Money, Technology Choice and Pattern of Exchanges in Search Equilibrium

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Abstract: This paper examines in a decentralized trading environment how money may generate real effects via technology choice (high vs. low) by modeling explicitly the pattern of exchanges within a random-matching framework. We inquire (i) whether the presence of trade frictions grants the high technology disadvantageous and (ii) whether money encourages the adoption of the high technology. While high-quality goods yield greater consumption value, they incur a production time delay and a greater production cost. We allow buyers to form their best responses to accepting different types of goods. In a complete information world, we characterize the steady-state monetary equilibrium with either instantaneous or non-instantaneous production. We show how the introduction of money affects the technology choice by mitigating the high technology's disadvantage in production time delay. We also show that when production takes time, the extent to which the high-technology equilibrium yields higher social welfare than the low-technology equilibrium depends crucially on the quantity of money in the economy. Additionally, we identify sources of social inefficiency caused by search frictions that can lead to under-investment in the advanced technology in decentralized equilibrium. (*JEL Classification Codes*: E00, D83, O33)

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"When the division of labour has been once thoroughly established, it is but a very small part of a man's wants which the produce of his own labour can supply. He supplies the far greater part of them by exchanging that surplus part of the produce of his own labour, which is over and above his own consumption, for such parts of the produce of other men's labour as he has occasion for." (Adam Smith, *The Wealth of Nations*, Book I, Chapter IV, paragraph 1)

1 Introduction

We develop a search-theoretic framework to study how money in a decentralized trading environment may affect *technology choice* and *decentralized exchange patterns* in the presence of trade frictions. Since the seminal work of Kiyotaki and Wright (1989,1993), there has been a growing literature on money in search equilibrium, emphasizing that the use of a medium of exchange minimizes the time/resource costs associated with searching for exchange opportunities, hence alleviating the "double coincidence of wants" problem with barter.[1] While the study of the role of money in facilitating trade has generated considerable insights towards understanding the origin and use of money, its roles in promoting higher production technology remain largely unexplored.

Different from the canonical Walrasian monetary growth models, the search-theoretic framework allows us to provide a deep structure to help understand more clearly how money affects technology choice in decentralized trade.[2] Under this setup, we can inquire (i) *whether the presence of trade frictions grants the high technology disadvantageous* and (ii) *whether money encourages adoption of the high technology.* In particular, our paper models explicitly the production *process* of quality-differentiated goods in a way that enables low-quality goods to be produced and traded in equilibrium *even under perfect observability of goods quality.* Money, by improving decentralized trades, can mitigate the high technology's cost-

[1] The literature of barter versus money with a formal model of exchange is pioneered by Jones (1976). In the prototypical search model of money following Kiyotaki and Wright, exchange is characterized by one-for-one swaps of goods and money, implying fixed prices, under which the optimal inflation issue can be studied using the arguments by Li (1995). More recent attempts to characterize pricing behavior and the distribution of cash permit divisible goods and money. For a brief survey, the reader is referred to Rupert, Schindler and Wright (2001) and papers cited therein.

[2] Our paper is thus in sharp contrast with the ad hoc setup of money-in-the-production-function.

disadvantage and hence encourage the implementation of a more advanced technology. Our paper therefore provides a plausible channel through which money can generate a real effect via technology choice.

More specifically, we consider a continuous-time search and random-matching model with three groups of agents: producers, goods traders and money traders. There are two clusters of goods: high quality and low quality, with each cluster consisting of a continuum of varieties. While high-quality goods yield greater consumption values, they incur a production time delay and a greater production resource cost. For tractability, we assume that goods and money are indivisible and that each non-producing agent has only one unit of space to store either goods or money. The indivisibility of goods simplifies the comparison between competing technologies to a great extent. Otherwise, if the output volume is negotiable between producers and buyers, the level of the technology cannot be measured by the net utility of the goods produced, because the quantity of low-quality products may be large enough to provide higher net utility than the high-quality products. At any point in time, each producer must choose between the two technologies and can only produce one unit of the good of a particular type. Upon a successful production, a producer becomes a goods trader with a commodity of a particular quality. The quality of goods is public information to all traders. Each buyer consumes only a subset of varieties, exclusive of those self-produced, and forms a *best response* to accepting goods of different quality within the desired subset.

The way through which money influences technology choice can be illustrated intuitively. Since the deepening of specialization entails some time for a consumer to buy the output from a producer, we have to consider the inventory costs incurred. If the use of money can save consumers' transactions time to search for desired commodities, the time costs of inventories can be reduced. This makes the high technology's disadvantage in manufacturing and time delay costs less significant, thus creating an intensive margin in favor of the high technology. Since only producers take into account the underlying inventory costs, this transactions time effect becomes more important when production takes longer time. In conducting the welfare analysis, we employ the autarky technology choice as the benchmark. Note that in the absence of search frictions, producers' *first-best* technology choice is the same as their autarky counterparts, which may thus be referred to as *autarky efficiency*.[3] It is in this regard that

[3] The reader should be warned that the term "autarky efficiency" is referred to technology choice rather

the good produced by the high technology is never the worse good in terms of consumer's valuation. The presence of search frictions distorts the producer's technology choice. By comparing this with the first-best outcome in a frictionless economy, we can then investigate whether the introduction of money can restore the efficient technology choice outcome.

A principal contribution of our paper to the existing literature on money and product quality is that low-quality goods may be produced and traded in equilibrium without relying on incomplete information. In particular, we establish the possible coexistence of two locally stable *pure-strategy* and one locally unstable *mixed-strategy* equilibria with active trade, depending on the society's initial endowment of money. By examining equilibrium and welfare outcomes, we obtain the following properties. First, when production is instantaneous, the mixed-strategy equilibrium, if it coexists with the pure-strategy high-technology equilibrium, is Pareto-dominated, and features a positive relationship between the fraction of high technology producers and the society's endowment of money. Moreover, the high-technology equilibrium Pareto dominates the low-technology one, if and only if the high technology would provide more net utility in a frictionless economy. Second, when production takes time, the high-technology equilibrium has higher level of social welfare than the low one if, in addition to autarky efficiency, the high technology's delay cost is not too large and the social endowment of money is sufficiently high. The introduction of money affects producers' technology choice, by mitigating the high technology's disadvantage in production time delay. Finally, by deriving the optimal quantity of money under each equilibrium, we identify an important source of social inefficiency caused by search frictions that can lead to an *under-investment* in the advanced technology in decentralized equilibrium.

Literature Review

In the money search literature, there are papers considering two types of traded goods, including Williamson and Wright (1994), Berentsen and Rocheteau (2004), and Trejos (1997). However, in these models, the low-quality good is always undesirable under perfect observability, as it is costless to produce and generates no consumption value, compared to a high-quality good yielding a strictly positive net utility gain. In order for both goods to be traded, private information about the goods quality is therefore assumed. In contrast, our paper shows the possibility of producing low-quality goods in equilibrium *even under perfect*

than the pattern of exchange.

observability of goods quality by modelling more explicitly the process of production of the two quality-differentiated goods, thus complementing previous studies by proposing an alternative and plausible underlying driving force that permits low-quality goods to be produced and traded in equilibrium.

There is a small but growing literature studying money and technology choice in search equilibrium. The first branch of this literature considers endogenous choice of the *horizontal* scopes of production specialization in the absence of quality differentiation. In Shi (1997), agents can produce desired goods at a higher cost than those for trade. Money enhances decentralized trade and thus creates a gain from specialization. Recently, Camera, Reed and Waller (2003) allow agents to choose whether to be a "jack of all trade" or a "master of one" in which money again advances individuals' specialization in a decentralized trading environment. In Laing, Li and Wang (forthcoming), a multiple-matching framework is developed where trade frictions manifest themselves in limited consumption variety and via a positive feedback between shopping and work effort decisions, money creation may have a positive effect on productive activity.

The second branch, to which our paper belongs, analyzes the endogenous choice between *vertically* differentiated production technologies under a given scope of specialization. The only previous study to our knowledge is by Kim and Yao (2001) who introduce money into a search model with *divisible* goods, instantaneous production and two competing technologies. Their focus is exclusively on the mixed-strategy equilibrium, whereas the proportions of high- and low-technology producers are exogenously given. Note that the amount of low-quality goods produced is always larger than that of the high-quality goods in their paper, but the *de facto* efficiency problem associated with the assumption of divisible goods is ignored. In contrast, our paper considers the more general case of *non-instantaneous production* to allow for different production time under different technologies by maintaining the simplifying assumption of *indivisible* goods. In our model setup, only pure-strategy equilibria are found to be stable; the mixed-strategy equilibrium is both *unstable* and *Pareto suboptimal*.[4] Moreover, we allow money traders to determine whether they would accept either type or both types of goods and, as a consequence, the proportion of producers using high/low technology is *endogenous*. Furthermore, we study the welfare implications under various equilibria and

[4] The finding of the instability of the mixed-strategy equilibrium is in line with that in Wright (1999).

with different initial social endowment of money.

2 The Basic Model

The basic structure extends that of Kiyotaki and Wright (1993). Time is continuous. There is a continuum of infinitely-lived agents whose population is normalized to one. Based on their activities, agents are divided into three different categories at any point in time: producers, goods traders and money traders. Both goods and money are indivisible. Each non-producing agent has only one unit of space that may be used to store either a unit of commodity or a unit of money.

There are two groups of goods: high quality (type-H) and low quality (type-L). Each group consists of a continuum of varieties whose characteristic location can be indexed on a unit circumference. At any point in time, each producer can produce only one unit of the good of a particular type. Upon producing a commodity, a producer becomes a goods trader instantaneously. Thus, producers can be classified as type-H or type-L, as are goods traders. The type of agents (and hence the quality of goods) is assumed to be public information to all traders.

Money is storable but cannot be consumed or produced. At the beginning of time, there are $M \in (0, 1)$ units of money in the economy, so we have a measure of M money traders due to the unit-storage-space assumption. Thus, letting N_0, N_H, N_L, and N_m, respectively, be the measures of producers, type-H goods traders, type-L goods traders, and money traders,[5] population identity implies:

$$N_m + N_H + N_L + N_0 = 1 \tag{1}$$

The proportion of type-H goods traders to all goods traders, denoted h, and the fraction of money traders to all traders, denoted μ, can thus be expressed as:

$$h = \frac{N_H}{N_H + N_L} \tag{2}$$

$$\mu = \frac{N_m}{N_m + N_H + N_L} = \frac{M}{1 - N_0}. \tag{3}$$

[5] Due to the assumption of unit storage space and the indivisibility of money, $N_m = M$.

Traders match with each other according to a Poisson process characterized by the arrival rate parameter, β. Because that the probability for a particular pair of traders to rematch is zero in our continuum economy and that no authority exists to enforce the repayment of credits or IOU's, sellers must accept money in the absence of double coincidence of wants. Throughout the main text of the paper, we follow Trejos (1997) and Kim and Yao (2001) to focus exclusively on pure monetary equilibrium, that is, barter exchanges are not allowed in the basic framework. A discussion of the pure barter economy is relegated to Appendix.

2.1 Production Technology

There are two types of technologies. The high technology can produce a unit of the high-quality good at a (utility) cost of ε, while the low technology incurs a lower manufacturing cost of $\delta\varepsilon$ (with $0 < \delta < 1$) to produce one unit of the low-quality good.

The two technologies also differ in the arrival rates of the respective outputs. Specifically, the product arrival of the low technology follows a Poisson process with arrival rate of α, while that of the high technology has an arrival rate of $\eta\alpha$ (with $0 < \eta < 1$).

2.2 Preferences

Following the convention of the money-search literature, we assume that no agent would consume the good he or she produces. Moreover, each agent gains positive utility only by consuming a subset of the varieties of each type (called a consumable set), whose measure is denoted by x. Thus, x can be regarded as a taste specialization index.

Despite their taste heterogeneity, all agents have identical utility functional forms. While the consumption of the first unit of a high-quality good within the consumable set yields a utility $U > 0$, any additional unit at a given point in time would not generate any extra value. Similarly, the consumption of the first unit of a low-quality good within the consumable set gives a utility of θU (with $0 < \theta < 1$).

To ensure non-trivial technological choice, we impose:

Assumption 1: $\theta U > \varepsilon$.

Assumption 1 implies that both types of products deliver positive net values to the economy since $U > \varepsilon$ and $\theta U > \delta\varepsilon$. Moreover, this assumption also guarantees the existence of mixed-strategy equilibrium, as we will show later.

