. For convenience, we use subscript $i \in \{L, M\}$ to indicate where a household resides: subscript L refers to a local city, whereas subscript M represents the metropolis). To support a core-periphery system of cities in equilibrium under which all unskilled workers reside in (a continuum of) local cities (of an identical size, L) and all skilled workers reside in the single metropolis, it is necessary to assume that high-tech workers are paid higher wages in the high-tech good sector than in the food sector: $W_{\mathrm{M}}(\delta) > W_{\mathrm{L}}$ for all $\delta \in [0, 1]$. Under this assumption, no skilled workers are willing to produce the food product.

We further assume that the skill distribution remains the same as the size of the high-tech city changes.[9] Given the mass of skilled workers M, if there is more than one high-tech city, then the population of skilled workers in each high-tech city must be lower. As a consequence, the probability for a high-tech firm-skilled worker pair to have a perfect match is lower. These considerations provide our spatial economy with a positive aggregation externality from skill matching in the sense that the concentration of skilled workers results in better matches and thus enhances the productivity of the high-tech good production.

For simplicity, we treat the location of each central business district (CBD) as given *a priori*.[10] To engage in production, workers residing in a given city are required to commute to the CBD. A worker's commuting cost (in units of z) is assumed to be a linearly increasing function of the distance from the residence to the CBD. This commuting cost discourages a city to expand unboundedly. Cities are formed due in part to increasing returns in the provision of public infrastructure on intracity transportation or public utility necessary for developing and operating a city. When a

[9] This assumption rules out the possibility that a perfectly matched high-tech firm-skilled worker pair alone form a high-tech city. This type of assumption is widely used in the anonymous Nash bargain literature in that given a large number of economic agents, collective bargaining or coalition formation are not allowed (for example, see Diamond [9]). In the context of urban economics, this assumption implies an open migration policy in that no pre-migration agreement can be imposed. It is realistic in particular if the total mass of the skilled labor is so large that firms cannot know a worker's skill type *ex ante*.

[10] For a general equilibrium treatment of endogenous CBD, see Wang [20].

two-level completely specialized hierarchical system is formed, we express the public infrastructure cost schedules for local cities and the metropolis, respectively, as to depend on their population mass in the following forms: FL^{ε} and FM^{ε}, where $\varepsilon \in (0,1)$ and $F > 0$ is the shift parameter of the public infrastructure cost. Without loss of generality, we simply assume that these public infrastructure services are produced using the high-tech commodity z (i.e., the *numeráire*). Since $\varepsilon < 1$, it can be easily verified that the (per capita) share of the infrastructure cost is decreasing in the size of the city.[11]

In equilibrium, the net marginal cost of city expansion (i.e., the marginal benefit from reduced public infrastructure costs net of the marginal cost of increased commuting expenses) must be zero in local cities. For high-tech cities, we assume that the positive external effect of matching from an increased mass of skilled workers is sufficiently large to dominate the net marginal cost of city expansion. Under this assumption, there must be a *single* metropolis, since a reduced city size under a multiple high-tech city structure would lead to lower consumer welfare. Moreover, this assumption, together with the condition ensuring that skilled workers receive higher wages in the high-tech industry, will be sufficient to guarantee that no skilled workers intend to reside in local cities. Therefore, the core-periphery system can be supported in equilibrium.

Finally, we assume that the total expenditure on infrastructure and commuting costs and the fixed entry costs for high-tech firms are not too large. These conditions will ensure the positiveness of equilibrium prices and the non-negativity of equilibrium consumption allocations. Both are necessary for the establishment of a spatial equilibrium.

3. THE MODEL

3.A. *Households*

Each consumer/worker (or household, in short) is endowed with one unit of labor. He/she is free to take one job and to reside in a lot with a fixed size (normalized to unity) in a particular city where the affiliated firm is located.[12] All households are identical in every aspect except for their skill types. Their (identical) preference over the two consumption goods (x

[11] With the infrastructure cost function specified above, our model economy in effect exhibits increasing returns in the context of city expansion, which will be balanced out on the margin by the increased transportation costs.

[12] The assumption of fixed lot-size (or inelastic demand for land) is common in the system of cities literature. It circumvents the difficulty associated with general equilibrium models with continuous location and variable lot size as pointed out by Berliant and ten Raa [5] and Berliant and Wang [6]. See Berliant and Fujita [4] for a general equilibrium structure of a monocentric city with elastic land in which land is modeled in a ''discrete'' manner.

and z) is assumed to take a simple Cobb–Douglas form:

$$u = Ax^{\alpha}z^{(1-\alpha)}, \tag{3.1}$$

where $A > 0$ and $\alpha \in (0, 1)$. Since this utility functional form satisfies the Inada condition (i.e., $\lim_{j \to 0} \partial u / \partial j = \infty$ and $\lim_{j \to \infty} \partial u / \partial j = 0$, $j \in \{x, z\}$), it follows that autarky cannot be an optimal decision for an individual household.

