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Occupational choice and dynamic indeterminacy

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Abstract

This paper constructs a two-sector model of two-period lived overlapping generations with endogenous occupational choice in which ability-heterogeneous agents choose whether to become educated when young and skilled when old. We show that endogenous occupational choice in this two-sector framework can result in dynamic indeterminacy without complicated preferences/technologies and without requiring the consumption-good production to be more capital-intensive.

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1. Introduction

Previous studies have established that dynamic indeterminacy can occur in two-sector overlapping-generations (OLG) models either with complicated preferences/technologies or with the consumption-good sector being more capital-intensive than the investment-good sector. In this paper, we argue that if agents select their occupation over skilled versus unskilled jobs according to their learning ability, a two-sector OLG economy may exhibit

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dynamic indeterminacy under simple preference and technology specifications, regardless of the factor-intensity rankings.

Specifically, we analyze endogenous occupational choice in a two-sector overlapping-generations model populated with two-period lived agents. In our economy, all agents are endowed with one unit of labor over the entirety of their lifetime. They are identical in every respect except for the ability to learn. We assume that the young who wish to be educated must self-finance the schooling expenses. As a result, those with higher ability (or lower disutility in schooling) borrow when young to accomplish higher education and become skilled workers when old. We further assume that agents only consume in the second period, a condition that implies forced savings in goods. Thus, under a positive market interest rate, those with lower ability only work when young and their savings facilitate both education and physical capital investments. Endogenous educational choice, therefore, gives rise to endogenous occupational choice between borrowers (the to-be skilled) and lenders (the unskilled). While the ratio of the productivity of the skilled to that of the unskilled is a constant exceeding one, the production technologies of both the consumption-good and investment-good sectors exhibit capital–skill complementarity.

Under the analytical framework described above, we show that equilibrium dynamics with occupational choice may feature multiple converging transition paths and hence extrinsic uncertainty may affect the dynamic behavior of the economy. More specifically, even with linear preferences, constant-returns Cobb–Douglas production technologies and a perfect credit market, we find that dynamic indeterminacy may arise, regardless of the factor-intensity rankings. Accordingly, occupational choice is the sole source creating dynamic indeterminacy, as long as the two goods are not homogeneous. In particular, we show that by allowing occupational choice in our two-sector framework, we add an additional choice variable that helps stabilize capital adjustments due to capital–skill complementarity and the associated changes in net savings. As a result, the conventional condition on the factor intensity ranking is no longer needed for local indeterminacy.

Related literature

Our paper contributes to the study of complex dynamics in overlapping generations models. Most literature in this area has derived complex dynamics such as indeterminacy, cycles and chaos, depending on complicated preferences or specific restrictions on production technologies. In the context of OLG models with capital accumulation, for example, Reichlin (1986) finds that if labor-leisure choice is allowed under a complex preference structure, indeterminacy and chaotic equilibrium may emerge with a constant-returns production technology, whereas Boldrin (1992) demonstrates that indeterminacy arises in the presence of external increasing returns.¹ In considering an OLG model involving a utility function that is non-separable intertemporally, Michel and Venditti (1997)

¹ In the exchange economy models with overlapping generations, the possibility of endogenous fluctuations depends entirely on the forms of the utility functions; see, for example, Benhabib and Day (1982) and Grandmont (1985). Jullien (1986) shows that the standard Diamond economy will exhibit cyclical behavior if there are multiple state variables (money and capital).

show that dynamic indeterminacy can occur under the standard neoclassical technology. The two-sector OLG frameworks with constant-returns technologies described by Galor (1992) and Reichlin (1992) are the most closely related to the model constructed here. Their models demonstrate that indeterminacy may occur if the consumption-good sector uses capital more intensively than the investment-good sector.² In contrast to all previous OLG studies, dynamic indeterminacy can emerge in our model without relying on any specific factor-intensity rankings, even under linear preferences and constant-returns Cobb–Douglas production technologies. As a consequence, local indeterminacy in our sector model mainly stems from the endogenous occupational choice of the young, an alternative source of indeterminacy that contributes to the existing literature.³

2. The model

Consider a two-sector model where sector 1 manufactures the investment good (regarded as the numeraire) and sector 2 produces the consumption good. The economy is populated with two-period overlapping generations. There is a continuum of individual agents of unit mass within each generational cohort, and the agents are identical in every respect except their ability (or disutility) to acquire education. Individual agents do not value leisure and they consume only in the second period of their lifetime. They have no initial wealth, but each is endowed with one unit of time that can be supplied in one of the two periods of his/her lifetime as a production input. In the absence of altruism, the utility function is simply assumed to be linear in the second-period consumption.⁴