2.3 Value Functions

Define π_i $(i = H, L)$ as the probability with which a money trader will accept type-i goods, and Π_i as the average probability of acceptability in the economy (which is taken as parametrically given by all individual traders). Denote the discount rate by r. Further define V_i as the asset value of a type-i agent, where $i = 0, H, L, m$ represents producers, type-H goods traders, type-L goods traders, and money traders, respectively.

We are now well equipped to set up the Bellman equations, displayed for simplicity by assuming steady states (as in the conventional money and search literature):

$$rV_0 = \max\{\alpha(V_L - V_0 - \delta\varepsilon), \eta\alpha(V_H - V_0 - \varepsilon)\} \tag{4}$$

$$rV_i = \beta\mu x\Pi_i(V_m - V_i), \qquad i = H, L \tag{5}$$

$$rV_m = \beta(1-\mu)x\left[h\max_{\pi_H}\{\pi_H(U + V_0 - V_m)\} + (1-h)\max_{\pi_L}\{\pi_L(\theta U + V_0 - V_m)\}\right]. \tag{6}$$

Equation (4) states that the flow value of a producer is the maximum incremental value, over the two technologies, from the producer state to the goods trader state net of the corresponding production cost, upon a successful arrival of the product (measured by α and $\eta\alpha$, respectively).

Recall that with a flow probability β, a type-i goods trader can meet another trader who will be a money trader with probability μ. The chance for this money trader to like the goods trader's product is x, which will be accepted with probability Π_i. Thus, as indicated by (5), the flow value of a type-i goods trader is the incremental value from exchanging the product for money, which is the differential, $V_m - V_i$, multiplied by the flow probability, $\beta\mu x\Pi_i$.

Similarly, the flow probability for a money trader to meet a type-H goods trader whose commodity is within the consumable set is $\beta(1-\mu)xh$ and that to meet a type-L goods trader is $\beta(1-\mu)x(1-h)$. The value of meeting a type-i goods trader is the utility (U and θU, for $i = H, L$, respectively) plus the incremental value from the money trader state to the producer state ($V_0 - V_m$). A money trader may stay put (by not accepting the good, i.e., $\pi_i = 0$) or accept the trade with probability $\pi_i > 0$ (which is the best response by the money trader, possibly less than one). Thus, this flow value must be multiplied by the corresponding acceptance probability, as displayed in (6).

It is convenient to define by Δ_i ($i = H, L$) the producer's effective discount factors over the expected span of the production process and by ρ_i ($i = H, L$) the goods trader's effective discount factors for the expected waiting period for sales.

$$\Delta_H \equiv \frac{\eta\alpha}{\eta\alpha + r}; \quad \Delta_L \equiv \frac{\alpha}{\alpha + r} \tag{7}$$

$$\rho_H \equiv \frac{\beta\mu x\Pi_H}{\beta\mu x\Pi_H + r}; \quad \rho_L \equiv \frac{\beta\mu x\Pi_L}{\beta\mu x\Pi_L + r}. \tag{8}$$

Given the Poisson process, $\frac{1}{\eta\alpha}$ is the average waiting time for production and $\frac{r}{\eta\alpha}$ is the discount rate over the expected span of the production process , thus yielding the producer's effective discount factors, Δ_H. Similar explanations apply to Δ_L, ρ_H and ρ_L.

Accordingly, we can rewrite the value functions (4) and (5) in a cleaner manner,

$$rV_0 = \max\{\Delta_L(V_L - \delta\varepsilon), \Delta_H(V_H - \varepsilon)\} \tag{9}$$

$$V_i = \rho_i V_m, \qquad i = H, L. \tag{10}$$

3 Equilibria with Instantaneous Production

We begin by considering a special case with instantaneous production ($\alpha \to \infty$), which enables a complete analytic analysis of the *steady-state monetary equilibrium*. With instantaneous production, we have $N_0 = 0$, and , from (3), $\mu = M$. Moreover, (7) implies $\Delta_H = \Delta_L = 1$ and hence (9) can be rewritten as:

$$V_0 = \max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\}. \tag{11}$$

3.1 Money Trader's Best Response

To solve the equilibrium under instantaneous production, first consider the money traders. A money trader's best responses π_H and π_L are determined according to the following:

$$\pi_H \begin{cases} = 0, & \text{if } \; U + V_0 - V_m < 0 \\ \in (0,1), & \text{if } \; U + V_0 - V_m = 0 \\ = 1, & \text{if } \; U + V_0 - V_m > 0 \end{cases} \tag{12}$$

$$\pi_L \begin{cases} = 0, & \text{if} \quad \theta U + V_0 - V_m < 0 \\ \in (0,1), & \text{if} \quad \theta U + V_0 - V_m = 0 \\ = 1, & \text{if} \quad \theta U + V_0 - V_m > 0 \end{cases} . \tag{13}$$

Thus, in the case of $U + V_0 - V_m = 0$ or $\theta U + V_0 - V_m = 0$, the corresponding best response (π_H or π_L) constitutes a mixed strategy.

In equilibrium, the individual's best response agrees with the average behavior in the economy, that is,

$$\pi_i = \Pi_i, \tag{14}$$

for $i = H, L$.

3.2 Existence

We focus on the case of nondegenerate equilibrium in which all agents in the economy participate in trade actively. Thus, a producer must have positive payoff,

$$\max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\} > 0 \tag{15}$$

Moreover, a money trader must buy at least one type of the commodities. This is valid under the following active equilibrium condition:

$$\max\{U + V_0 - V_m, \theta U + V_0 - V_m\} > 0 \tag{16}$$

The strict inequality is required in order to ensure the validity of condition (15).

Since $\theta < 1$, this condition requires $U + V_0 - V_m > 0$, and thus $\pi_H = 1$, which means the money trader will fully accept the type-H goods. Based on the three different best responses towards the acceptability of the type-L goods, we can have three equilibria: (A) $\pi_L^A = 0$; (B) $\pi_L^B \in (0,1)$; and (C) $\pi_L^C = 1$. We use superscript A, B, and C to denote each equilibrium whenever it is necessary. Also, we can define the effective discount factor for the purchasing period (when always accepting a good) as:

$$\rho_m \equiv \frac{\beta(1-\mu)x}{\beta(1-\mu)x + r}. \tag{17}$$

	Equilibrium A	**Equilibrium B**	**Equilibrium C**
V_0	$\dfrac{\rho_H^A\rho_m^A U-\varepsilon}{1-\rho_H^A\rho_m^A}$	$\dfrac{\rho_H^B\theta U-\varepsilon}{1-\rho_H^B}$, or $\dfrac{\rho_L^B\theta U-\delta\varepsilon}{1-\rho_L^B}$	$\dfrac{\rho_L^C\rho_m^C\theta U-\delta\varepsilon}{1-\rho_L^C\rho_m^C}$
V_H	$\dfrac{\rho_H^A\rho_m^A(U-\varepsilon)}{1-\rho_H^A\rho_m^A}$	$\dfrac{\rho_H^B(\theta U-\varepsilon)}{1-\rho_H^B}$	V_L^C
V_L	0	$\dfrac{\rho_L^B(\theta U-\delta\varepsilon)}{1-\rho_L^B}$	$\dfrac{\rho_L^C\rho_m^C(\theta U-\delta\varepsilon)}{1-\rho_H^C\rho_m^C}$
V_m	$\dfrac{\rho_m^A(U-\varepsilon)}{1-\rho_H^A\rho_m^A}$	$\dfrac{\theta U-\varepsilon}{1-\rho_H^B}$, or $\dfrac{\theta U-\delta\varepsilon}{1-\rho_L^B}$	$\dfrac{\rho_m^C(\theta U-\delta\varepsilon)}{1-\rho_L^C\rho_m^C}$
Π_L	0	π_L^B	1
h	1	h^B	0
M	S^A	S^B	S^C

Table 1: Solutions for the Instantaneous Production Case

It is not difficult to solve the asset values (V_0, V_H, V_L, V_m) from the linear equation system (6), (10) and (11), and the solutions are summarized in Table 1. We can interpret the solutions intuitively with the effective discount factors defined in (8) and (17). In equilibrium A, for example, the producer bears the manufacturing cost instantaneously but should wait for both the selling and purchasing periods to consume, so his net utility in one production-trade-consumption cycle is $\rho_H\rho_m U-\varepsilon$. Since the effective discount factor for one production-trade-consumption cycle is $\rho_H\rho_m$, the summation of infinite geometric series yields the solution in the first cell in Table 1, where other cells can be derived in an analogous fashion.[6]

Utilizing these asset values, we next turn to deriving the best responses of the agents and checking the corresponding conditions on the parameters. Define $Q \equiv \frac{(\beta x+r)r\varepsilon}{\beta^2x^2(U-\varepsilon)}$ and impose:

Assumption 2: $Q\max\left\{\dfrac{\delta U-\delta\varepsilon}{\theta U-\delta\varepsilon},1\right\}<\dfrac{1}{4}.$

Assumption 3: $\dfrac{1}{\theta U-\varepsilon}+\dfrac{\theta}{1-\theta}<\dfrac{\beta x}{r}.$

The usefulness of these two assumptions will be discussed later. We first examine the two pure-strategy equilibria (A and C). In equilibrium A, we can show from (8) and (10) that $V_L=0$. Hence, no producer would choose the low technology ($h=1$) since it yields negative

[6] Note that ρ_H^A, ρ_H^B, and ρ_L^C have the same functional form with respect to μ and exogenous parameters, so do ρ_m^A and ρ_m^C.

flow value to producers ($V_L - \delta\varepsilon < 0$). From (13), we know that $\pi_L = 0$, if $\theta U + V_0 - V_m < 0$, or equivalently, $\theta U < \rho_m^A U - (1 - \rho_m^A)V_0$, as $V_m = \rho_m^A(U + V_0)$ when the high technology is chosen. We now define:

$$M_1 \equiv \max\{1 - \frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x(U - \varepsilon)}, 0\} \tag{18}$$

and $M_2 \leq 0.5$ such that

$$M_2(1 - M_2) = \frac{(\beta x + r)r\varepsilon}{\beta^2 x^2(U - \varepsilon)}, \tag{19}$$

which has real roots under Assumption 2. We can then establish:

Lemma 1: (Equilibrium A) *Equilibrium A exists if $S^A \equiv (0, M_1) \cap (M_2, 1 - M_2) = (M_2, \min(1 - M_2, M_1)) \neq \emptyset$ and $M \in S^A$, where $M \in (0, M_1)$ iff $\theta U < \rho_m^A U - (1 - \rho_m^A)V_0^A$.*

Proof. *All proofs are in Appendix.*

Note that by accepting the type-L goods, a money trader can enjoy the utility of θU. By rejecting the trade and waiting for type-H goods in the next trade, the money trader will have the discounted utility, $\rho_m^A U$, at the cost of delayed production, $(1 - \rho_m^A)V_0$. Hence when $M \in (0, M_1)$, or equivalently, $\theta U < \rho_m^A U - (1 - \rho_m^A)V_0$, it is a money trader's best response to reject a trade with a type-L producer. Intuitively, in an economy swamped by too much money, money traders would buy any type of goods as soon as possible since they cannot afford the long waiting period for the second chance. This is particularly important when the difference in the quality is not large enough to make the waiting worthwhile. This effect is due primarily to the presence of search frictions in which too much money can crowd out the advanced technology. Thus, it may be referred to as the *nonselectivity effect*.

The requirement that $M \in (M_2, 1 - M_2)$ is to ensure positive producer payoffs. If the amount of initial money endowment is either too small or too large, then the probability of successful trades is too low. This *transactions cost effect* caused by search frictions implies that no producers would find profitable to use the high technology given the high manufacturing cost. Thus, if the economy has insufficient amount of money initially, the introduction of money can serve to mitigate the transactions cost effect and hence encourage producers to adopt the advanced technology. It may be noted that the transactions cost effect is also present in Shi (1997), where an insufficient endowment of money may discourage agents from trading in a decentralized market and instead make them stay in autarky. By mitigating search frictions, money enlarges the extent of the market, encouraging horizontal production specialization in Shi (1997) while enhancing vertical production quality in ours.