Define P as the relative price of x in units of z. In addition to commuting costs, each household must incur expenses on land rent. The aggregate land rent in a local city and in the metropolis are denoted by ALR_{L} and ALR_{M}, respectively. Under the assumption of public ownership of land, the aggregate land rent in a particular city net of the correspondent public infrastructure cost will be shared equally by the residents in that city.[13] Denote R_i as the unit land rent, t as the amount of good z used for commuting a unit distance, and W_i and τ_i as the correspondent wage and non-wage incomes, respectively, for $i \in \{L, M\}$. Then the budget constraint facing a representative household who resides at distance r from the CBD in a particular city $i \in \{L, M\}$ can be written as

$$z(r) + Px(r) + R_{\mathrm{L}}(r) = Y_{\mathrm{L}} - tr, \tag{3.2}$$

$$z(r) + Px(r) + R_{\mathrm{M}}(r) = Y_{\mathrm{M}} - tr, \tag{3.3}$$

where $Y_i = W_i + \tau_i$ measures household income in units of z and is the sum of wage and non-wage incomes (the latter is acquired from the redistributed profit of city development if it is not zero).

Each household seeks to maximize its utility as specified in (3.1) subject to the correspondent budget constraint, (3.2) or (3.3). Straightforward differentiation yields the uncompensated demand for x and z in a city $i \in \{L, M\}$:

$$x_i(r) = \alpha P^{-1}[Y_i - tr - R_i(r)], \tag{3.4}$$

$$z_i(r) = (1 - \alpha)[Y_i - tr - R_i(r)]. \tag{3.5}$$

As expected, the uncompensated demand for a particular good is increasing in the household's net income but decreasing in the price of that good.

3.B. *Locational Equilibrium and City Formation*

Assume that there is no idle land in any given city. Since each household inelasticly consumes one unit of land, the circular-city structure implies

[13] This assumption is common in the general-equilibrium models of system of cities because it allows us to close the model without including a separate group of land owners. An alternative is to introduce absentee landlords, where they are assumed to live in the hinterland.

that the total population of a representative local city with border b is equal to

$$L = \int_0^b 2\pi r\, dr = \pi b^2. \tag{3.6}$$

Hence, the number of local cities in the spatial system is given by $n = N/L$. Similarly, the population of the single metropolis with fringe f can be written as

$$M = \int_0^1 M(T)\, dT = \int_0^f 2\pi r\, dr = \pi f^2. \tag{3.7}$$

Notably, the first equality in (3.7) is a direct consequence of the assumption of uniformly distributed skill types (T) over $[0, 1]$, while the second equality is due to the assumptions that ensure a single metropolis (formal conditions will be provided in Section 4 below).

Locational equilibrium requires that within each city, all households of a particular type must achieve the same utility level. With a Cobb–Douglas preference, it can be easily shown that for any location $r \le b$ in a local city and $r \le f$ in the metropolis, we need $Y_L - tr - R_L(r) = Y_L - tb - R_L(b)$ and $Y_M - tr - R_M(r) = Y_M - tb - R_M(f)$, respectively. Following the conventional wisdom, we normalize the opportunity cost of land to be zero such that $R_L(b) = R_M(f) = 0$. Then, the land rent schedules under locational equilibrium become $R_L(r) = t(b - r)$ and $R_M(r) = t(f - r)$. Utilizing (3.6) and (3.7) and integrating the above land rent schedules, we can derive the aggregate land rent in a given city $i \in \{L, M\}$ as

$$ALR_L = \int_0^b R_L(r) 2\pi r\, dr = \lambda L^{3/2} \tag{3.8}$$

$$ALR_M = \int_0^f R_M(r) 2\pi r\, dr = \lambda M^{3/2}, \tag{3.9}$$

where $\lambda \equiv t/(3\pi^{1/2})$ is an increasing function of the unit commuting cost, t. From (3.8) and (3.9), the non-wage income of a representative household residing in city $i \in \{L, M\}$ can be derived as

$$\tau_L = \left(ALR_L - FL^{\varepsilon}\right)/L; \qquad \tau_M = \left(ALR_M - FM^{\varepsilon}\right)/M. \tag{3.10}$$

The equilibrium values of τ_L and τ_M will be determined endogenously in Section 4 below.

3.C. *Food Production*

The food industry is perfectly competitive. Given the assumption ensuring that no skilled workers are willing to engage in food production, unskilled workers are the only input in this industry. The production is assumed to take a simple constant-returns-to-scale form:

$$X = \gamma L, \tag{3.11}$$

where $\gamma > 0$ measures the productivity of the unskilled labor.[14] Then production efficiency implies that a food producer will hire unskilled workers at a wage rate equal to their marginal product:

$$W_{\mathrm{L}} = \gamma P. \tag{3.12}$$

Within this framework, it is obvious that there is no income disparity among unskilled workers.

3.D. *High-Tech Good Production*

The high-tech industry employs skilled workers of different skill types. As mentioned above, a particular high-tech firm needs to hire not only perfectly matched but also closely matched skill workers in order to undertake production. Let D denote the maximal distance of skill mismatch acceptable to the particular firm, which will be endogenously determined in a Nash equilibrium. We can now specify the output of a high-tech firm of type S as

$$Z(S) = 2\int_{S}^{S+D} (a - C\delta) M\, d\delta - K = 2\int_{0}^{D} (a - C\delta) M\, d\delta - K, \tag{3.13}$$

where the second equality is a result of symmetry. Since each firm employs skilled workers over an interval of measure $2D$, the total number of firms in the high-tech industry will be $m = 1/(2D)$. Under the symmetry assumption, the profit of a type-S firm is then given by

$$J(S) = 2\int_{0}^{D} \left[(a - C\delta) - W_{\mathrm{M}}(\delta)\right] M\, d\delta - K. \tag{3.14}$$

Unrestricted entry implies that each high-tech firm will earn zero profit in equilibrium.