Agents are heterogeneous *ex ante* only in the disutility costs they incur in acquiring education at the behest of their innate abilities. Consider a particular agent born at time $t - 1$, whose consumption occurs at time t . We denote the disutility costs by α_{t-1} (in units of the numeraire investment good), which are assumed to be uniformly distributed, i.e., $\alpha_{t-1} \in U[-\epsilon, \epsilon]$ with $0 < \epsilon < \infty$. Thus, a more able person will incur less disutility from acquiring education. We assume a pecuniary education cost of $\eta \geq \epsilon > 0$ per person (in units of the investment good), where $\eta \geq \epsilon$ ensures that no one will undertake an education for fun. Since an individual agent is not endowed with initial wealth or provided with a bequest, the pecuniary cost of education must be financed by borrowing against his/her future income. We assume throughout the paper that the credit market is perfect. Hence an individual agent who wishes to borrow is always granted an education loan.

² While Galor (1992) assumes that both production sectors have the standard neoclassical production functions, Reichlin (1992) examines a two-sector model in the absence of factor substitution.

³ The literature of occupational choice within the dynamic general-equilibrium framework includes Banerjee and Newman (1993), Aghion and Bolton (1997), Fender and Wang (2003), and many papers cited therein. This literature builds upon a one-sector framework in which the dynamic properties of the equilibrium have not been completely characterized.

⁴ The assumption of one unit lifetime-endowment of labor is innocuous. The structure of two-period lived agents who consumes only when old implies a one-to-one relationship between educational choice and occupational choice (borrowers versus lenders). We will discuss the implications for relaxing this assumption in the concluding section.

Denote the market (real) interest rate from period $t - 1$ to t by r_t and the corresponding gross rate of interest by $R_t \equiv 1 + r_t$. Given a linear felicity function in the second-period consumption, an individual agent born at time $t - 1$ will undertake an education to become skilled when old if the benefit from earning a high-skilled wage in the second period, $w_{H,t}$, subtracting the disutility cost of education, α_{t-1} , outweighs the costs from the sum of the foregone earnings for an unskilled job in the first period, $R_t w_{L,t-1}$, and the (interest payment included) costs of becoming educated, $R_t \eta$. Thus, the optimal schooling decision can be expressed as the decision to undertake an education whenever: $w_{H,t} - \alpha_{t-1} \geq R_t(\eta + w_{L,t-1})$. Under a proper assumption (to be specified later), there is a critical value of α_{t-1} , denoted by α_{t-1}^* , at which the agent is indifferent between being educated and remaining unskilled:

$$w_{H,t} - \alpha_{t-1}^* = R_t(\eta + w_{L,t-1}). \quad (1)$$

Should there exist such a critical point, agents of type $\alpha_{t-1} \in [-\epsilon, \alpha_{t-1}^*]$ would undertake an education and become skilled (type H) in the second period and those with $\alpha_{t-1} \in (\alpha_{t-1}^*, \epsilon]$ would remain uneducated and work without skills (type L) in the first period. This gives rise to an endogenous occupational choice under which a nontrivial fraction of agents become borrowers (the educated) and the remainder become lenders (the unskilled). Although those who decide to work without skills when young save their entire wage income (forced savings), we note that this saving decision is endogenously determined by occupational choice.

Within this stylized framework, skilled and unskilled workers are assumed to be fractional substitutes with one unit of skilled labor equivalent to $\delta > 1$ units of unskilled labor. Thus, letting $\ell_{i,t}$ represent aggregate employment of type i worker in period t ($i = L, H$), the aggregate “effective labor” can be expressed as: $N_t = \ell_{L,t} + \delta \ell_{H,t}$. Free mobility of labor between the two sectors implies that the unskilled and skilled wage rates must satisfy:

$$w_{H,t} = \delta W_t = \delta w_{L,t}. \quad (2)$$

Denote by K_t the amount of capital available at the beginning of period t . Assume competitive factor markets and full depreciation of the capital stock. Both consumption and investment goods are produced using labor and capital with Cobb–Douglas technologies that exhibit constant returns. Under this fractionally substitutable labor setup, we can easily see that our production technologies feature capital–skill complementarity (as an increase in δ leads to a higher marginal product of capital).