The solution of equilibrium C is quite similar to that of equilibrium A. Observe that when $\pi_L = \Pi_L = 1$, equation (5) results in $V_H^C = V_L^C$, as well as $\rho_H^C = \rho_L^C$. The producer would definitely choose the low technology to minimize his cost, which means $h = 0$. After solving for the values, we find that since $U > \theta U > V_m^C - V_0^C$, for any $M \in (0, 1)$, equilibrium C exists as long as $V_0^C > 0$. Define $M_3 \leq 1/2$ such that

$$M_3(1 - M_3) = \frac{(U - \varepsilon)\,\delta Q}{\theta U - \delta\varepsilon}, \tag{20}$$

which has real roots under Assumption 2. Then we have:

Lemma 2: (Equilibrium C) *Equilibrium C exists if $M \in S^C \equiv (M_3, 1 - M_3)$. Moreover, $S^C \supseteq S^A$ if $0 < \delta \leq \theta < 1$.*

Equilibrium B is a bit more complicated. The money trader's mixed strategy implies $\theta U + V_0^B - V_m^B = 0$. Based on the fact that the producers are indifferent between the two technologies, we can solve the money trader's acceptability of low-quality goods,

$$\pi_L^B = \Pi_L^B \equiv 1 - \frac{(1 - \delta)\varepsilon}{\rho_H(\theta U - \delta\varepsilon)}, \tag{21}$$

and the equilibrium proportion of type-H goods in the market,

$$h^B \equiv \frac{(\beta\mu x + r)(\theta U - \varepsilon)}{\beta(1 - \mu)x(1 - \theta)U}. \tag{22}$$

It is easily seen that π_L^B is increasing in μ and thus M. Moreover, h^B is increasing in μ and thus M, which implies as the amount of money increases in the economy, there are more people holding type-H goods. Define,

$$M_4 \equiv \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)}, \tag{23}$$

we obtain:

Lemma 3: (Equilibrium B) *Equilibrium B exists if $S^B \equiv (M_4, M_1) \neq \emptyset$ and $M \in S^B$. Moreover, $S^B \subseteq S^A$.*

Under Assumptions 2 and 3, $S^j \neq \emptyset$ ($j = A, B, C$) and with the aid of Lemmas 1-3 and Proposition $A1$ in Appendix, we can establish:

Proposition 1: (Existence, Stability and Characterization) *Under Assumptions 1-3, a steady-state monetary equilibrium exists, which possesses the following properties:* $\pi_H = 1$ and

(i) $\pi_L = 0$, *only if* $M \in S^A$ *(equilibrium A);*

(ii) $\pi_L \in (0, 1)$, *only if* $M \in S^B$ *(equilibrium B);*

(iii) $\pi_L = 1$, *only if* $M \in S^C$ *(equilibrium C).*

Moreover, multiple equilibria may arise. Among the three equilibria, equilibrium A and C are locally stable, while equilibrium B is locally unstable. Furthermore, by mitigating the transactions cost effect, the introduction of money encourages investment in the advanced technology and the emergence of equilibrium A. By contrast, only type-L technology will be chosen in a pure barter economy with instantaneous production.

As to the existence, Assumptions 2 and 3 ensure the nonemptiness of S^C and S^B, respectively, whereas both assumptions together guarantee that S^A is nonempty. From Lemma 3, when $M \in S^B$, the mixed-strategy equilibrium B always co-exists with the pure-strategy equilibrium A (as $S^B \subseteq S^A$). Moreover, when $0 < \delta \leq \theta < 1$, both pure-strategy equilibria co-exist if $M \in S^A$ (as $S^A \subseteq S^C$) while all three types of equilibria co-exist if $M \in S^B$.

The possibility for low-quality goods to be produced and traded contributes to the existing literature on money and product quality in which only high-quality goods arise in equilibrium unless information is asymmetric between buyers and sellers. The co-existence of these various types of equilibria with active trade is also new to the literature. Notably, for some given sets of social endowment of money, the equilibrium outcome is indeterminate where the underlying equilibrium selection mechanism is due entirely to *self-fulfilling prophecies*. For example, should a producer expect that traders are less selective in good quality (animal spirits), he or she would be more inclined toward employing the low technology. As a result, there will be more traders with low-quality goods and the probability for money holders to locate a high-quality good becomes lower. This implies that money holders would tend to be more generous toward accepting a low-quality good, so the expectations are self-fulfilled.

The two pure-strategy equilibria are both locally stable, since small disturbances in the acceptability of the type-L goods cannot affect the producer's choice. However, equilibrium

B is locally unstable. To see this we can simply disturb Π_L. If the agents believe Π_L to be a bit larger (smaller), V_L would be higher (lower). Thus the producer will prefer the low (high) technology, thereby leading to equilibrium C (A). This is consistent with the finding in Wright (1999) where mixed-strategy equilibria are always unstable in an evolutionary sense in the search-theoretic model of money with indivisible goods and indivisible money.

Equilibrium B in our model can be compared with the mixed-strategy equilibrium in Kim and Yao (2001): When both types of products co-exist, the share of type-H goods (h) and the level of social welfare are increasing in the quantity of money supply (M).

3.3 Welfare Implications

Note that in the absence of search frictions, the producers would make the same technology choice (*first-best*) as their autarky counterparts, while the existence of search frictions can distort the producer's choices. Hence we employ the first best choice as the benchmark of our welfare analysis, and investigate whether the introduction of money can reinstall the *autarky efficiency.*

Due to the assumption of instantaneous production, only the goods and money traders are considered in the commonly used equally weighted steady-state social welfare function. Observe that $M \in [0, M_1]$ is equivalent to $V_m^A > V_m^B$, which implies $V_H^A > V_H^B > V_L^B$, and $V_0^A > V_0^B$, *pointwise* with respect to M. The fact that $S^B \subseteq S^A$ implies, for any value of $M \in S^B$, there is always an equilibrium with $\pi_L = 0$ (equilibrium A) that Pareto dominates the mixed-strategy equilibrium. Since this equilibrium is locally unstable and Pareto-dominated in its existence region, we put more effort on comparing the two pure-strategy equilibria, A and C.

By examining these two equilibria, we find that both goods traders and money traders prefer (pointwise with respect to M) the technology with autarky efficiency, i.e., the one with the higher net-of-cost utility. The Pareto ranking in this case is straightforward because the producers are of measure zero. In general, it may be useful to compare the steady-state social welfare instead of Pareto rankings:

$$Z = N_0 V_0 + N_L V_L + N_H V_H + N_m V_m. \tag{24}$$

	Equilibrium A	Equilibrium C
Monetary	$\beta x M(1-M)(U-\varepsilon)$	$\beta x M(1-M)(\theta U-\delta\varepsilon)$
Autarky	$(U-\varepsilon)$	$(\theta U-\delta\varepsilon)$

Table 2: Flow Values of Welfare in the Instantaneous Production Case

We assume that a social planner can set the initial amount of money M to maximize Z. Hence we compare the maximal welfare in equilibria A and C.

The flow values of social welfare for both equilibria are shown in Table 2.[7] The *optimal quantity of money* can be easily solved as $\min\{1/2, M_1\}$ for equilibrium A and $1/2$ for equilibrium C.[8] If $M_1 > 1/2$ (which holds when θ is sufficiently small), the welfare comparison between Equilibrium A and C is again equivalent to autarky efficiency. Otherwise, the social planner would choose the high technology only when it provides sufficiently more net utility than the low technology, such that

$$\frac{U-\varepsilon}{\theta U-\delta\varepsilon} \geq \frac{1/4}{M_1(1-M_1)} > 1.$$

From (18), M_1 is decreasing in θ and independent of δ. Therefore, when the quality difference is sufficiently small, the social planner could still support the production of the type-L good, even when the type-H good provides more net utility. On the contrary, the production cost differential (captured by δ) does not play any role, which is a result of the take-it-or-leave-it offer to buyers whose only concern is the quality of the goods. Under instantaneous production, the social planner can do no better than the autarky efficiency outcome, with a frictional exchange process being introduced. This conclusion would no longer be true if goods production also takes time (see Section 4 below).

Proposition 2: (Welfare and Optimal Quantity of Money) *Equilibrium B is always Pareto dominated by equilibrium A either pointwise with respect to M or in the sense of equally weighted social welfare maximization. Concerning equilibrium A and C:*

[7] We use flow values here just because the stock values of social welfare for autarky economy are infinite, and hence incomparable, due to instantaneous production.

[8] Rigorously speaking, since the admissible set is not closed, the optimal quantity of money does not exist if $M_1 < 1/2$. For illustrative purposes, however, we will refer to the welfare maximizer over the closure of the admissible set as the optimal quantity of money.

(i) *under pointwise Pareto criterion, the dominance of one over the other is completely determined by autarky efficiency;*

(ii) *under social welfare maximization,*

a. *the dominance of one over the other is determined by autarky efficiency if* $M_1 > 1/2$,

b *the social planner is less likely to adopt the high technology than autarky efficiency if* $M_1 \leq 1/2$;

(iii) *the socially optimal quantity of money is* $\min\{1/2, M_1\}$ *for equilibrium* A *and* $1/2$ *for equilibrium* C.

Recall the nonselectivity effect that too much money may discourage the adoption of the high technology. By accounting for this, the social planner must set the optimal quantity of money for equilibrium A at a lower level than for equilibrium C. Also due to the presence of the nonselectivity effect, the optimal quantity of money in our paper may be lower than that obtained by Kiyotaki and Wright (1993) under an exogenously given production technology (which is $1/2$). Particularly, money in our benchmark economy affects welfare via its effects on the adoption of the more advanced technology and the pattern of exchange: too much money may crowd out the high technology due to the nonselectivity effect, while the amount of money should be appropriate to minimize the transaction cost associated with decentralized trade.

4 Non-instantaneous Production

When production is not instantaneous, *i.e.*, when α is finite, there is a nontrivial steady-state mass of producers. Thus, $\mu > M$ and this creates great algebraic complexity. Nonetheless, this exercise allows us to gain additional insights into how the introduction of money could improve technological development.

4.1 Steady-State Monetary Equilibrium

Based on the active equilibrium condition (16) we once more obtain: $\pi_H = 1$, which means money trader will fully accept the type-H goods in equilibrium. Based on the three different best responses to accepting type-L goods, we again have three equilibria: (AA): $\pi_L^{AA} = 0$; (BB): $\pi_L^{BB} \in (0,1)$; and (CC): $\pi_L^{CC} = 1$, where the labelings AA, BB, and CC correspond to A, B, and C, in the instantaneous production case.

To solve the population distribution in the steady state, we equate the outflows and inflows from and to the population of goods and money traders to yield:

$$\Lambda\eta\alpha N_0 = \beta\mu x \Pi_H N_H \tag{25}$$

$$(1-\Lambda)\alpha N_0 = \beta\mu x \Pi_L N_L \tag{26}$$

$$\beta\mu x(\Pi_L N_L + \Pi_H N_H) = \beta(1-\mu)x[h\Pi_H + (1-h)\Pi_L]N_m \tag{27}$$

where Λ is the proportion of producers employing the high technology. From equation (25) and (26), and using $\pi_H = \Pi_H = 1$, we can derive:

$$\Lambda = \frac{h}{h + \eta(1-h)\Pi_L} \tag{28}$$

Observe that Λ is strictly increasing in h, satisfying: $\lim_{h\to 0}\frac{\Lambda}{h} = \frac{1}{\eta}$, and $\lim_{h\to 1}\frac{\Lambda}{h} = 1$.