3.E. *Non-Symmetric Nash Bargain*

We depart from the simple symmetric Nash bargain framework of Diamond [9] to allow the division fraction of the matching surplus to depend on the relative bargaining power between a high-tech firm and a

[14] It is assumed that labor is more productive in a local city than in the hinterland, which provides the incentive for forming a local city.

skilled worker.[15] Consider the bargaining situation facing a skilled worker with a characteristic distance δ away from the most adjacent firm. To bargain with the most adjacent firm, a worker needs to take the best alternative (i.e., the threat point) into account. In principle, the best alternative could be the potential earning from employment in the food industry or by the second most adjacent high-tech firm. The latter is possible because a skilled worker can be hired by any high-tech firms *ex ante*. When the unskilled wage is sufficiently low (see Assumption 1 in Section 4.C below), the best alternative will be the potential earning from employment by the second most adjacent firm. The surplus of matching between this particular worker and the most adjacent firm is therefore the productivity differential between the most productive match, $a - C\delta$, and the best alternative, $a - C(2D - \delta)$. We plot these two productivity schedules for $\delta \in [0, 2D]$ in Fig. 2. Notably, when the two productivity schedules intersect with each other, the area of employment over the skill space (D) is determined. Therefore, *ex post* upon matching, it is the best interest for a firm of type S to employ skill workers only over the interval $[S - D, S + D]$.

It can be easily seen from Fig. 2 that the matching surplus is measured by the difference between the two productivity schedules (for a particular skill worker matching with the most adjacent and the second most adjacent firms): $G = (a - C\delta) - [a - C(2D - \delta)] = 2[(a - C\delta) - (a - CD)] = 2C(D - \delta)$. Let θ denote the fraction of the surplus going to the firm. Then the wage schedule of a skilled worker of type δ is $W_M(\delta) = a - C\delta - \theta G$. Utilizing the expression for G, we have

$$W_M(\delta) = a - 2\theta CD - (1 - 2\theta)C\delta, \qquad \delta \in [0, D]. \tag{3.15}$$

It is evident that when the bargaining power of high-tech firms is higher (i.e., a larger θ), the wage rate for skilled workers of type δ will be lower (recall that $\delta < D$). For the remainder of this paper, we are interested in the case where an improvement in skill matching (i.e., a lower δ) affects wages in a non-negative manner, i.e., $dW_M/d\delta \leq 0$. This is guaranteed when workers have relatively stronger bargaining power so that $0 \leq \theta \leq 1/2$.[16]

[15] More complicated search, matching and wage bargaining processes can be found in Burdett and Mortensen [7], Rubinstein and Wolinsky [17], and Laing *et al.* [16].

[16] Observe that if $\theta = 1/2$ this wage schedule reduces to the uniform wage as appeared in Kim [15] and Abdel-Rahman and Wang [2] in which within-the-core income disparities are absent.

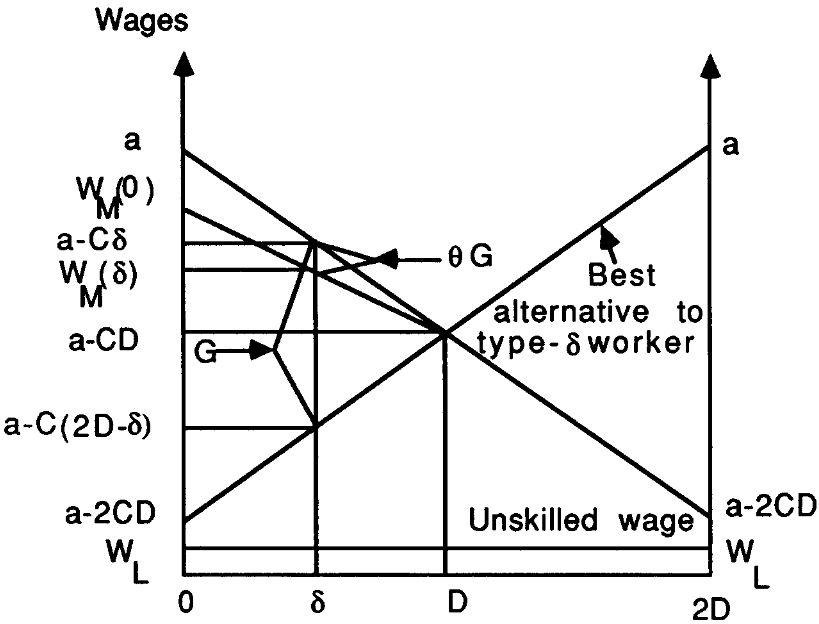

FIG. 2. Nash wage bargain.