Denote the relative price of the consumption good in units of the investment good by p . Competitive profit conditions that equate the price with the unit cost in each sector yield:

$$1 = R_t^{\theta_1} W_t^{1-\theta_1}, \quad (3)$$

$$p_t = R_t^{\theta_2} W_t^{1-\theta_2}, \quad (4)$$

where θ_i , $i = 1, 2$, are constant capital cost shares that take values between 0 and 1. Assuming that $\theta_1 \neq \theta_2$, we can solve (3) and (4) to obtain a unique pair of factor prices (R_t, W_t) for any nonnegative p_t :

$$R_t = R(p_t) \quad \text{and} \quad W_t = W(p_t). \quad (5)$$

The effects of the relative price of goods on factor returns depend crucially on the factor-intensity rankings. When the consumption good is produced using capital (labor) more intensively, a higher relative price of the consumption good results in a higher (lower) return on capital and lower (higher) wage rates for both the unskilled and the skilled. This is in fact a straightforward application of the Stolper–Samuelson theorem to a three-factor model with two factors that are fractionally substitutable (skilled labor and unskilled labor).

Let x_{t-1} denote the proportion of the generation born at time $t - 1$ who become educated. It follows that in labor market equilibrium, we have:

$$\ell_{H,t} = x_{t-1}, \quad (6)$$

$$\ell_{L,t-1} = 1 - x_{t-1}. \quad (7)$$

It is straightforward to show that with a uniform distribution of disutilities of education across the population, $x_{t-1} = (\alpha_{t-1}^* + \epsilon)/(2\epsilon)$. From this we obtain:

$$\alpha_{t-1}^* = -(1 - 2x_{t-1})\epsilon. \quad (8)$$

This provides a linear relationship between the proportion of the labor force becoming educated and the critical value of disutility cost of education.

To close the model, we need to specify the goods market-clearing conditions. The aggregate consumption of type i ($i = H, L$) at time t is given by

$$C_{H,t} = (w_{H,t} - R_t\eta)x_{t-1}/p_t, \quad (9)$$

$$C_{L,t} = w_{L,t-1}R_t(1 - x_{t-1})/p_t. \quad (10)$$

Utilizing (2), (5), (9) and (10), we can express the demand for the consumption good in period t as

$$C_t^d = \{R(p_t)(1 - x_{t-1})W(p_{t-1}) + [\delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t)]\}/p_t. \quad (11)$$

Using the duality concepts, the supply of the consumption good at time t is equal to

$$C_t^s = R'(p_t)K_t + W'(p_t)(1 - x_t + \delta x_{t-1}). \quad (12)$$

Thus, the market-clearing condition for the consumption good at period t , $C_t^s = C_t^d$, implies

$$\begin{aligned} p_t [R'(p_t)K_t + W'(p_t)(1 - x_t + \delta x_{t-1})] \\ = R(p_t)(1 - x_{t-1})W(p_{t-1}) + [\delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t)]. \end{aligned} \quad (13)$$

Finally, investment goods market-clearing is captured by

$$K_t + \eta x_{t-1} = W(p_{t-1})(1 - x_{t-1}). \quad (14)$$

This is equivalent to equating the demand for loanable funds (the left-hand side) with the supply of loanable funds (the right-hand side).

3. Steady-state equilibrium

We are now ready to define dynamic competitive equilibrium and non-degenerate steady-state equilibrium in our two-period OLG economy with endogenous occupational choice.

Definition 1. A dynamic competitive equilibrium (DCE) is a tuple of positive quantities $\{C_{H,t}, C_{L,t}, \ell_{H,t}, \ell_{L,t}, K_t, x_t\}$, a tuple of positive prices $\{w_{H,t}, w_{L,t}, R_t, p_t\}$, and a critical value $\alpha_t^* \in [-\epsilon, \epsilon]$, such that

- (i) schooling is optimal: type $\alpha_{t-1} \in [-\epsilon, \alpha_{t-1}^*]$ become educated and type $\alpha_{t-1} \in (\alpha_{t-1}^*, \epsilon]$ remain uneducated, where α_{t-1}^* satisfies (1);
- (ii) aggregate consumption of the skilled and unskilled are determined by (9) and (10), respectively;
- (iii) factor price and competitive profit conditions are given by (2), (3) and (4);
- (iv) allocation of labor across sectors and labor market equilibrium are given by (6), (7) and (8);
- (v) goods market equilibrium is achieved as in (13) and (14).

Definition 2. A non-degenerate steady-state equilibrium (NSSE) is a DCE, represented by a tuple $\{C_H, C_L, \ell_H, \ell_L, K, x, w_H, w_L, R, p, \alpha^*\}$, with all variables being constant over time.

In case a variable takes on its steady-state value, we drop the time subscript.