Now μ is no longer equal to M. Since the expressions in terms of M are complex, our strategy is to establish the existence of various types of equilibria based on the values of μ. Once this is done, we can derive the corresponding values of M by utilizing the following monotone increasing relationship between M and μ, which can be obtained by combining equations (25) and (27) to yield, $\Lambda\eta\alpha(1-\frac{M}{\mu}) = \beta\mu x h(\frac{M}{\mu} - M)$, or,

$$M = \frac{\mu\eta\alpha\,(\Lambda/h)}{\beta x\mu(1-\mu) + \eta\alpha\,(\Lambda/h)} \tag{29}$$

The expression could be simplified with the aid of the limiting properties under equilibrium AA or CC. As a result, the population distribution will be determined by only three endogenous variables, h, μ, and Π_L, since from (1), (2), (3), (28) and (29), all population

	Equilibrium AA	Equilibrium BB	Equilibrium CC
V_0	$\Delta_H \dfrac{\rho_H^{AA}\rho_m^{AA}U-\varepsilon}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\Delta_H \dfrac{\rho_H^{BB}\theta U-\varepsilon}{1-\rho_H^{BB}\Delta_H}$, or $\Delta_L \dfrac{\rho_L^{BB}\theta U-\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$	$\Delta_L \dfrac{\rho_L^{CC}\rho_m^{CC}\theta U-\delta\varepsilon}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$
V_H	$\dfrac{\rho_H^{AA}\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\dfrac{\rho_H^{BB}(\theta U-\Delta_H\varepsilon)}{1-\rho_H^{BB}\Delta_H}$	V_L^{CC}
V_L	0	$\dfrac{\rho_L^{BB}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{BB}\Delta_L}$	$\dfrac{\rho_L^{CC}\rho_m^{CC}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{CC}\rho_m^{CC}}$
V_m	$\dfrac{\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\dfrac{\theta U-\Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H}$, or $\dfrac{\theta U-\Delta_L\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$	$\dfrac{\rho_m^{CC}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$
Π_L	0	π_L^{BB}	1
h	1	h^{BB}	0
μ	S^{AA}	S^{BB}	S^{CC}

Table 3: Solutions for the Non-instantaneous Production Case

masses can be expressed in terms of h, μ and M.[9] As before, we can solve the system using the discount rates Δ_H and Δ_L (see Table 3; the details of derivation are in Appendix.).[10]

Repeating the same steps as in the previous section, one can derive parameter regions for μ (instead of M) to support each type of equilibrium. As shown in the Appendix, we have: $S^{AA} = (0, \mu_1) \cap (M_2, 1-M_2)$, where μ_1 solves:

$$(1-\theta)U = \frac{(\beta\mu x + r)r(U-\Delta_H\varepsilon)}{\beta^2 x^2 \mu(1-\mu)(1-\Delta_H) + r\beta x + r^2}; \tag{30}$$

$S^{BB} = (M_4, \mu_1)$, and, $S^{CC} = S^C$. Based on the proofs in Appendix, we can establish the existence regions of these equilibria:

Proposition 3: (Existence) *Under Assumptions 1-3, a steady-state monetary equilibrium exists, which possesses the following properties:* $\pi_H = 1$ and

[9] Actually we express all the values in terms of μ. In contrast to the instantaneous production case, μ is now endogenously determined. It seems that it is no longer suitable to use μ to characterize the existence conditions. However, equation (29) enables us to relate μ with the exogenous variable M in each equilibrium. All the conditions in terms of μ can be transformed into expressions in M accordingly.

[10] Like Table 1, we have ρ_H^{AA}, ρ_H^{BB}, and ρ_L^{CC} with the same functional form. ρ_m^{AA} and ρ_m^{CC} also have same functional form. The argument μ may differ.

(i) $\pi_L = 0$, *only if* $\mu \in S^{AA}$ *(equilibrium AA);*

(ii) $\pi_L \in (0, 1)$, *only if* $\mu \in S^{BB}$ *(equilibrium BB);*

(iii) $\pi_L = 1$, *only if* $\mu \in S^{CC}$ *(equilibrium CC);*

where multiple equilibria may arise.

4.2 Welfare Implications

Due to its complexity, we will not conduct welfare analysis based on the endowment of money (M). Rather, we restrict our attention to the case where the fraction of money traders (μ) is identical in different types of equilibria. With this modification, we still have equilibrium AA Pareto dominates equilibrium BB. However the welfare comparison between equilibria AA and CC is a bit more sophisticated now. Let us derive the social welfare for the respective equilibria as follows:

$$Z^{AA} = \frac{\eta\alpha b}{b + \eta\alpha}\left(\frac{U - \varepsilon}{r}\right) \tag{31}$$

$$Z^{CC} = \frac{\alpha b}{b + \alpha}\left(\frac{\theta U - \delta\varepsilon}{r}\right). \tag{32}$$

where $b \equiv \beta x \mu(1 - \mu)$.

Obviously the optimal fraction of money traders satisfies $\mu = 0.5$ in each case, provided that $\mu_1 \geq 0.5$, from which the optimal quantity of money can be derived using equation (29). For pointwise comparison of social welfare between different types of equilibria with respect to μ, we still have the net utility terms, $U - \varepsilon$ versus $\theta U - \delta\varepsilon$ as in the instantaneous production case. However, the slow production process makes the high technology less attractive than the low technology as the multiplier on the right-hand side of (31) is less than that of (32) if $\eta < 1$. When the net utility gain from undertaking the high technology is positive and sufficiently large to overcome the disadvantage from a non-instantaneous production process, the welfare under equilibrium AA is greater than that under equilibrium CC.

Meanwhile, the autarkic values in the respective equilibria are

$$W^{AA} = \frac{U - \Delta_H\varepsilon}{1 - \Delta_H} = \frac{(\eta\alpha + r)U - \eta\alpha\varepsilon}{r} \tag{33}$$

	Equilibrium AA	Equilibrium CC
Monetary	$\dfrac{\eta\alpha b(U-\varepsilon)}{r(b+\eta\alpha)}$	$\dfrac{\alpha b(\theta U-\delta\varepsilon)}{r(b+\alpha)}$
Autarky	$\dfrac{(\eta\alpha+r)U-\eta\alpha\varepsilon}{r}$	$\dfrac{(\alpha+r)\theta U-\alpha\delta\varepsilon}{r}$

$b\equiv\beta x\mu(1-\mu)$

Table 4: Present Values of Welfare in the Non-Instantaneous Production Case

$$W^{CC}=\frac{\theta U-\Delta_L\delta\varepsilon}{1-\Delta_L}=\frac{(\alpha+r)\theta U-\alpha\delta\varepsilon}{r}. \tag{34}$$

Again, the comparison between the two values depends crucially on the net utility gain versus the loss in a non-instantaneous production process. Formally, we define $q\equiv\frac{\theta U-\delta\varepsilon}{U-\varepsilon}$ and calculate two critical values for η,

$$\eta_Z(\theta,\delta)=q-\frac{q\alpha(1-q)}{\alpha+b-\alpha q}\quad\text{and}\quad\eta_W(\theta,\delta)=q-\frac{r(1-\theta)U}{\alpha(U-\varepsilon)},$$

such that $Z^{AA}>Z^{CC}$ iff $\eta>\eta_Z$, and that $W^{AA}>W^{CC}$ iff $\eta>\eta_W$. Note that when $q\geq 1$, type-L technology is always chosen in both monetary and autarky economies, since it provides more net utility and requires less production time.[11] In the case where $q<1$, we can show that $\eta_Z>\eta_W$. As a result, the introduction of money can completely restore the first-best technology choice only when $\mu_1>1/2$ and $\eta>\eta_Z$.

Nonetheless, the introduction of money does improve the efficiency of technology choices if $\frac{\beta x^2}{\beta x^2+r}<\theta<\frac{\beta x(1-\mu)}{\beta x(1-\mu)+r}$, $\mu<\mu_1$ and $\eta>\eta_Z$ (see Proposition $A2$ in Appendix and the proof therein). Under $\frac{\beta x^2}{\beta x^2+r}<\theta$, only can the type-$L$ technology be chosen in a pure barter economy, as the utility gain from consuming the high-tech good is not sufficient to overcome the time delay cost. While the inequalities, $\theta<\frac{\beta x(1-\mu)}{\beta x(1-\mu)+r}$ and $\mu<\mu_1$, ensure that the type-H technology can be adopted in a monetary economy, the condition $\eta>\eta_Z$ guarantees that adopting the high type technology results in higher welfare than adopting the low one. In this case, search frictions grant too much disadvantage for producers to adopt the high technology under barter; the introduction of money can fully mitigate such disadvantage to encourage the employment of the advanced technology in equilibrium. Summarizing,

[11] It is also true in the pure barter economy.

Proposition 4: (Money and Technology Choice) *When $\eta > \eta_Z$, technology choice in a monetary economy is autarky efficient. By further assuming $\frac{\beta x^2}{\beta x^2 + r} < \theta < \frac{\beta x(1-\mu)}{\beta x(1-\mu)+r}$ and $\mu < \mu_1$, the introduction of money improves the efficiency of technology choice by encouraging the adoption of the high technology that cannot be chosen in the pure barter economy.*

As long as the type-H goods provide more utility and the nonselectivity effect is sufficiently small ($\mu_1 \geq 0.5$), autarky efficiency is a *sufficient* (but not necessary) condition for equilibrium AA to dominate CC in social welfare sense. As a consequence, the monetary economy can improve technological development.

Proposition 5: (Welfare under Non-instantaneous Production) *While equilibrium AA always Pareto dominates equilibrium BB, it leads to a higher welfare than equilibrium CC if $\eta > \eta_Z$, and $\mu_1 \geq 0.5$. The optimal quantity of money is* $\min\{[2 + \frac{\beta x}{2\eta\alpha}]^{-1}, M(\mu_1)\}$ *for equilibrium AA with $M(\mu) = \frac{\mu\eta\alpha}{\beta x\mu(1-\mu)+\eta\alpha}$ and $[2 + \frac{\beta x}{2\alpha}]^{-1}$ for equilibrium CC.*

Notice that the results of social welfare comparison are essentially driven by the values of goods and money traders. Provided that the two technologies provide the same values to producers in autarky, the sellers and buyers in the monetary exchange economy would prefer the high one (pointwise with respect to μ), since

$$\frac{1-\Delta_H}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} > \frac{1-\Delta_L}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}.$$

However, in terms of Pareto criteria, we must also examine the welfare of producers, whose relative gain from employing the high technology can be written as:

$$\begin{aligned} V_0^{AA} - V_0^{CC} &= \left(\frac{1-\Delta_H}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}W^{AA} - \frac{1-\Delta_L}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}W^{CC}\right) - (1-\theta)U \\ &= \left[\frac{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}\frac{U-\Delta_H\varepsilon}{\theta U-\Delta_L\delta\varepsilon} - 1\right]\frac{\theta U - \Delta_L\delta\varepsilon}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L} - (1-\theta)U. \end{aligned}$$

The term in the square bracket is similar to the value comparison for goods and money traders, but the last term may upset such a comparison if θ is sufficiently lower than one. This last term can be viewed as the difference in inventory costs per unit of goods, which is driven by the time-consuming trading period in the monetary economy with search frictions. Thus, even when the high technology provides a higher autarkic value, the producers may still prefer the low technology when frictional exchanges are taken into account.

Another interesting finding is that the producer's gains from employing the high technology relevant for decentralized equilibrium ($V_0^{AA} - V_0^{CC}$) need not be maximized at the welfare-optimizing quantity of money. In particular, we can identify a *time-saving effect* from $\frac{1}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$, which is increasing in $\mu(1-\mu)$. In fact, it is the only effect in the case of instantaneous production, since $\Delta_H = \Delta_L = 1$. When production takes time, there also exists a *mitigation effect*, which is decreasing in $\mu(1-\mu)$ as long as it takes more time to produce the type-H goods ($\Delta_H < \Delta_L$).[12] Intuitively, a longer waiting period to trade would mitigate the disadvantage of the high technology in production time to a greater extent.

When the expected trading period approaches to its minimum (μ tends to 0.5), the mitigation effect may be strong enough to dominate the time-saving effects under some parameter values. Figure 2 illustrates a numerical example, in which the sign of producers' gain depends on the quantity of money and the mitigation effect dominates the time saving effect near the optimal quantity of money. The resultant social inefficiency from the presence of a strong mitigation effect leads to a negative gain from employing the high technology and hence an under-investment in that technology. Summarizing,

Proposition 6: (Under-investment in the High Technology under Non-instantaneous Production) *With a sufficiently short expected trading period and an endowment of money near its optimal level, producers tend to under-invest in the high technology in decentralized equilibrium.*

5 Conclusion

An interesting message our model has delivered is that the use of money affects producers' technology choices in favor of the high technology in the instantaneous production model and that such an effect is reinforced if production takes time. Moreover, we identify a social inefficiency caused by search frictions leading to under-investment in the advanced technology in decentralized equilibrium. Furthermore, in the case of mixed-strategy equilibrium, the share of high-technology output is increasing in the quantity of money.

The implication of our model could go beyond the technology choice issue. Should we

[12] This effect is via the term, $\frac{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} = \frac{\Delta_L}{\Delta_H} + \frac{1}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}(1-\frac{\Delta_L}{\Delta_H})$.

regard the high technology as a production plan of high volume, and the low technology as one with low volume, it becomes a binary output quantity model, where the utilities, manufacturing costs and production times are all increasing in the scale of production. This may shed light on the possibility of multiple equilibria in the multiple consumption units or divisible goods setup. For instance, in a simple case with constant return and cost to scale, the highest possible volume of output is best in the sense of social welfare. The optimal volume of output will be determined by the relevant set of parameters (similar to S^A), which depends on the quantity of money in the economy.