4. EQUILIBRIUM

4.A. *Equilibrium Size of Local Cities*

Local cities are formed by a local government whose objective is to maximize the utility level achieved by the representative household for a given relative price schedule. Since all households in each local city are identical in every aspect, they must achieve the same utility level in equilibrium. In order to solve for the equilibrium size of a local city, we first substitute (3.2), (3.4), (3.5), (3.6), (3.8), and (3.10) into (3.1) to obtain the indirect utility for a representative local-city household as

$$U_L = A_0 P^{-\alpha}\left[\gamma P - 2\lambda L^{1/2} - FL^{\varepsilon - 1}\right]. \tag{4.1}$$

Note that all individual households are price takers. By maximizing the indirect utility for a given P, the desired or "equilibrium" size of each local city must satisfy the following first-order condition: $\lambda L^{1/2} = (1 - \varepsilon)FL^{\varepsilon-1}$ or, multiplying both sides by L and rearranging terms, $FL^{\varepsilon} = \lambda L^{3/2} + \varepsilon FL^{\varepsilon}$. This latter equation implies that the size of a given local city is chosen such that the cost of infrastructure (FL^{ε}) is exactly financed by the aggregate land rent [$\lambda L^{3/2}$, as derived in (3.8) above] and a "congestion tax" ($\varepsilon F L^{\varepsilon}$), which can be regarded as a modified version of the *Henry George Theorem*.[17] Straightforward manipulations of the first-order condition yield

$$L^{*} = \left[3(1-\varepsilon)F\pi^{1/2}/t\right]^{2/(3-2\varepsilon)}, \tag{4.2}$$

implying that the equilibrium size of each local city is decreasing in the unit commuting cost (t) and increasing in the shift parameter of the infrastructure cost required for the formation of a city (F).

4.B. *Equilibrium Entry and Determination of Equilibrium Skilled Wages*

As in the case of local cities, the metropolis is formed by a city government. However, the role of the city government, in this case, is more passive—it is limited to the provision of public infrastructure required for the formation of the metropolis by imposing the necessary tax to finance it. This is because in equilibrium, the system of cities contains only one metropolis with skilled workers and its size is thus fixed by the population of the skilled labor.

By unrestricted entry, each firm in the high-tech industry will achieve zero profit in equilibrium, i.e., $J(S) = 0$ for all S. Thus, from (3.14) and (3.15), the equilibrium area of skilled worker employment and the equilibrium number of high-tech firms in the metropolis can be derived as

$$D^{*} = \left[K/(\theta C M)\right]^{1/2} \tag{4.3}$$

$$m^{*} = \left[\theta C M/(4K)\right]^{1/2}. \tag{4.4}$$

These results indicate that the equilibrium number of high-tech firms is increasing in the bargaining power of high-tech firms (θ), the mass of skilled workers (M), and the marginal productivity loss due to mismatch (C), but decreasing in the fixed entry cost (K).

[17] The congestion tax is larger the greater is the value of ε. Notice also that the second-order condition is $(1/6)\lambda L^{-1/2} < (1-\varepsilon)(2-\varepsilon)FL^{\varepsilon-1}$, which is automatically satisfied at L^{*}. In addition, it is noteworthy that our equilibrium concept regarding the determination of the size of each local city involves partially social optimality. In particular, while all other decisions are decentralized, the choice of the city size requires coordination among individual households (i.e., a welfare-maximizing government is under operation). Similar concepts have been used in Anas [3] in characterizing the size of a city under "laissez faire" and in Wang [20] in determining uniquely the location of marketplaces. This modification of the equilibrium concept is due to the fact that purely competitive setups will generally lead to indeterminacy of spatial equilibria.

Substituting (4.3) into (3.15), we get the equilibrium wages of a skilled worker of type δ:

$$W_{\mathrm{M}}(\delta) = a - 2(\theta CK/M)^{1/2} - (1-2\theta)C\delta, \qquad \delta \in [0, D]. \quad (4.5)$$

A straightforward comparative-static analysis provides a characterization of the wages of skilled workers:

PROPOSITION 1. *The skilled wage rate is increasing in the level of maximal productivity and the mass of skilled workers, but decreasing in the bargaining power of high-tech firms, the fixed entry cost, the mismatch cost, and the scale of skill mismatch. In the special case with a symmetric Nash bargain where the matching surplus is equally divided between firms and workers* (*i.e.*, $\theta = 1/2$), *the skilled wage rate is independent of each worker's skill characteristic.*

4.C. *Material Balance and Determination of Equilibrium Prices*

Substitution of (3.12) and the expression for rental and commuting expenses into (3.4) and (3.5) (with $i = L$) yields an unskilled worker's demand for goods:

$$x_{\mathrm{L}} = x_{\mathrm{L}}(r) = \alpha P^{-1}(\gamma P - \beta) \quad (4.6)$$

$$z_{\mathrm{L}} = z_{\mathrm{L}}(r) = (1-\alpha)(\gamma P - \beta), \quad (4.7)$$

where $\beta \equiv F^{1/(3-2\varepsilon)}(3-2\varepsilon)[3(1-\varepsilon)\pi^{1/2}/t]^{-(2-2\varepsilon)/(3-2\varepsilon)}$ represents an unskilled worker's expenditure on land rent and commuting net of non-wage income. Using the expressions for the land rent and manipulating (3.4) and (3.5) (with $i = M$), (3.10) and (4.5), we can derive a type-δ skilled worker's demand for goods:

$$x_{\mathrm{M}}(\delta) = x_{\mathrm{M}}(\delta, r) = \alpha P^{-1}\Big[a - 2(\theta CK/M)^{1/2} - (1-2\theta)C\delta - 2\lambda M^{1/2} - FM^{\varepsilon-1}\Big], \quad (4.8)$$

$$z_{\mathrm{M}}(\delta) = z_{\mathrm{M}}(\delta, r) = (1-\alpha)\Big[a - 2(\theta CK/M)^{1/2} - (1-2\theta)C\delta - 2\lambda M^{1/2} - FM^{\varepsilon-1}\Big]. \quad (4.9)$$