Although there are 11 endogenous variables in our system, the recursive nature of the model enables us to summarize the system in terms of the sequence of the fraction of the population becoming skilled and the relative price, $\{x_t, p_t\}$, alone. On the one hand, we can combine the market-clearing condition for both the consumption and investment goods (13) and (14) to eliminate the sequence of the capital stock $\{K_t\}$ and derive:

$$\begin{aligned} p_t \{ R'(p_t) [(1 - x_{t-1})W(p_{t-1}) - \eta x_{t-1}] + W'(p_t)(1 - x_t + \delta x_{t-1}) \} \\ = R(p_t)(1 - x_{t-1})W(p_{t-1}) + \delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t). \end{aligned} \quad (15)$$

Under the Cobb–Douglas production technologies, the factor price–output price elasticities,

$$\frac{p_t R'(p_t)}{R(p_t)} = -\frac{(1 - \theta_1)}{\theta_1 - \theta_2} \equiv \Theta_r \quad \text{and} \quad \frac{p_t W'(p_t)}{W(p_t)} = \frac{\theta_1}{\theta_1 - \theta_2} \equiv \Theta_w, \quad (16)$$

are constant. If the consumption good is more capital (respectively labor) intensive than the investment good, i.e., if $\theta_1 >$ (respectively $<$) θ_2 , then clearly $\Theta_r < 0$ and $\Theta_w > 1$ (respectively $\Theta_r > 1$ and $\Theta_w < 0$). Using Θ_r and Θ_w , we can rewrite (15) to obtain a “goods market equilibrium condition” (hereafter referred to as the “*EE* locus”):

$$\begin{aligned} (\Theta_r - 1)R(p_t) [(1 - x_{t-1})W(p_{t-1}) - \eta x_{t-1}] + \Theta_w W(p_t)(1 - x_t) \\ = (1 - \Theta_w)\delta W(p_t)x_{t-1}. \end{aligned} \quad (17)$$

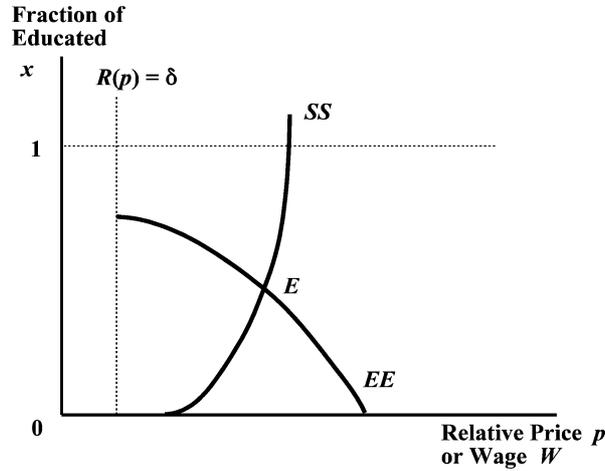


Fig. 1. Steady-state equilibrium.

On the other hand, we can substitute (2), (5), and (8) into (1) to derive an “optimal schooling relationship” (hereafter referred to as the “SS locus”):

$$\delta W(p_t) + \epsilon(1 - 2x_{t-1}) = R(p_t)[\eta + W(p_{t-1})]. \quad (18)$$

The *EE* and *SS* loci, (17) and (18), govern the dynamical system in $\{x_t, p_t\}$. In the steady state, $x_t = x$ and $p_t = p$, which satisfy:

$$x = \left\{ 1 + \frac{[(1 - \Theta_w)\delta W(p) + \eta(\Theta_r - 1)R(p)]}{W(p)[R(p)(\Theta_r - 1) + \Theta_w]} \right\}^{-1}, \quad (19)$$

$$x = \frac{1}{2} + \frac{1}{2\epsilon} \{ [\delta - R(p)]W(p) - \eta R(p) \}. \quad (20)$$

In general, an NSSE may not exist. However, if ϵ and η are sufficiently small and δ is sufficiently large, then the steady-state equilibrium values of (p, x) are uniquely determined by (19) and (20). We plot in Fig. 1 the steady-state *EE* and *SS* loci for the case of $\Theta_r < 0$ and $\Theta_w > 1$ where the *EE* locus is (locally) downward-sloping and the *SS* locus is upward-sloping. For the case of $\Theta_r > 1$ and $\Theta_w < 0$, the *EE* locus becomes (locally) upward-sloping and the *SS* locus is downward-sloping. Substituting the steady-state equilibrium values of (p, x) into (2), (3), (4), (6), (7) and (8), we next obtain the steady-state equilibrium values of factor prices (w_H , w_L and R), labor demand for each type (ℓ_H and ℓ_L) and the critical value of the disutility cost of education (α^*). Finally, utilizing (9), (10) and (14), we obtain steady-state equilibrium consumption (C_H and C_L) and capital (K) in a recursive manner. We can establish:

Theorem 1. *If both ϵ and η are sufficiently small and $\delta > \max\{\Theta_w/(1 - \Theta_r), 1\}$, the non-degenerate steady-state equilibrium of the dynamical system (17) and (18) exists and is unique.*