Due to the decentralized exchange mechanism, we have multiple equilibria while the one with coexistence of both technologies is unstable and Pareto-dominated. One may wonder whether this finding is robust under an alternative, directed-search framework (with a high- and a low-quality submarket). Our preliminary results suggest that we may have a unique stable equilibrium with coexistence of both technologies under proper conditions. Moreover, when production is instantaneous, an increase in the quantity of money tends to encourage high-quality goods production, but does not affect the thickness of each market.

As one of the central features of the model, perfect observability is assumed throughout. To another extreme, if buyers cannot detect the quality of the commodities trade at all, then V_H always equals V_L and producers will always choose the cost-saving technology without investing in the high technology. In the case of partial observability, we expect similar results as in Trejos (1997). In particular, if the high technology has adequate relative efficiency over the low, then the buyers would prefer type-H goods whenever they are able to identify its quality. It is therefore straightforward to conclude that the presence of private information will not eliminate the positive role of money in production efficiency as long as partial observability is preserved.

References

[1] Berentsen, Aleksander and Guillaume Rocheteau (2004), "Money and information," *Review of Economic Studies*, 71, 915-944.

[2] Camera, Gabriele, Rob Reed and Chris Waller (2003), "A jack of all trade or a master of one? Specialization, trade and money," *International Economic Review*, 44, 1275-1294.

[3] Diamond, Peter and Joel Yellin (1990), "Inventories and money holdings in a search economy," *Econometrica*, 58, 929-950.

[4] Jones, Robert (1976), "The origin and development of media exchange," *Journal of Political Economy*, 84, 757-776.

[5] Kim, Young Sik, and Shuntian Yao (2001), "Liquidity, quality, production cost, and welfare in a search model of money," *Economic Journal*, 111, 114-127.

[6] Kiyotaki, Nobuhiro, and Randall Wright (1989), "On money as a medium of exchange," *Journal of Political Economy*, 97, 927-954.

[7] Kiyotaki, Nobuhiro, and Randall Wright (1993), "A search-theoretic approach to monetary economics," *American Economic Review*, 83, 63-77.

[8] Laing Derek, Li, Victor E., and Ping Wang (forthcoming), "Inflation and productive activity in a multiple-matching model of money," *Journal of Monetary Economics*.

[9] Li, Victor E. (1995), "The optimal taxation of fiat money in search equilibrium," *International Economic Review*, 36, 927-942.

[10] Rupert, Peter, Martin Schindler and Randall Wright (2001), "Generalized search-theoretic models of monetary exchange," *Journal of Monetary Economics*, 48, 605-622.

[11] Shi, Shouyong (1997), "Money and specialization," *Economic Theory*, 10, 99-113.

[12] Trejos, Alberto (1997), "Incentives to produce quality and the liquidity of money," *Economic Theory*, 9, 355-365

[13] Trejos, Alberto, and Randall Wright (1995), "Search, bargaining, money, and prices," *Journal of Political Economy*, 103, 118-141.

[14] Williamson, Stephen, and Randall Wright (1994), "Barter and monetary exchange under private information," *American Economic Review*, 84, 104-123.

[15] Wright, Randall (1999), "A note on asymmetric and mixed-strategy equilibria in the search-theoretic model of fiat money," *Economic Theory*, 14, 463-471

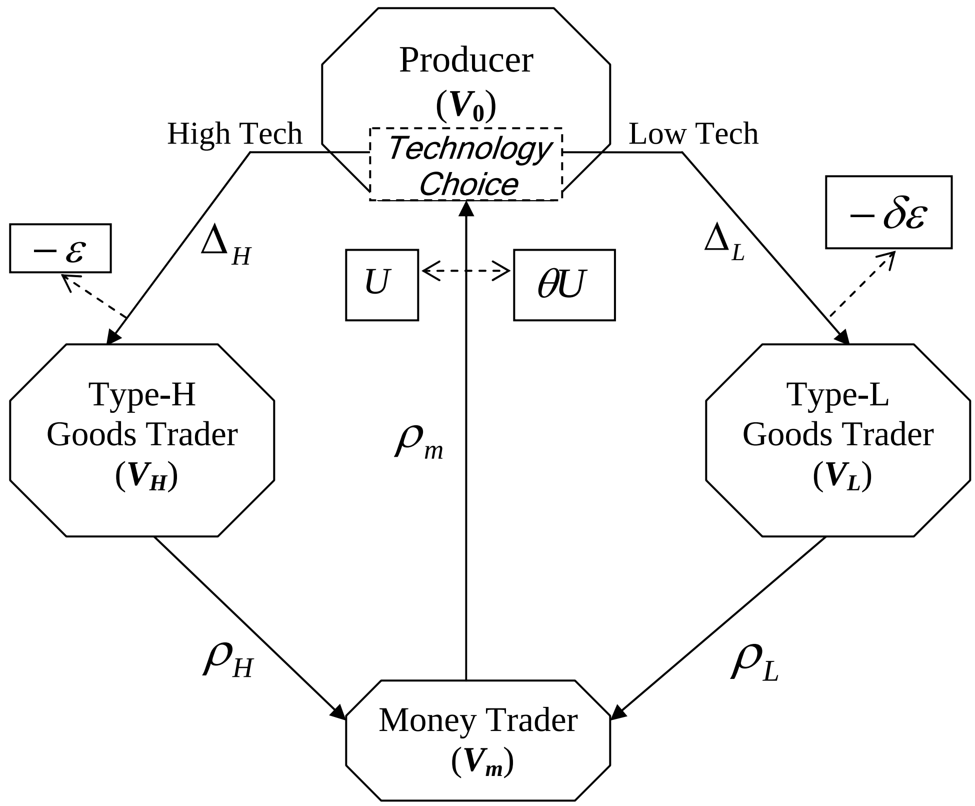

Figure 1: Steady-State Flow Chart

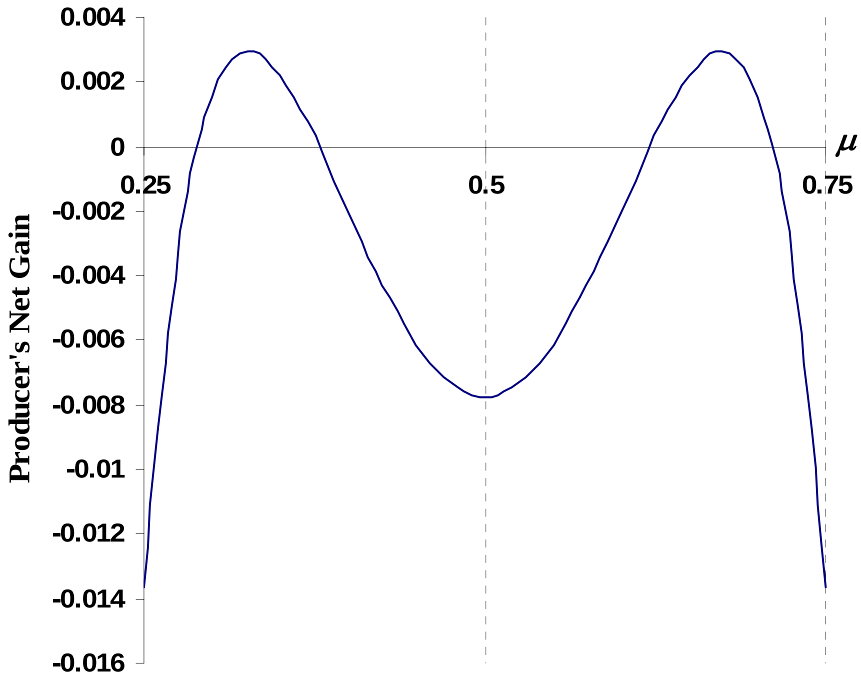

Figure 2: Producers' Net Gains from Investing in High Technology

Appendix

(Not intended for publication)

A. Technology Choice in a Pure Barter Economy

In this appendix, we investigate the technology choice issue in the scenario of a pure barter economy. On the basis of the notation we employ in Section II, we can set up the related values functions:

$$rV_0 = \max\{\alpha(V_L - V_0 - \delta\varepsilon), \eta\alpha(V_H - V_0 - \varepsilon)\}, \quad \text{(A1)}$$

$$rV_H = \beta x^2[h\Pi_{HH} \max_{\pi_{HH}}\{\pi_{HH}(U + V_0 - V_H)\} + (1-h)\Pi_{LH} \max_{\pi_{HL}}\{\pi_{HL}(\theta U + V_0 - V_H)\}], \quad \text{(A2)}$$

$$rV_L = \beta x^2[h\Pi_{HL} \max_{\pi_{LH}}\{\pi_{LH}(U + V_0 - V_L)\} + (1-h)\Pi_{LL} \max_{\pi_{LL}}\{\pi_{LL}(\theta U + V_0 - V_L)\}], \quad \text{(A3)}$$

where $\pi_{i,j}$ indicates the probability for i-type goods trader to accept j-type commodities. The equilibrium population equations are

$$\Lambda\eta\alpha N_0 = \beta x^2[h\Pi_{HH}\pi^*_{HH} + (1-h)\Pi_{LH}\pi^*_{HL}]N_H, \quad \text{(A4)}$$

$$(1-\Lambda)\alpha N_0 = \beta x^2[h\Pi_{HL}\pi^*_{LH} + (1-h)\Pi_{LL}\pi^*_{LL}]N_L. \quad \text{(A5)}$$

The active equilibrium condition similar to condition (16) yields

$$\Pi_{HH} = \pi^*_{HH} = \Pi_{LH} = \pi^*_{LH} = 1. \quad \text{(A6)}$$

As a result, we can rewrite equation (A2) and (A3) as

$$rV_H = \beta x^2[h(U + V_0 - V_H) + (1-h)\pi^*_{HL}(\theta U + V_0 - V_H)], \quad \text{(A7)}$$

$$rV_L = \beta x^2[h\Pi_{HL}(U + V_0 - V_L) + (1-h)\Pi_{LL}\pi^*_{LL}(\theta U + V_0 - V_L)], \quad \text{(A8)}$$

and solve Λ as a function of h

$$\Lambda = \frac{h[h + (1-h)\pi^*_{HL}]}{h[h + (1-h)\pi^*_{HL}] + (1-h)\eta[h\Pi_{HL} + (1-h)\Pi_{LL}\pi^*_{LL}]}. \quad \text{(A9)}$$

Instantaneous Production

In the instantaneous production case, $V_0 = \max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\}$. Observe that $\theta U + V_0 - V_H \geq \theta U - \varepsilon > 0$ under Assumption 1. Therefore $\Pi_{HL} = \pi^*_{HL} = 1$. Similarly $\theta U + V_0 - V_L \geq \theta U - \delta\varepsilon > 0$, and $\Pi_{LL} = \pi^*_{LL} = 1$. From (A7) and (A8), we can find that $V_L = V_H$, which means only the low technology would be chosen, since $V_L - \delta\varepsilon > V_H - \varepsilon$. The results can be summarized as follows:

Proposition A1 (Pure Barter with Instantaneous Production) *Under instantaneous production with pure barter exchange and Assumption 1, only the type-L technology will be chosen even when it provides less net utilities than the type-H technology.*

	Equilibrium A^b	Equilibrium B^b	Equilibrium C^b
V_0	$\max\{\Delta_H(V_H-\varepsilon), \Delta_L(V_L-\delta\varepsilon)\}$	$\Delta_H\dfrac{\theta U-\varepsilon}{1-\Delta_H}$	$\Delta_L\dfrac{\rho_b\theta U-\delta\varepsilon}{1-\rho_b\Delta_L}$
V_H	$\dfrac{h\beta x^2(U-\Delta_H\varepsilon)}{h\beta x^2(1-\Delta_H)+r}$	$\dfrac{\theta U-\Delta_H\varepsilon}{1-\Delta_H}$	$\dfrac{\rho_b(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_b\Delta_L}$
V_L	$\dfrac{(1-h)\beta x^2(\theta U-\Delta_L\delta\varepsilon)}{(1-h)\beta x^2(1-\Delta_L)+r}$	$\dfrac{\alpha+r}{r}\eta(\theta U-\varepsilon)+\delta\varepsilon$	$\dfrac{\rho_b(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_b\Delta_L}$
π^*_{HL}	0	π_b	1
π^*_{LL}	1	1	1
h	h_s or 1	h_b	0

Table 5: Solutions for Pure Barter Economy with Non-Instantaneous Production

Note that, as described in Proposition 2, the social planner will choose the technologies providing more utilities in the monetary economy under Assumption 1-3 and the condition $M_1 > 1/2$, it is obvious that the introduction of money does improve the efficiency in technology choice.