The material balance condition for the food product, x, requires that total demand be equal to total supply. Since the measure of each type of skilled workers is M/D, total demand for x is the sum of food demand by unskilled workers (Nx_{L}) and that by skilled workers $[(M/D)\int_0^D x_{\mathrm{M}}(\delta)d\delta]$. On the other hand, given the constant-returns-to-scale technology as specified in (3.11), total food supply by the entire food industry can be derived as γN. The material balance condition can thus be written as: $Nx_{\mathrm{L}} + (M/D)\int_0^D x_{\mathrm{M}}(\delta)\,d\delta = \gamma N$, which, together with (4.6) and (4.8), determines the equilibrium price of x in units of z:

$$P^* = \alpha[(1-\alpha)\gamma N]^{-1}[(a - 2K - \rho)M - \beta N - \sigma], \quad (4.10)$$

where $\rho \equiv 2\lambda M^{1/2} + FM^{\varepsilon-1}$ and $\sigma \equiv [2 + (1/2 - \theta)/\theta](\theta CK/M)^{1/2}$. By totally differentiating (4.10), we obtain:

PROPOSITION 2. *The equilibrium price of the food product in units of the high-tech good is increasing in the level of maximal productivity of skilled workers, but decreasing in the mass of unskilled workers, the mismatch cost, the fixed entry cost, the city formation cost, the unit commuting cost, and the productivity of unskilled workers. The effect of the mass of skilled workers on the equilibrium price is, however, ambiguous.*

An increase in the mass of unskilled workers (N) enlarges the supply of good x, thus lowering the equilibrium price of x. But the effect of the mass of skilled workers (M) on P^* is ambiguous in sign due to the opposing effects from the positive matching externality and the changes in commuting/infrastructure costs.[18] An increase in M results in higher aggregate productivity for skilled workers due to better matches; however, it also leads to higher commuting/infrastructure costs in the metropolis. The former effect tends to increase the relative price of x by raising the supply of good z, whereas the latter suppresses P^* via an enlarged demand for z. Therefore, the net effect will depend on the sign of $[a - (3\lambda M^{1/2} + \varepsilon FM^{\varepsilon-1} + \sigma/2)]$; if the level of maximal productivity of skilled workers is sufficiently large as compared to the cost parameters, one can expect an increase in the mass of skilled workers to result in a higher relative price of x. Moreover, an increase in the level of maximal productivity of skilled workers or a decrease in the mismatch cost or the fixed entry cost of the high-tech firm raises the supply of good z and hence increases the relative price of x. Furthermore, recall that both commuting and public infrastructure require the use of the high-tech commodity; as a consequence, a higher cost of forming a city or commuting increases the demand for z, thus reducing the relative price of x.

4.D. *Existence of Spatial Equilibrium*

By substituting (4.6)–(4.10) into (3.1), we can now obtain the equilibrium utility levels for households residing, respectively, in a given local city and

[18] From (4.10) we can derive: $dP^*/dN = -\alpha[(1-\alpha)\gamma N]^{-1}\beta - P^*N^{-1} < 0$ and $dP^*/dM = \alpha[(1-\alpha)\gamma N]^{-1}[a - 2K - 3\lambda M^{1/2} - \varepsilon FM^{\varepsilon-1} - \sigma/2]$ (which is ambiguous in sign).

in the metropolis:

$$U_L^* = A_0(P^*)^{-\alpha}(\gamma P^* - \beta) \quad (4.11)$$

$$U_M^*(\delta) = A_0(P^*)^{-\alpha}\left[a - \rho - 2(\theta CK/M)^{1/2} - (1 - 2\theta)C\delta\right], \quad (4.12)$$

where $A_0 \equiv A\alpha^{\alpha}(1 - \alpha)^{1-\alpha}$ is a constant scaling factor.

We are now ready to establish sufficient conditions for supporting the Nash bargaining outcome as well as the two-level hierarchical system in spatial equilibrium. Recall that our Nash solution is obtained under the assumption that the unskilled wage rate is *not* the best alternative to skilled workers of all types. Since the minimal productivity of a skilled worker employed by the second most adjacent firm is $a - 2CD$, it is sufficient to assume that $a - 2CD > W_L$. Define $\Delta \equiv (1 - \alpha)N - \alpha M$. Utilizing (3.12), (4.3), and (4.10), this sufficient condition can then be written as $\Delta a > \{2(1 - \alpha)N[CK/(\theta M)]^{1/2} - \alpha[(\rho + \sigma)M + \beta N]\}$, which can be guaranteed under,

Assumption 1.

(a) $\Delta > 0$,

(b) $a > \Delta^{-1}\{2(1 - \alpha)N[CK/(\theta M)]^{1/2} - \alpha[(\rho + \sigma)M + \beta N]\}$.

Assumption 1a requires a sufficiently large mass of unskilled workers relative to skilled workers, which is not necessary but made for convenience. To be more specific, under Assumption 1a, if the level of maximal productivity of skilled workers is sufficiently high such that Assumption 1b holds, then the unskilled wage rate will be too low to be the best alternative to any skilled workers.