Proof. First, as ϵ converges to zero, the locus of (20) in (p, x) space converges to the vertical line $p = p^e > 0$, where p^e is the solution to

$$\frac{R(p)\eta}{W(p)} + R(p) = \delta. \quad (21)$$

Since both Θ_r and Θ_w are constant and either $\Theta_r < 0$ and $\Theta_w > 1$, or $\Theta_r > 1$ and $\Theta_w < 0$ holds, p^e uniquely exists, irrespective of the factor-intensity rankings. Substituting p^e into (19), we see that under $\delta > \max\{\Theta_w/(1 - \Theta_r), 1\}$, the steady state x also uniquely exists in the open interval $(0, 1)$ for any p^e satisfying $0 < R(p^e) < \delta$. Finally, we note that (21) implies that $0 < R(p^e) < \delta$ and that as η converges to zero, $R(p^e)$ converges to δ which is greater than one. Therefore, $R(p^e)$ is guaranteed to be greater than one as long as both ϵ and η are sufficiently small. \square

Theorem 1 implies that, in order to ensure an NSSE, the dispersion of heterogeneity and the pecuniary costs of education cannot be too large whereas the productivity differential between the skilled and unskilled cannot be too small. One may have thought that these assumptions are restrictive. Standard numerical analyses (summarized in panels A and B of Table 1), however, suggest that, over a large range of plausible parameter values, we can obtain a unique NSSE even by allowing ϵ to be as high as 0.2 (recall that ϵ must be bounded by η) and allowing η to be as high as 0.3 (or more than 35 percent of the unskilled wage), where the value of δ can be taken from 1.25 to 3.5 (or for the skilled wage to exhibit a 25 to 250 percent markup over the unskilled wage).

Table 1
Two-sector model

Parameters	x	R	λ_1	λ_2	
<i>A. Consumption-good sector is more capital-intensive ($\theta_1 = 0.2, \theta_2 = 0.3$)</i>					
ϵ, η	0.05, 0.10	0.59	2.22	0.178	12.589
	0.10, 0.10	0.59	2.21	0.179	12.680
	0.10, 0.20	0.53	2.01	0.164	12.709
	0.20, 0.20	0.53	2.01	0.165	12.920
	0.20, 0.30	0.48	1.86	0.153	12.963
	1.25	0.50	1.04	0.170	6.478
δ	2.00	0.52	1.63	0.165	10.211
	2.50	0.53	2.01	0.164	12.709
	3.00	0.53	2.40	0.162	15.210
	3.50	0.53	2.78	0.161	17.712
<i>B. Consumption-good sector is more labor-intensive ($\theta_1 = 0.3, \theta_2 = 0.2$)</i>					
ϵ, η	0.05, 0.10	0.66	2.17	0.264	8.353
	0.10, 0.10	0.66	2.15	0.265	8.375
	0.10, 0.20	0.58	1.96	0.241	8.421
	0.20, 0.20	0.58	1.94	0.244	8.512
	0.20, 0.30	0.52	1.80	0.226	8.575
	1.25	0.56	1.03	0.254	4.278
δ	2.00	0.58	1.59	0.245	6.758
	2.50	0.58	1.96	0.241	8.421
	3.00	0.58	2.31	0.237	10.088
	3.50	0.58	2.67	0.233	11.756

What are the consequences of a more favorable educational environment (captured by either higher δ or lower η)? For brevity, consider the simple case with ϵ converging to zero, where the steady-state values of (x, p) can be solved in a recursive manner (where p is determined by (21) alone). From (21), a more favorable educational environment leads to a lower (higher) relative price of the consumption good in units of the investment good if the consumption-good sector uses labor (capital) more intensively than the investment-good sector. Regardless of the factor intensity rankings, however, the return on capital always rises and the unskilled wage always falls in response to a favorable change in undertaking an education.

4. Equilibrium dynamics

We are now prepared to characterize the dynamic properties of the steady-state equilibrium. In order to gain intuition, let us start with two degenerate cases, one with no occupational choice and another with only one sector (homogeneous consumption and investment goods).