Non-Instantaneous Production

When we have non-instantaneous production, it is a bit more complicated. If $V_H \leq V_L$, the producers will choose only the low technology, which requires less production cost and shorter production time. From equation (A7) and (A8) as well as $h = 0$, we can find that

$$V_H = \frac{\beta x^2\pi^*_{HL}(\theta U + V_0)}{r + \beta x^2\pi^*_{HL}} \quad \text{and} \quad V_L = \frac{\beta x^2\Pi_{LL}\pi^*_{LL}(\theta U + V_0)}{r + \beta x^2\Pi_{LL}\pi^*_{LL}}$$

Hence $\pi^*_{HL} \leq \Pi_{LL}\pi^*_{LL}$. Meanwhile, we must have $\theta U + V_0 - V_H \geq \theta U + V_0 - V_L$, which implies that $\pi^*_{HL} \geq \pi^*_{LL} \geq \Pi_{LL}\pi^*_{LL}$. Since $\pi^*_{HL} = \pi^*_{LL} = 0$ leads to $V_L = 0$ and $V_0 < 0$, the only possible case is $\pi^*_{HL} = \pi^*_{LL} = 1$, where $V_H = V_L$, and the producers only choose the low technology ($h = 0$).

If $V_H > V_L$, we have $\theta U + V_0 - V_H < \theta U + V_0 - V_L$, and thus $\pi^*_{HL} \leq \pi^*_{LL}$. Note that we cannot have both mixed strategies at the same time. Therefore, we have only four cases to discuss: (1) $\pi^*_{HL} = \pi^*_{LL} = 1$; (2) $0 < \pi^*_{HL} < \pi^*_{LL} = 1$; (3) $0 = \pi^*_{HL} \leq \pi^*_{LL} < 1$; and (4) $0 = \pi^*_{HL} < \pi^*_{LL} = 1$.

Case 1: $\pi^*_{HL} = \pi^*_{LL} = 1$. It implies $V_H = V_L$, and the producers only choose the low technology ($h = 0$). The solutions are provided as equilibrium C^b in Table 6 with

$$\rho_b = \frac{\beta x^2}{\beta x^2 + r}. \tag{A10}$$

In this case, we need $\theta U + V_0 - V_L > 0$, which always holds under Assumption 1. Meanwhile, $V_0 > 0$ requires

$$\frac{\delta\varepsilon}{\theta U} < \frac{\beta x^2}{\beta x^2 + r}.$$

Case 2: $0 < \pi^*_{HL} < \pi^*_{LL} = 1$. The immediate implication is

$$\theta U + V_0 - V_H = 0. \tag{A11}$$

Based on equation (A11), we can rewrite the value functions as

$$V_H = \frac{\beta x^2 h(1-\theta)U}{r}, \tag{A12}$$

$$V_0 = \frac{\beta x^2 h(1-\theta)U}{r} - \theta U, \tag{A13}$$

$$V_L = \frac{\beta x^2[h\Pi_{HL} + (1-h)] + r\Pi_{HL}}{\beta x^2[h\Pi_{HL} + (1-h)] + r} V_H. \tag{A14}$$

Observe from (A13) that $V_0 > 0$ implies $h > 0$ and consequently $rV_0 = \eta\alpha(V_H - V_0 - \varepsilon) = \eta\alpha(\theta U - \varepsilon)$ in the case of positive production time. We can combine it with equation (A13) to obtain the proportion of type-H goods

$$h_b = \frac{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}{\beta x^2(1-\theta)U}. \tag{A15}$$

If $h = h_b < 1$, we can substitute (A15) into the expressions of V_H and V_0

$$V_H = \frac{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}{r} = \frac{\theta U - \Delta_H\varepsilon}{1 - \Delta_H} \tag{A16}$$

and

$$V_0 = \frac{\eta\alpha(\theta U - \varepsilon)}{r} = \Delta_H \frac{\theta U - \varepsilon}{1 - \Delta_H}. \tag{A17}$$

In order to make the producers indifferent between the two technologies, we need

$$V_L - V_0 - \delta\varepsilon = \eta(V_H - V_0 - \varepsilon). \tag{A18}$$

With the help of equations (A11), (A14) (A16), and (A17) we can convert equation (A18) into

$$\frac{1 - \Pi_{HL}}{\beta x^2[h_b\Pi_{HL} + (1-h_b)] + r} = \frac{(1-\eta)\theta U - (\delta - \eta)\varepsilon}{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}.$$

and compute the solution for the cross-type acceptability, denoted as π_b. Note that $\pi_b < 1$ as long as $\theta U > \delta\varepsilon$. Actually this equilibrium is unstable if we disturb the acceptability Π_{HL} slightly away from its equilibrium level. Note that $\Pi_{HL} \geq 0$ only when

$$\frac{(1-\theta)U}{(\beta x^2 + r)(1-\theta)U - (\eta\alpha + r)\theta U + \eta\alpha\varepsilon} \geq \frac{(1-\eta)\theta U - (\delta - \eta)\varepsilon}{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon} \tag{A19}$$

$$\frac{(1-\theta)U}{(1-\eta)\theta U-(\delta-\eta)\varepsilon}\geq\frac{(\beta x^2+r)(1-\theta)U}{(\eta\alpha+r)\theta U-\eta\alpha\varepsilon}-1$$

The other subcase with $h_b=1$ requires some particular cost-utility ratio to satisfy equation (A15). Moreover, we need $\Pi_{HL}<\pi_b$ to discourage the producers from choosing the low technology. As a consequence, this equilibrium does not hold generically.

Case 3: $0=\pi^*_{HL}\leq\pi^*_{LL}<1$. Now we have $V_L=0$ and $V_H=\frac{\beta x^2(U-\Delta_H\varepsilon)}{\beta x^2(1-\Delta_H)+r}$. Hence the producers will only choose the high technology. Note that we need $\theta U+V_0<V_H$, which requires $\theta U<\rho_b U-(1-\rho_b)V_0$, or

$$\Delta_H<1-\frac{r(\theta U-\varepsilon)}{\beta x^2(1-\theta)U-r\varepsilon},\text{ or }\eta\alpha<\frac{\beta x^2(1-\theta)U-r\theta U}{\theta U-\varepsilon}. \tag{A20}$$

Similar to the discussion following Lemma 1, one must guarantee that it is not too costly to wait for the next trade, instead of producing right now. Observe that, given Assumption 1 and $\theta<\rho_b$, we have $\rho_b U>\varepsilon$, which implies $V_0>0$.

Case 4: $0=\pi^*_{HL}<\pi^*_{LL}=1$. It demands $V_L<\theta U+V_0<V_H$. While the cross-type acceptability is zero, we have separating equilibrium with

$$V_H=\frac{h\beta x^2(U-\Delta_H\varepsilon)}{h\beta x^2(1-\Delta_H)+r},$$

$$V_L=\frac{(1-h)\beta x^2(\theta U-\Delta_L\delta\varepsilon)}{(1-h)\beta x^2(1-\Delta_L)+r},$$

and

$$V_0=\max\{\Delta_H(V_H-\varepsilon),\Delta_L(V_L-\delta\varepsilon)\}$$

The condition $V_L<\theta U+V_0<V_H$ requires $\theta U+\Delta_H(V_H-\varepsilon)<V_H$, or

$$h>h_0\equiv\frac{r(\theta U-\Delta_H\varepsilon)}{\beta x^2(1-\Delta_H)(1-\theta)U}.$$

Note that $h_0<1$ iff the conditions given in Case 3 are satisfied.

Moreover, we need $V_0>0$, which implies

$$h>h_1\equiv\frac{\varepsilon r}{\beta x^2(U-\varepsilon)}$$

when the type-H technology is chosen, or

$$h<h_2\equiv1-\frac{\delta\varepsilon r}{\beta x^2(\theta U-\delta\varepsilon)}$$

when the producers employ the low technology. So one of the necessary condition for the coexistence of both technologies is

$$h_2\geq\max\{h_0,h_1\}. \tag{A21}$$

Note that $h_2 \geq h_1$ implies

$$\frac{\delta\varepsilon}{\theta U} \leq \frac{\beta x^2 U - (\beta x^2 + r)\varepsilon}{(\beta x^2 + r)U - (\beta x^2 + 2r)\varepsilon},$$

where the right-hand side is less than $\frac{\beta x^2}{\beta x^2+r}$. As a result, being a subset of the existence region for high-technology only equilibrium, the existence region for coexistence in Case 4 is also a subset of the existence region in Case 1.

Since an increase in h leads to bigger V_H and smaller V_L, the function $f(h) = \Delta_H(V_H - \varepsilon) - \Delta_L(V_L - \delta\varepsilon)$ is strictly increasing in h. Moreover, it is easy to find that, under the necessary condition (A21), $f(h_2) > 0$ and $f(h_1) < 0$. Consequently, there exists a unique $h_s \in (h_1, h_2)$, such that $f(h_s) = 0$. This solution is only valid when

$$h_s \geq h_0. \tag{A22}$$

As a result, there are two possible equilibria conditional on the parameters. The separating equilibrium exists only when conditions (A20), (A21) and (A22) are all satisfied, while the producers would choose high technology when we have both (A20) and $\varepsilon r \leq \beta x^2(U - \varepsilon)$, or $\frac{\varepsilon}{U} < \frac{\beta x^2}{\beta x^2+r}$. Obviously, the existence region of the high-technology only equilibrium is a subset of that of the low-technology only equilibrium when $0 < \delta \leq \theta < 1$, which resembles Lemma 2.

Proposition A2 (Pure Barter with Non-instantaneous Production) *Under instantaneous production with pure barter exchange and Assumption* 1, *there exist multiple equilibria where the admissible sets of equilibria vary with different primitives of the economy. Equilibrium C^b (low technology only) exists as long as $\frac{\delta\varepsilon}{\theta U} < \frac{\beta x^2}{\beta x^2+r}$. The mixed-strategy equilibrium and the separating equilibrium are unstable. Parameters satisfying $\theta < \frac{\beta x^2}{\beta x^2+r}$ and inequality (A20) feature the adoption of the high technology.*

In case 1, the number of producers and goods holder satisfies $\alpha N_0 = \beta x^2 N_L$. Hence $N_0 = \frac{\beta x^2}{\alpha+\beta x^2}$, and $N_L = \frac{\alpha}{\alpha+\beta x^2}$. As a result, social welfare becomes

$$W_b^C = \alpha \frac{\beta x^2(\theta U - \delta\varepsilon)}{r(\alpha + \beta x^2)}.$$

Similarly, social welfare in the case of high-technology only equilibrium is

$$W_b^A = \eta\alpha \frac{\beta x^2(U - \varepsilon)}{r(\eta\alpha + \beta x^2)}.$$

When both equilibria coexist, we must have

$$\eta > \eta_A \equiv \frac{q\beta x^2}{\alpha + \beta x^2 - \alpha q},$$

where $q = \frac{\theta U - \delta\varepsilon}{U - \varepsilon}$, as previously defined.

B. Proofs

In this appendix, we provide detailed mathematical derivations of some fundamental relationships and propositions presented in the main text.

Proof of Lemma 1:

In equilibrium A, we need $\pi_L = 0$, and hence $\theta U + V_0 - V_m < 0$. Using the solutions provided in Table 1, we can obtain

$$\theta U + \frac{\rho_H^A \rho_m^A U - \varepsilon}{1 - \rho_H^A \rho_m^A} - \frac{\rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} < 0$$

or

$$\theta U - \varepsilon + \frac{\rho_H^A \rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} - \frac{\rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} < 0.$$

Therefore,

$$\frac{\theta U - \varepsilon}{U - \varepsilon} < \frac{\rho_m^A (1 - \rho_H^A)}{1 - \rho_H^A \rho_m^A}$$

Employing the definition of (8) and (17), we can multiply $(\beta \mu x + r)[\beta(1-\mu)x + r]$ to both the numerator and the denominator. Now we have

$$\frac{\theta U - \varepsilon}{U - \varepsilon} < \frac{\beta(1-\mu)x}{\beta x + r}$$

or

$$M < M_1 \equiv 1 - \frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x (U - \varepsilon)} \tag{B1}$$

where we use the equilibrium result that $\mu = M$.