Notably, given Assumption 1 and the presence of a positive matching surplus distributed to skilled workers [i.e., $(1 - \theta)G > 0$], it is obvious that skilled workers always earn higher wages than the unskilled, i.e., $W_M(\delta) > W_L$ for all $\delta \in [0, D]$. That is, a skilled worker would never engage in food production, despite his/her ability in doing so. This assumption immediately results in an inter-regional earned income inequality. However, rather than explaining why the inter-regional income disparity is present, we wish to focus on the main factors affecting the "magnitude" of such an inequality.

Recall that all individual households are price takers. In order to guarantee that only a single metropolis can exist in equilibrium, we must establish a condition ensuring that, given the same relative price, P^*, there will be no incentive for any skilled worker to move out of the existing metropolis to form a new city. This is guaranteed if the indirect utility of any skilled worker achieved is strictly increasing in the mass of skilled workers [i.e., $dU^*_{\mathrm{M}}(\delta)/dM > 0$ for all δ], which can be formally written as:

Assumption 2. $M^{1/2}(\theta cK)^{1/2} > \lambda M^{3/2} - (1 - \varepsilon)FM^{\varepsilon}$.

This assumption requires that as the city expands, the marginal benefit from positive matching externalities exceeds the marginal cost of increased transportation costs net of the marginal benefit from reduction in per capita city formation costs. This therefore supports a single high-tech metropolis.

To complete the analysis regarding the existence of a spatial equilibrium, it is also necessary to find conditions ensuring the positiveness of the equilibrium relative price as well as the non-negativity of the equilibrium consumption allocations: $P^* > 0$, $x_i \geq 0$, and $z_i \geq 0$ for $i \in \{L, M\}$. We have

Assumption 3.

(a) $\beta < (\alpha M/N)(a - \rho - \sigma)$,

(b) $K \leq [1 + (1/2 - \theta)]^{-2}[\theta/(CM)](\rho M + \beta N)^2$.

It is straightforward to show that when the total expenditure on infrastructure and commuting costs are not too large (Assumption 3a), the equilibrium relative price will be always positive. On the other hand, when the fixed entry costs of the high-tech firms are not too high (Assumption 2b), the equilibrium consumption allocations by unskilled workers will be always non-negative. Given Assumption 1 which ensures that skilled workers receive higher wages than unskilled workers, Assumption 3b also guarantees the non-negativity of x_{M} and z_{M}.

Given any set of parameters $(\alpha, a, F, t, K, N, M)$ satisfying Assumptions 1–3, it guarantees the existence of a unique two-level equilibrium hierarchical system in which the core is inhabited by skilled workers who produces only the high-tech good (z) and all periphery cities are populated by unskilled workers who produces only the food product (x). The equilibrium wage rates for unskilled and skilled workers $(W_{\mathrm{M}}$ and $W_{\mathrm{L}})$ are governed by (3.12) and (4.5), respectively. Under Assumption 1, the former is strictly lower than the latter. The equilibrium size of any local city (L^*) is given by (4.2), while the equilibrium employment area (D^*) and the equilibrium number of high-tech firms (m^*) are determined by (4.3) and (4.4). The equilibrium relative price of x in units of z is derived as in (4.10), whereas the equilibrium consumer welfare levels (equilibrium utilities) achieved by unskilled and skilled workers are presented in (4.11) and (4.12), respectively. It can be seen that while all households residing in local cities (unskilled workers) attain the same equilibrium utility level, households in the metropolis (skilled workers) achieve different utility levels in equilibrium. The latter is due to the considerations of labor-skill heterogeneities and non-symmetric Nash wage bargain. As a consequence, our model allows for both inter-regional and intra-regional (within-the-core) income inequalities.

5. INCOME DISPARITIES AND WELFARE IMPLICATIONS

5.A. *Income Inequality Measures*

There are, in general, several ways to measure income disparity. In our paper, we restrict our attention to the *earned income inequalities* both within-the-core and between the core and the periphery regions.[19] In particular, we can specify the within-the-core income inequality as

$$I \equiv (1/D^*) \int_0^{D^*} \left[W_{\mathrm{M}}^*(\delta) - (a - CD^*) \right] d\delta, \tag{5.1}$$

where $a - CD^*$ reflects the lowest skilled wage rate when the mismatch reaches the maximal acceptable critical value. This is a measure similar in spirit to the Gini coefficient. On the other hand, the inter-regional inequality can be measured by

$$Q \equiv (1/D^*) \int_0^{D^*} \left[W_{\mathrm{M}}^*(\delta) - W_{\mathrm{L}}^* \right] d\delta, \tag{5.2}$$

which evaluates the *average* difference between a skilled worker and an unskilled worker.

5.B. *Determinants of Income Disparities*

Combining (4.3), (4.5), and (5.1), the within-the-core income inequality measure is

$$I = 3\left[(1/2 - \theta)/\theta \right] (\theta CK/M)^{1/2}. \tag{5.3}$$

Analogously, substitution (3.12) and (4.5) into (5.2) yields the inter-regional inequality measure

$$Q = a - \left[2 - (1/2 - \theta)/\theta \right] (\theta CK/M)^{1/2} - \gamma P^*. \tag{5.4}$$

[19] Since all households residing in the metropolis receive an identical net non-wage income, the consideration of non-wage income disparity will not affect the within-the-core inequality measure. We recognize that non-wage income differentials will affect the inter-regional income inequality. However, it is worth noting that non-wage income disparity is an artifact of the setup of the city government and that there are no clear-cut comparative-static results regarding the non-wage income inequality measure. Thus, we will focus only on characterizing earned income inequalities.