4.1. The absence of occupational choice

Without occupational choice, we simply set $x_t = 0$ (together with $\epsilon = \eta = 0$). Under this condition, the dynamical system reduces to:

$$K_t = W_{t-1}, \quad (22)$$

$$(\Theta_r - 1)\Gamma(W_t)W_{t-1} = -\Theta_w W_t, \quad (23)$$

where $R_t = \Gamma(W_t) \equiv (W_t)^{-(1-\theta_1)/\theta_1}$ is derived from the competitive profit condition (3). By substituting (22) into (23) and manipulating, we obtain a single dynamic equation:

$$K_{t+1} = \left(\frac{1 - \Theta_r}{\Theta_w} \right)^{\theta_1} (K_t)^{\theta_1}. \quad (24)$$

Since $\theta_1 \in (0, 1)$ and $(1 - \Theta_r)/\Theta_w > 0$ regardless of the factor intensity rankings, capital evolves monotonically and is stable. We can thereby establish the existence and uniqueness of a non-degenerate steady-state equilibrium.⁵ Moreover, as K_t is a non-jumping state variable, we can conclude:

Theorem 2. *The steady-state equilibrium of the dynamical system (22) and (23) uniquely exists and is locally determinate.*

⁵ The steady-state capital stock can be easily derived:

$$K = \left(\frac{1 - \Theta_r}{\Theta_w} \right)^{\theta_1/(1-\theta_1)} > 0.$$

In (W, x) space, the steady-state equilibrium is determined by the intersection of the *EE* locus and the horizontal axis $x = 0$ (see Fig. 1).

The result suggests that endogenous occupational choice plays an essential role for the steady state to be locally indeterminate. The reader should be alerted that our framework differs from Galor (1992)—the endogenous saving decision is absent under our framework once occupational choice is removed. Thus, in this degenerate case, the steady state is locally determinate, even when the consumption-good sector is more capital intensive.

4.2. The case of one-sector production

The next question is whether a steady state can be locally indeterminate when the economy degenerates to one sector (i.e., $\theta_i = \theta$ and $p_t = 1$). The competitive profit condition becomes: $1 = R_t^\theta W_t^{1-\theta}$, which can be used to express R_t as a decreasing and convex function of W_t :

$$R_t = \Omega(W_t) \equiv (W_t)^{-(1-\theta)/\theta}, \quad (25)$$

Moreover, within this one-sector framework, the capital–labor ratio is given by

$$\frac{K_t}{(1-x_t) + \delta x_{t-1}} = -\frac{1}{\Omega'(W_t)}. \quad (26)$$

We can now rewrite the *EE* locus as:

$$\Omega'(W_t)[W_{t-1} - (\eta + W_{t-1})x_{t-1}] + [(1-x_t) + \delta x_{t-1}] = 0, \quad (27)$$

whereas the *SS* becomes:

$$\delta W_t + \epsilon(1 - 2x_{t-1}) - \Omega(W_t)(\eta + W_{t-1}) = 0. \quad (28)$$

These two equations constitute the dynamical system for $\{x_t, W_t\}$.

It is tedious but straightforward to show that if $1/\theta > \delta > \max\{\theta/(1-\theta), 1 + \eta - \epsilon\}$, then there exists a unique non-degenerate steady-state equilibrium in which a nontrivial fraction of agents become unskilled workers and a nontrivial fraction of agents undertake an education to become skilled workers.⁶ The steady-state *EE* locus is (locally) downward-sloping and the steady-state *SS* locus is upward-sloping in (W, x) space (see Fig. 1). To characterize the dynamics of this one-sector model of occupational choice, we begin by noting that under this one-sector framework, $\{x_t\}$ and $\{W_t\}$ are tied by the factor market equilibrium relationship, (26), for any $t \geq 0$. More specifically, for historically given K_0 and x_{-1} , (26) implies:

$$K_0 = -\frac{1}{\Omega'(W_0)}[(1-x_0) + \delta x_{-1}]. \quad (29)$$

That is, once x_0 is chosen, the associated factor price W_0 is determined by (29). Thus, the system features only one free jump variable. If the steady state is a saddle point, we have a unique equilibrium path converging to the steady state.

⁶ This can be done by manipulating (27) and (28) in the steady state to obtain a single equation in terms of W :

$$F(W) = \frac{1}{2} \left(1 + \frac{\delta W}{\epsilon} \right) - \left[\frac{1 - \Omega(W)(1-\theta)/\theta}{1 - \Omega(W)(1-\theta)/\theta + \eta\Omega'(W) - \delta} + \frac{\Omega(W)(\eta + W)}{2\epsilon} \right] = 0.$$

We can then use the mean value theorem to prove the existence of a unique root of $F(W) = 0$ under the required conditions. The detailed proof is available upon request.