In addition, we also need the producer's value to be positive, *i.e.*

$$\frac{\rho_H^A \rho_m^A U - \varepsilon}{1 - \rho_H^A \rho_m^A} > 0.$$

Hence

$$\frac{\varepsilon}{U} > \rho_H^A \rho_m^A = \frac{\beta^2 x^2 \mu(1-\mu)}{\beta^2 x^2 \mu(1-\mu) + (\beta x + r) r}$$

or

$$\mu(1-\mu) > Q \equiv \frac{(\beta x + r) r \varepsilon}{\beta^2 x^2 (U - \varepsilon)}. \tag{B2}$$

Observe that the quadratic equation given by the equality in (B2) has two real roots within the interval $(0, 1)$, if Assumption 2 holds. To differentiate the two roots, we define the smaller root to be M_2. As a result, condition (B2) can be written as $M_2 < M < 1 - M_2$ in equilibrium.

Thus, the existence region for equilibrium A is given by $M < M_1$ and $M_2 < M < 1 - M_2$.

Proof of Lemma 2:

The derivation of the existence region is analogous to that of condition (B2). We only have to replace U and ε with θU and $\delta\varepsilon$ respectively. In addition, if $0 < \delta \leq \theta < 1$ and Assumption 1 holds,

$$\frac{(\beta x + r)r\varepsilon}{\beta^2 x^2 (U-\varepsilon)} = \frac{(\beta x + r)r\delta\varepsilon}{\beta^2 x^2 (\delta U-\delta\varepsilon)} \geq \frac{(\beta x + r)r\delta\varepsilon}{\beta^2 x^2 (\theta U-\delta\varepsilon)}.$$

As a result, $S^A \subseteq S^C$.

Derivation of h^B and π^B*:*

Since $\theta U + V_0^B - V_m^B = 0$, we can rewrite the money holder's value (6) as

$$rV_m = \beta(1-\mu)xh(1-\theta)U.$$

Based on the solution listed in Table 1, we have

$$h^B = \frac{r}{\beta(1-\mu)x(1-\theta)U}\frac{\theta U - \varepsilon}{1-\rho_H^B} = \frac{(\beta\mu x + r)(\theta U - \varepsilon)}{\beta(1-\mu)x(1-\theta)U}$$

While the producers are indifference between the two technologies, the two solutions of V_0^B listed in Table 1 should be the same, *i.e.*

$$\frac{\rho_H^B \theta U - \varepsilon}{1-\rho_H^B} = \frac{\rho_L^B \theta U - \delta\varepsilon}{1-\rho_L^B} = \frac{\rho_L^B(\theta U - \delta\varepsilon)}{1-\rho_L^B} - \delta\varepsilon.$$

Note that

$$\frac{\rho_L^B}{1-\rho_L^B} = \frac{\beta\mu x \Pi_L}{r} = \frac{\rho_H^B}{1-\rho_H^B}\Pi_L.$$

Therefore

$$\frac{\rho_H^B \theta U - \varepsilon}{1-\rho_H^B} = \frac{\rho_H^B(\theta U - \delta\varepsilon)}{1-\rho_H^B}\Pi_L - \delta\varepsilon$$

$$\pi^B = \Pi_L = \frac{\rho_H^B \theta U - \varepsilon + (1-\rho_H^B)\delta\varepsilon}{\rho_H^B(\theta U - \delta\varepsilon)} = 1 - \frac{(1-\delta)\varepsilon}{\rho_H^B(U-\delta\varepsilon)}$$

Proof of Lemma 3:

The conditions for existence come from the requirement of $V_0 > 0$, and $h^B, \pi^B \in (0,1)$, where h^B and π^B are given by equation (22) and (21), respectively. Assumption 1 implies that $h^B > 0$, while the condition $h^B < 1$ is equivalent to $\mu = M < M_1$. The latter comes from the fact that

$$\begin{aligned}(\beta x M_1 + r)(\theta U - \varepsilon) &= \left[\beta x - \frac{(\beta x + r)(\theta U - \varepsilon)}{U-\varepsilon} + r\right](\theta U - \varepsilon) \\ &= (\beta x + r)\frac{(1-\theta)U}{U-\varepsilon}(\theta U - \varepsilon) \\ &= \beta x(1-M_1)(1-\theta)U\end{aligned}$$

and that h^B is increasing in μ.

Meanwhile, $V_0 > 0$ iff

$$\rho_H^B = \frac{\beta\mu x}{\beta\mu x + r} > \frac{\varepsilon}{\theta U}$$

or

$$\mu > M_4 \equiv \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)}. \tag{B3}$$

Observe that condition (B3), along with Assumption 1, implies that

$$\pi^B > 1 - \frac{(1-\delta)\theta U}{\theta U - \delta\varepsilon} = \frac{\delta(\theta U - \varepsilon)}{\theta U - \delta\varepsilon} > 0,$$

while Assumption 1 also implies that $\pi^B < 1$.

Now consider the relationship between S^B and S^A. We know that S^B is non-empty, iff $M_4 < M_1$.Observe that, with $Q \equiv \frac{(\beta x+r)r\varepsilon}{\beta^2 x^2 (U-\varepsilon)}$, we have

$$M_4(1 - M_1) = \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)} \frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x(U - \varepsilon)} = Q. \tag{B4}$$

Hence $M_1(1 - M_1) > M_4(1 - M_1) = Q$, and $M_4(1 - M_4) > M_4(1 - M_1) = Q$. By Lemma 1, $M_1 \in S^A$, and $M_4 \in S^A$. Consequently, $S^B = (M_4, M_1) \subseteq S^A$.

Proof of Proposition 1:

Since the stability is proved in the body text, only remaining work is to show that all the existence regions are non-empty under Assumption 1-3. Given Assumption 2, we know that $\frac{1}{2} \in (M_2, 1 - M_2)$, and $\frac{1}{2} \in S^C$. Now we need to establish $M_2 < M_1$. One sufficient condition is that $Q < M_1(1 - M_1)$, which boils down to

$$(U - \varepsilon)r\varepsilon < (\theta U - \varepsilon)[\beta x(1 - \theta)U - r(\theta U - \varepsilon)],$$

or

$$\frac{1}{\theta U - \varepsilon} + \frac{\theta}{1 - \theta} < \frac{\beta x}{r}.$$

Note that $Q < M_1(1 - M_1)$ and equation (B4) imply $M_4 < M_1$. As a result, Assumption 1-3 guarantee that $S^B \neq \emptyset$.

Derivation of the social welfare in the instantaneous production case:

In equilibrium A, the social welfare

$$\begin{aligned} Z^A &= MV_m^A + (1 - M)V_H^A \\ &= M\frac{\rho_m^A(U - \varepsilon)}{1 - \rho_H^A\rho_m^A} + (1 - M)\frac{\rho_H^A\rho_m^A(U - \varepsilon)}{1 - \rho_H^A\rho_m^A} \\ &= \frac{U - \varepsilon}{1 - \rho_H^A\rho_m^A}\rho_m^A[\rho_H^A + (1 - \rho_H^A)M] \\ &= \frac{U - \varepsilon}{(\beta x + r)r}\beta(1 - \mu)x(\beta\mu x + rM) \\ &= \frac{\beta x M(1 - M)(U - \varepsilon)}{r}, \end{aligned}$$

where the last equality employs the equilibrium result that $\mu = M$. Analogously, we can derive

$$Z^B = \frac{\beta x M(1-M)(\theta U - \delta\varepsilon)}{r}.$$

Proof of Proposition 2:

For each $M \in S^B$, $M < M_1$ and $\rho_H^A = \rho_H^B$. We have

$$\frac{V_m^A}{V_m^B} = \frac{\rho_m^A(1-\rho_H^B)}{1-\rho_H^A\rho_m^A}\frac{U-\varepsilon}{\theta U-\varepsilon} = \frac{\beta(1-\mu)xr}{(\beta x + r)r}\frac{U-\varepsilon}{\theta U-\varepsilon} > 1$$

and hence $V_H^A = \rho_H^A V_m^A > \rho_H^B V_m^B = V_H^B$. While the producers are indifferent between the two technologies, $V_H^B - \varepsilon = V_L^B - \delta\varepsilon$. Consequently $V_H^B > V_L^B$. So the goods trader's value in equilibrium A is always higher than that in equilibrium B. To the producers, we also have $V_0^A = V_H^A - \varepsilon > V_H^B - \varepsilon = V_0^B$. With the knowledge that $S^B \subseteq S^A$, we can conclude that equilibrium A Pareto dominates equilibrium B either for same M or at the optimal quantity of money. The other parts are straightforward.

Derivation of h^{BB} and π^{BB}*:*

Since $\theta U + V_0^B - V_m^B = 0$, we can rewrite the money holder's value (6) as

$$rV_m = \beta(1-\mu)xh(1-\theta)U.$$

Based on the solution listed in Table 3, we have

$$h^{BB} = \frac{r}{\beta(1-\mu)x(1-\theta)U}\frac{\theta U - \Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H}$$

While the producers are indifference between the two technologies, two solutions for V_0^{BB} listed in Table 3 should be the same. Since $\theta U + V_0^{BB} - V_m^{BB} = 0$, we can also equate two solutions for money holder's value

$$\frac{\theta U - \Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H} = \frac{\theta U - \Delta_L\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$$

Therefore

$$\rho_L^{BB}\Delta_L = \rho_H^{BB}\Delta_H + \frac{\Delta_H\varepsilon - \Delta_L\delta\varepsilon}{\theta U - \Delta_H\varepsilon}(1-\rho_H^{BB}\Delta_H)$$

$$\frac{\rho_H^{BB}}{1-\rho_H^{BB}}\Pi_L = \frac{\rho_L^{BB}\Delta_L}{\Delta_L - \rho_L^{BB}\Delta_L} = \frac{(\theta U - \Delta_H\varepsilon)\rho_H^{BB}\Delta_H - (\Delta_H\varepsilon - \Delta_L\delta\varepsilon)(1-\rho_H^{BB}\Delta_H)}{(\theta U - \Delta_H\varepsilon)(\Delta_L - \rho_H^{BB}\Delta_H) + (\Delta_H\varepsilon - \Delta_L\delta\varepsilon)(1-\rho_H^{BB}\Delta_H)}$$

$$\begin{aligned}\pi^{BB} &= \Pi_L = \frac{r}{\beta\mu x}\frac{(\theta U - \Delta_H\varepsilon)\rho_H^{BB}\Delta_H - (1-\rho_H^{BB}\Delta_H)(\Delta_H - \Delta_L\delta)\varepsilon}{(\theta U - \Delta_H\varepsilon)(\Delta_L - \rho_H^{BB}\Delta_H) + (1-\rho_H^{BB}\Delta_H)(\Delta_H - \Delta_L\delta)\varepsilon} \\ &= \frac{r}{\beta\mu x}\frac{\rho_H^{BB}\Delta_H\theta U - (\Delta_H - \Delta_L\delta + \rho_H^{BB}\Delta_H\Delta_L\delta)\varepsilon}{(\Delta_L - \rho_H^{BB}\Delta_H)\theta U + (\Delta_H - \Delta_L\delta + \rho_H^{BB}\Delta_H\Delta_L\delta - \Delta_H\Delta_L)\varepsilon}\end{aligned}$$

After substituting the expressions of the effective discount factors, we can obtain the result:

$$\pi_L^{BB}(\mu)=\frac{1}{\beta\mu x}\frac{\beta\mu x\eta(\alpha+r)\theta U-\{(\beta\mu x+r)\eta\alpha+r[(\beta\mu x+r)(\eta-\delta)-\delta\eta\alpha]\}\,\varepsilon}{[(\beta\mu x+r)+\eta\,(\alpha-\beta\mu x)]\theta U+[(\beta\mu x+r)(\eta-\delta)-\delta\eta\alpha]\varepsilon}.$$

Note that when $\Delta_H=\Delta_L=1$,

$$\begin{aligned}\pi^{BB} &= \frac{r}{\beta\mu x}\frac{\rho_H^{BB}\theta U-(1-\delta+\rho_H^{BB}\delta)\varepsilon}{(1-\rho_H^{BB})\theta U-(1-\rho_H^{BB})\delta\varepsilon}\\ &= \frac{r}{\beta\mu x}\frac{(\theta U-\delta\varepsilon)\rho_H^{BB}-(1-\delta)\varepsilon}{(\theta U-\delta\varepsilon)(1-\rho_H^{BB})}\\ &= \frac{(\theta U-\varepsilon)\rho_H^{BB}-(1-\delta)\varepsilon}{(\theta U-\delta\varepsilon)\rho_H^{BB}}=\pi^B\end{aligned}$$

Although h^{BB} is increasing in μ, the relationship between π_L^{BB} and μ is no longer monotone. The solution of π_L^{BB} reduces to π_L^B with $\alpha\to\infty$ and $\eta\to 1$.