Totally differentiating (5.3) and (5.4) and utilizing Proposition 2 (which characterizes P^*), we can study the determinants of these two income inequality measures and summarize the main results in the following:

PROPOSITION 3.

(a) *The within-the-core income inequality is increasing in the cost of entry or mismatch, but decreasing in the bargaining power of high-tech firms as well as the mass of skilled workers.*

(b) *The inter-regional income inequality is increasing in the commuting or city formation cost as well as in the mass of unskilled workers and the level of maximal productivity of skilled workers.*

(c) *If the bargaining power of high-tech firms is sufficiently high* ($1/6 < \theta \leq 1/2$) *and the productivity of unskilled workers* (γ) *is sufficiently low, then the inter-regional income inequality is decreasing in the cost of entry or mismatch; otherwise* ($\theta < 1/6$), *the inter-regional income inequality is increasing in these cost parameters.*

Thus, the cost and the productivity parameters may have different effects on inter-regional income inequality, depending on the bargaining power of high-tech firms as well as on the productivity of unskilled workers.

In summary, the main determinants of geographical income inequalities include: (i) traditional economic factors, such as labor productivity (for both skilled and unskilled workers) and the mass of unskilled workers, (ii) spatial factors, consisting of the costs to commute to and to form a city, and (iii) search and matching factors, such as entry and mismatch costs, relative bargaining power, and the mass of skilled workers (which creates a positive matching externality).

5.C. *Welfare versus Income Inequality*

Let us first define the social welfare function by uniformly aggregating all households' utilities achieved in equilibrium for both the core and the periphery regions:[20]

$$\Omega = NU_{\mathrm{L}}^* + (M/D)\int_0^D U_{\mathrm{M}}^*(\delta)\, d\delta. \tag{5.5}$$

From (4.11), (4.12), and (5.5), it is straightforward to derive

$$\Omega = A_0 P^{-\alpha}\Big[N(\gamma P - \beta) + aM - 2\lambda M^{2/3} - FM^{\varepsilon} - \{2 + [1/2 - \theta]/\theta\}(\theta cKM)^{1/2}\Big]. \tag{5.6}$$

[20] See Scotchmer [18] for a proposition relating this welfare maximization problem with competitive equilibrium.

Total differentiation of (5.6) and application of the material balance condition yield:[21]

PROPOSITION 4. *Aggregate social welfare is decreasing in all of the cost parameters* (c, K, F, t) *and increasing in the maximum productivity of skilled workers, the mass of skilled and unskilled workers, and the bargaining power of high-tech firms; it is, however, independent of the productivity of unskilled workers.*

From Propositions 3 and 4, we can compare the comparative-static results for aggregate social welfare with those for income inequality measures.

PROPOSITION 5.

(a) *A reduction in the cost of entry or mismatch, or an increase in the bargaining power of high-tech firms or in the mass of skilled workers, reduces the within-the-core income inequality and enhances social welfare.*

(b) *A decrease in the cost of commuting or city formation reduces the inter-regional income inequality and increases social welfare.*

(c) *A lower level of maximal productivity of skilled workers or a smaller mass of unskilled workers decreases the inter-regional income inequality, but it reduces social welfare.*

(d) *When the bargaining power of high-tech firms is sufficiently high* $(1/6 < \theta \leq 1/2)$ *and the productivity of unskilled workers is sufficiently low, an increase in the cost of entry or mismatch narrows inter-regional income inequality, but it suppresses social welfare.*

(e) *Although the effect of a larger mass or a higher level of maximal productivity of skilled workers, or of a lower cost of commuting or city formation is to improve social welfare, it may be accompanied by a widened within-the-core income inequality.*

This latter proposition can shed light on the distributional and welfare implications of many practically relevant public policy programs. First, a loose immigration policy that encourages unskilled labor migration can result in higher aggregate social welfare, but it may also widen both intra- and inter-regional income disparity. Second, policies such as a more progressive wage/corporate income tax schedule or a higher tollway user's fee can reduce the inter-regional income inequality; however, it is unambiguously welfare-distorting and their effect on the within-the-core income

[21] From the material balance condition, $\partial\Omega/\partial P = A_0 P^{-(\alpha+1)}[PN\gamma(1-\alpha) - \alpha\{aM - 2\lambda M^{3/2} - FM^{\varepsilon} - \beta N - \sigma\}] = 0$. This together with (5.7) implies: $d\Omega/dN = (\partial\Omega/\partial P)(dP/dN) + (\gamma P - \beta) > 0$. The result of $\partial\Omega/\partial P = 0$ is also used in deriving other comparative-static results in Propositions 4 and 5.

disparity is ambiguous. Third, a stronger labor union or an unemployment compensation program that reduces the bargaining power of high-tech firms has a negative effect on social welfare; but it may improve the inter-regional inequality by suppressing the wages of skilled workers. Finally, depending crucially on the bargaining power of high-tech firms and the productivity of unskilled workers, education, on-the-job training, and financial policies that can be used to reduce the cost of mismatch or firm entry is clearly welfare enhancing, though it may be at the expense of an increased inter-regional income inequality.