Theorem 3 (Characterization of the Dynamics). *For sufficiently small ϵ or η , the steady-state equilibrium of the dynamical system (27) and (28) is locally determinate, featuring a unique one-dimensional saddle path.*

Proof. By totally differentiating (27) and (28), the Jacobian matrix of the one-sector model evaluated at the steady-state value of (x, W) is given by:

$$\tilde{J} = \begin{bmatrix} \frac{2\epsilon}{Z}[(1-x)W - \eta x]\Omega'' + Z & \frac{\Omega}{Z}[(1-x)W - \eta x]\Omega'' + (1-x)\Omega' \\ \frac{2\epsilon}{Z} & \frac{\Omega}{Z} \end{bmatrix}.$$

where $Z \equiv \delta - (\eta + W)\Omega' > \Omega > 1$ (noting that $\Omega' < 0 < \Omega''$). In the neighborhood of the steady state, $(1-x)W - \eta x = -[(1-x) + \delta x]/\Omega' > 0$. Straightforward manipulations give the trace and the determinant:

$$\text{Tr}(\tilde{J}) = \frac{2\epsilon}{Z}[(1-x)W - \eta x]\Omega'' + Z + \frac{\Omega}{Z} > 0,$$

$$\text{Det}(\tilde{J}) = \Omega - \frac{2\epsilon}{Z}(1-x)\Omega' > \Omega > 1.$$

Evaluating the characteristic function at $\{-1, 1\}$, we have:

$$\Lambda(-1) = 1 + \text{Tr}(\tilde{J}) + \text{Det}(\tilde{J}) > 0,$$

$$\Lambda(1) = -\frac{1}{Z}\{(Z - \Omega)(Z - 1) + 2\epsilon[(1-x)W - \eta x]\Omega''\}.$$

Therefore, if ϵ is sufficiently small or if η is not too large, then $\Lambda(1) < 0$ because $Z > \Omega > 1$. If this is the case, $\Lambda(\tilde{\lambda}) = 0$ has two positive real roots, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, satisfying $0 < \tilde{\lambda}_1 < 1 < \tilde{\lambda}_2$. Since only one of $\{x_t, W_t\}$ can jump freely, the steady state is locally determinate. \square

Although Theorem 3 is proved based on small values of ϵ or η , our numerical exercises reported in Table 2 suggest that even with fairly large values ϵ or η over a large range of plausible parameter values, the saddle-point stability property is quite robust. Therefore, despite endogenous occupational choice, dynamic indeterminacy in general cannot arise if the OLG economy features only one sector that produces a single homogeneous good.

Table 2
One-sector model

Parameters	x	R	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	
ϵ, η	0.05, 0.10	0.56	2.19	0.264	8.415
	0.10, 0.10	0.56	2.18	0.265	8.498
	0.10, 0.20	0.49	1.97	0.240	8.521
	0.20, 0.20	0.49	1.98	0.244	8.708
	0.20, 0.30	0.44	1.82	0.225	8.738
	1.25	0.45	1.05	0.255	4.365
δ	2.00	0.49	1.61	0.245	6.853
	2.50	0.49	1.97	0.240	8.521
	3.00	0.49	2.33	0.237	10.19
	3.50	0.49	2.68	0.233	11.86

4.3. The general setup

Our next task is to examine the stability properties in the general two-sector setup with endogenous occupational choice. To begin, we claim that given the initial value K_0 , both x_0 and p_0 can be chosen freely in our two-sector dynamical system. To see this, notice that at $t = 0$, (14) does not restrict the choice of x_0 and p_0 (because only x_{-1} and p_{-1} are involved).⁷ By expressing (14), (17) and (18) in $t = 1$, we have:

$$\begin{aligned} K_1 + \eta x_0 &= W(p_0)(1 - x_0), \\ (1 - \Theta_w)\delta W(p_1)x_0 &= (\Theta_r - 1)R(p_1)[(1 - x_0)W(p_0) - \eta x_0] + \Theta_r W(p_1)(1 - x_1), \\ \delta W(p_1) + \epsilon(1 - 2x_0) &= R(p_1)[\eta + W(p_0)]. \end{aligned}$$

There are obviously five endogenous variables, x_0 , p_0 , x_1 , p_1 and K_1 , implying that these three equations do not restrict the choice of x_0 and p_0 , either. Throughout $t = 2, 3, \dots$, this argument continues to hold true. In other words, x and p are “jump variables” whose initial values can be freely chosen.

Denote by J the Jacobian matrix of the linearized 2×2 dynamical system evaluated at the steady-state value of (x, p) , and by J_{ij} the (i, j) element of J . Total differentiation of (17) and (18) gives:

$$\begin{aligned} J_{11} &= \frac{1}{\Theta_w W} \left[\frac{2\epsilon}{B} Q - (1 - \Theta_w)\delta W - (\Theta_r - 1)(\eta + W)R \right], \\ J_{12} &= \frac{1}{\Theta_w W} \frac{RW'}{B} [Q + B(\Theta_r - 1)(1 - x)], \\ J_{21} &= \frac{2\epsilon}{B}, \\ J_{22} &= \frac{RW'}{B}, \end{aligned}$$

where $Q \equiv (\Theta_r - 1)[(1 - x)W - \eta x]R' + [\Theta_w(1 - x) - (1 - \Theta_w)\delta x]W' > 0$ and $B \equiv \delta W' - (\eta + W)R' < (>) 0$ if $\theta_1 < (>) \theta_2$.