The values in equilibria AA and CC listed in Table 3 can be explained intuitively. Note that the effective discount factors indicate the time costs over the respective waiting periods (production, selling, and buying). Take V_0^{AA} as an example. As the producers must wait for all the three waiting periods, the utility should be discounted by all the three factors, Δ_H, ρ_H, and ρ_m. Meanwhile, the production cost is generated at the end of the production period, so only Δ_H is attached to it. This provides the producer's value in one cycle, $\Delta_H\rho_H^{AA}\rho_m^{AA}U-\Delta_H\varepsilon$. The value is then obtained by simply dividing the one-cycle value by one minus the discount factor for a cycle, $\Delta_H\rho_H^{AA}\rho_m^{AA}$.

Proof of Proposition 3:

By comparing the solution for producer's values (V_0) in Table 1 and 3, we can find that the condition for $V_0>0$ would not change in the non-instantaneous production case. However, in Equilibrium AA, the condition $\theta U+V_0-V_m<0$ leads to

$$\theta U+\Delta_H\frac{\rho_H^{AA}\rho_m^{AA}U-\varepsilon}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}-\frac{\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}<0$$

or

$$\frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}<(1-\theta)U$$

Note that the left-hand side is strictly increasing in μ, since

$$\begin{aligned}\frac{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}{1-\rho_m^{AA}} &= 1+\frac{\rho_m^{AA}-\rho_H^{AA}\rho_m^{AA}\Delta_H}{1-\rho_m^{AA}}=1+\frac{\rho_m^{AA}(1-\rho_H^{AA}\Delta_H)}{1-\rho_m^{AA}}\\ &= 1+\frac{\rho_m^{AA}(1-\Delta_H)}{1-\rho_m^{AA}}+\frac{\rho_m^{AA}(1-\rho_H^{AA})\Delta_H}{1-\rho_m^{AA}}\\ &= 1+\frac{\beta(1-\mu)x}{r}(1-\Delta_H)+\frac{\beta(1-\mu)x}{\beta\mu x+r}\Delta_H.\end{aligned}$$

Denote $\mu_1 = \mu_1(\Delta_H)$ as the solution for

$$(1-\theta)U = \frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} = \frac{(\beta\mu x+r)r(U-\Delta_H\varepsilon)}{\beta^2x^2\mu(1-\mu)(1-\Delta_H)+r\beta x+r^2}. \tag{B5}$$

Hence we need $\mu < \mu_1$ to guarantee $\theta U + V_0 - V_m < 0$. (Analogous to Lemma 1, $\mu \in (0,\mu_1)$ iff $\theta U < \rho_M^{AA}U - (1-\rho_M^{AA})V_0^{AA}$.) By Assumption 1, $\theta U > \varepsilon$. Hence $\mu_1 < 1$. When $\Delta_H = 1$,

$$1 - \frac{\theta U - \varepsilon}{U-\varepsilon} = \frac{(1-\theta)U}{U-\varepsilon} = \frac{\beta\mu x + r}{\beta x + r} = 1 - \frac{\beta(1-\mu)x}{\beta x + r}.$$

Hence $\mu_1(1) = M_1$. Moreover,

$$\begin{aligned}
& \frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} - \frac{(1-\rho_m^{AA})\varepsilon}{\rho_H^{AA}\rho_m^{AA}} \\
= \;& (1-\rho_m^{AA})\frac{\rho_H^{AA}\rho_m^{AA}(U-\Delta_H\varepsilon) - (1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\varepsilon}{(1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\rho_H^{AA}\rho_m^{AA}} \\
= \;& (1-\rho_m^{AA})\frac{\rho_m^{AA}\rho_H^{AA}U - \varepsilon}{(1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\rho_H^{AA}\rho_m^{AA}} \geq 0
\end{aligned}$$

as long as $V_0^{AA} > 0$. It means the right-hand side of (B5) is just a constant plus a term that is increasing in Δ_H. Recall that this term is also strictly increasing in μ. Therefore the implicit function $\mu_1(\Delta_H)$ given by (B5) is decreasing in Δ_H, and $\mu_1(\Delta_H) \geq \mu_1(1) = M_1$ in non-instantaneous production case, where $\Delta_H < 1$.

As a consequence, Assumptions 1-3 are sufficient for all the existence regions to be non-empty in the case of non-instantaneous production.

Derivation of the social welfare in the non-instantaneous production case:

Consider equilibrium AA with $h = 1$ first. From equation (25)-(29), along with the population identity $N_m + N_H + N_L + N_0 = 1$ and $N_m = M$ in equilibrium, we can solve

$$N_0 = \frac{\mu - M}{\mu} \text{ and } N_H = \frac{M(1-\mu)}{\mu}.$$

Based on the equation (9), (10) and the solutions listed in Table 3, we have

$$\begin{aligned}
Z^{AA} &= \frac{\mu-M}{\mu}V_0^{AA} + \frac{M(1-\mu)}{\mu}V_H^{AA} + MV_m^{AA} \\
&= \frac{\mu-M}{\mu}\Delta_H(\rho_H^{AA}V_m^{AA} - \varepsilon) + \frac{M(1-\mu)}{\mu}\rho_H^{AA}V_m^{AA} + MV_m^{AA} \\
&= V_m^{AA}\left[\frac{\mu-M}{\mu}\Delta_H\rho_H^{AA} + \frac{M(1-\mu)}{\mu}\frac{\beta\mu x}{\beta\mu x + r} + M\right] - \frac{\mu-M}{\mu}\Delta_H\varepsilon \\
&= \frac{\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}\left[\frac{\mu-M}{\mu}\Delta_H\rho_H^{AA} + M\frac{\beta x + r}{\beta\mu x + r}\right] - \frac{\mu-M}{\mu}\Delta_H\varepsilon \\
&= \frac{\mu-M}{\mu}\frac{\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}\left[\Delta_H\rho_H^{AA} + \frac{M\mu}{\mu-M}\frac{\beta x + r}{\beta\mu x + r}\right] - \frac{\mu-M}{\mu}\Delta_H\varepsilon
\end{aligned}$$

Recall that, when $h = 1$, $M = \dfrac{\mu\eta\alpha}{\beta x\mu(1-\mu)+\eta\alpha}$, and hence,

$$\frac{\mu - M}{\mu} = \frac{\beta x\mu(1-\mu)}{\beta x\mu(1-\mu)+\eta\alpha} \quad \text{and} \quad \frac{M\mu}{\mu - M} = \frac{\eta\alpha}{\beta x(1-\mu)}$$

As a consequence,

$$\begin{aligned}
\frac{\mu}{\mu - M} Z^{AA} &= \frac{\rho_m^{AA}(U - \Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}\left[\Delta_H\rho_H^{AA} + \frac{\eta\alpha}{\beta x(1-\mu)}\frac{\beta x + r}{\beta\mu x + r}\right] - \Delta_H\varepsilon \\
&= \frac{(U-\Delta_H\varepsilon)\eta\alpha\left[\beta\mu x\beta x(1-\mu) + (\eta\alpha + r)(\beta x + r)\right]}{\beta^2 x^2\mu(1-\mu)r + (\eta\alpha + r)(r\beta x + r^2)} - \Delta_H\varepsilon \\
&= \frac{(U-\Delta_H\varepsilon)\eta\alpha}{r} - \Delta_H\varepsilon \\
&= \frac{\eta\alpha U - \Delta_H\varepsilon(\eta\alpha + r)}{r} \\
&= \frac{\eta\alpha(U-\varepsilon)}{r}
\end{aligned}$$

and

$$Z^{AA} = \frac{\mu - M}{\mu}\frac{\eta\alpha(U-\varepsilon)}{r} = \frac{\beta x\mu(1-\mu)}{\beta x\mu(1-\mu)+\eta\alpha}\frac{\eta\alpha(U-\varepsilon)}{r}.$$

We can compute Z^{BB} analogously.

Derivation of the social welfare in barter economy:

Recall that in the low-technology only equilibrium, the number of producers and goods holder satisfies $\alpha N_0 = \beta x^2 N_L$, which implies $N_0 = \frac{\beta x^2}{\alpha+\beta x^2}$, and $N_L = \frac{\alpha}{\alpha+\beta x^2}$. Hence the social welfare becomes

$$\begin{aligned}
W_b^C &= \frac{\beta x^2}{\alpha+\beta x^2}\Delta_L\frac{\rho_b\theta U - \delta\varepsilon}{1-\rho_b\Delta_L} + \frac{\alpha}{\alpha+\beta x^2}\frac{\rho_b(\theta U - \Delta_L\delta\varepsilon)}{1-\rho_b\Delta_L} \\
&= \frac{\beta x^2\Delta_L(\rho_b\theta U - \delta\varepsilon) + \alpha\rho_b(\theta U - \Delta_L\delta\varepsilon)}{(\alpha+\beta x^2)(1-\rho_b\Delta_L)} \\
&= \frac{(\beta x^2\Delta_L + \alpha)\rho_b\theta U - (\beta x^2 + \alpha\rho_b)\Delta_L\delta\varepsilon}{(\alpha+\beta x^2)(1-\rho_b\Delta_L)} \\
&= \alpha\frac{(\beta x^2 + \alpha + r)\rho_b\theta U - (\beta x^2 + \alpha\rho_b)\delta\varepsilon}{(\alpha+\beta x^2)(\alpha + r - \alpha\rho_b)} \\
&= \alpha\frac{(\beta x^2 + \alpha\rho_b)(\theta U - \delta\varepsilon)}{(\alpha+\beta x^2)(\alpha + r - \alpha\rho_b)} = \alpha\frac{\beta x^2(\theta U - \delta\varepsilon)}{r(\alpha+\beta x^2)}
\end{aligned}$$

Analogously, the social welfare in the high-technology only equilibrium

$$W_b^A = \eta\alpha\frac{\beta x^2(U-\varepsilon)}{r(\eta\alpha+\beta x^2)}.$$

Proof of $\eta_Z > \eta_b > \eta_W$ given $\beta x\mu(1-\mu) > \beta x^2$, and $q < 1$:

It is easy to show that $\eta_Z > \eta_b$ iff $\beta x\mu(1-\mu) > \beta x^2$.

Comparing η_b and η_W,

$$\begin{aligned}\eta_b - \eta_W &= \frac{\beta x^2 q}{\beta x^2 + \alpha - \alpha q} - q + \frac{r(1-\theta)U}{\alpha(U-\varepsilon)} \\ &= -\frac{\alpha(1-q)q}{\beta x^2 + \alpha - \alpha q} + \frac{r(1-\theta)U}{\alpha(U-\varepsilon)}\end{aligned}$$

Given $q < 1$, we know $\eta_b > \eta_W$ iff

$$\begin{aligned}k(q) &= -\alpha(1-q)q + x(b+\alpha-\alpha q) \\ &= \alpha q^2 - \alpha q(1+x) + x(b+\alpha) > 0,\end{aligned}$$

where we employ the short-hand notation $x \equiv \frac{r(1-\theta)U}{\alpha(U-\varepsilon)} > 0$. Observe that $k(1) > 0$, $k(0) > 0$. To have at least a real root between 0 and 1, we need $0 < \frac{1+x}{2} < 1$, and a positive discriminant. However, when $0 < x < 1$, the discriminant

$$\begin{aligned}D &= \alpha^2(1+x)^2 - 4\alpha(b+\alpha) \\ &= \alpha[\alpha(1+x)^2 - 4(b+\alpha)] < 0.\end{aligned}$$

As a consequence, $\eta_b > \eta_W$, for all $0 < q < 1$.