6. CONCLUSIONS

This paper develops a two-sector, general-equilibrium spatial model of a system of cities allowing for labor skill heterogeneities. The model generates a core-periphery spatial structure with an equilibrium configuration characterized by complete specialization. That is, skilled workers and high-tech firms locate in a single metropolis, while unskilled workers and food producers reside in local cities. When the wages of the heterogeneous skilled labor are determined via a non-symmetric Nash bargain, not only inter-regional (between the core and the periphery regions) but intra-regional (within-the-core) income inequalities can emerge. We find that the main determinants of geographical income inequality include economic forces as well as spatial and search/matching factors (in particular, commuting and city formation costs, firm entry and mismatch costs and the mass of skilled workers). Moreover, we show that factors enlarging income disparities may often lead to higher aggregate social welfare. Therefore, public policies designed to reverse the observed urbanization trend of widening geographical income inequality may not be necessarily welfare improving.

The spatial dichotomy between skilled and unskilled workers provides us an analytic characterization of social welfare and income inequality in a core-periphery system of cities. As a trade-off of its analytic simplicity, this present paper ignores inter-regional labor migration and thus the formation of a city with mixed (skilled and unskilled) labor. In future research, one may consider that each firm bargains with skilled and unskilled workers to set wages and unequal earnings is supported in equilibrium by the benefit of matching as well as the cost of migration. Moreover, one may allow for endogenous learning so that the unskilled can become skilled upon paying an education or training cost. Thus, each type of worker could reside in the same city and the cost of learning would account for a fraction of the income disparity. Of course, these generalizations will require further simplification of the spatial structure and the optimization problems of consumers/firms in order to reach theoretical conclusions.

REFERENCES

1. H. M. Abdel-Rahman, Agglomeration economies, types and sizes of cities, *Journal of Urban Economics*, **27**, 25–45 (1990).
2. H. M. Abdel-Rahman and P. Wang, Toward a general-equilibrium theory of a core-periphery system of cities, *Regional Science and Urban Economics*, **25**, 529–546 (1995).
3. A. Anas, On the birth and growth of cities: laissez-faire and planning compared, *Regional Science and Urban Economics*, **22**, 243–258 (1992).
4. M. Berliant and M. Fujita, Alonso's discrete population model of land use: Efficient allocations and competitive equilibria, *International Economic Review*, **33**, 535–66 (1993).
5. M. Berliant and T. ten Raa, On the continuum approach of spatial and some public good or product differentiation models: Some problems, *Journal of Economic Theory*, **55**, 95–120 (1991).
6. M. Berliant and P. Wang, Endogenous formation of a city without agglomerative externalities or market imperfections: Market places in a regional economy, *Regional Science and Urban Economics*, **23**, 121–144 (1993).
7. K. Burdett and D. T. Mortensen, Search, layoffs, and labor market equilibrium, *Journal of Political Economy*, **88**, 652–672 (1980).
8. W. Christaller, "Die zentralen orte in Suddeutschland," Fischer, Jena (1933); Translated by C. W. Bashin, to "Central Place in Southern Germany," Prentice-Hall, Englewood Cliffs, NJ (1966).
9. P. Diamond, Aggregate demand in search equilibrium, *Journal of Political Economy*, **89**, 881–894 (1982).
10. M. Fujita, A monopolistic competition model of spatial agglomeration: differentiated product approach, *Regional Science and Urban Economics*, **18**, 87–124 (1988).
11. M. Fujita, "Urban Economic Theory: Land Use and City Size," Cambridge Univ. Press, London, England (1989).
12. M. Fujita and R. Ishii, "Global Economic Restructuring and Urbanization Processes in East Asia: An Overview," Working Paper, University of Pennsylvania, Philadelphia, PA, (1990).
13. R. W. Helsley and W. C. Strange, Matching and agglomeration economies in a system of cities, *Regional Science and Urban Economics*, **20**, 189–222 (1990).
14. J. V. Henderson and H. M. Abdel-Rahman, Efficiency through decentralization with product diversity, *Regional Science and Urban Economics*, **21**, 491–510 (1991).
15. S. Kim, Heterogeneity of labor markets and city size in an open spatial economy, *Regional Science and Urban Economics*, **21**, 109–126 (1991).
16. D. Laing, T. Palivos, and P. Wang, Learning, matching and growth, *Review of Economic Studies*, **62**, 115–129 (1995).
17. A. Rubinstein and A. Wolinsky, Decentralized trading, strategic behavior and the Walrasian outcome, *Review of Economic Studies*, **57**, 63–78 (1990).
18. S. Scotchmer, Hedonic prices and cost/benefit analysis, *Journal of Economic Theory*, **37**, 55–75 (1985).
19. J.-F. Thisse and D. E. Wildasin, Public facility location and urban spatial structure: Equilibrium and welfare analysis, *Journal of Public Economics*, **48**, 83–118 (1992).
20. P. Wang, Competitive equilibrium formation of marketplaces with heterogeneous consumers, *Regional Science and Urban Economics*, **20**, 295–304 (1990).
21. D. E. Wildasin, Income redistribution in a common labor market, *American Economic Review*, **81**, 757–774 (1991).