As ϵ converges to zero, J_{21} does the same, implying that the characteristic equation converges to

$$\Lambda(x) \equiv \begin{vmatrix} \lambda - J_{11} & -J_{12} \\ 0 & \lambda - J_{22} \end{vmatrix} = 0. \quad (30)$$

We can now establish:

Theorem 4. *If a non-degenerate steady-state equilibrium of the dynamical system (17) and (18) exists, then for sufficiently small ϵ , it is locally indeterminate, regardless of the factor intensity rankings.*

⁷ At $t = 0$, the behavior of the initial old is passive where the good market equilibrium condition and the optimal schooling relationship are not well-defined.

Proof. From (30), as ϵ converges to zero, one root converges to J_{11} and another converges to J_{22} . Using the definition of Θ_w and Θ_r and applying the steady-state *SS* locus, we can derive:

$$\begin{aligned} J_{22} &= \frac{\Theta_w p R W}{\delta \Theta_w p W - (\eta + W) \Theta_r p R} \\ &= \frac{\theta_1 R W}{\theta_1 \delta W - (\theta_2 - 1)(\eta + W) R} \\ &= \frac{\theta_1 W}{\eta + W} \\ &\in (0, 1). \end{aligned}$$

Thus, one root (denoted by λ_1) is positive and within the unit circle. Similarly, taking $\epsilon \rightarrow 0$, we can manipulate J_{11} to obtain:

$$\begin{aligned} J_{11} &= \frac{-[(1 - \Theta_w)\delta W + (\Theta_r - 1)(\eta + W)R]}{\Theta_w W} \\ &= \frac{\theta_2 \delta W + (1 - \theta_2)(\eta + W)R}{\theta_1 W} \\ &= \frac{\delta}{\theta_1} \\ &> 1, \end{aligned}$$

implying another root (denoted by λ_2) is positive but outside the unit circle. Since both x and p are jump variables, the dynamical system is locally indeterminate. \square

When ϵ is not small, the underlying dynamic properties become too complicated to characterize analytically. Accordingly, we perform numerical exercises to show the robustness of our results. While we have examined many cases with various parameter values, we report here only the most informative ones, grouped by the factor-intensity rankings (see Table 1, panels A and B, respectively).⁸ In the benchmark cases, we select the skill markup as 150% ($\delta = 2.5$), and the ability heterogeneity and the pecuniary cost of education as $2\epsilon = \eta = 0.2$. The capital cost shares in the two sectors are chosen as 0.2 and 0.3, respectively. These parameter values give a steady-state fraction of the skilled as $x = 0.53$ (0.58) and the steady-state gross rate of interest as 2.01 (1.96) for the case where the consumption-good production uses capital (labor) more intensively. Again, we impose the constraints: $\eta \geq \epsilon$ and $\delta > R > 1$. Thus, we perturb 2ϵ from 0.1 to 0.4, η from 0.1 to 0.3 (with $\eta \geq \epsilon$) and δ from 1.25 to 3.5 (consistent with dynamic efficiency). Our results indicate that for these plausible parameters, the dynamical system is always locally indeterminate, with a positive stabilizing root and another positive root outside the unit circle.

In summary, we have established the local indeterminacy property regardless of the factor intensity rankings. This contrasts sharply with the stability condition obtained in

⁸ For example, we have experimented a wide range of $\{\theta_1, \theta_2\}$, with $|\theta_1 - \theta_2|$ as small as 0.001 and as large as 0.5, and found that our results are robust.

the standard two-sector OLG model without occupational choice, where the steady state is indeterminate only when the consumption-good sector is more capital intensive than the investment good sector (cf. Galor, 1992 and Reichlin, 1992). A natural question arises: what are the underlying forces giving rise to dynamic indeterminacy?

From Theorems 2 and 3, we clearly see that in a standard OLG framework without complicated preference/technology specifications, dynamic indeterminacy cannot arise if occupational choice is absent or if the economy degenerates to only one sector that produces a single homogeneous good. In the one sector case, the framework lacks sufficient reinforcing forces in which the price dynamics (factor prices) are purely driven by the dynamics of the capital stock and the steady state is locally determinate. Under a two-sector setup with independent price dynamics, the conventional condition on the factor intensity ranking (requiring the consumption-good sector to be more capital intensive) ensures that not only the price but the quantity dynamics are stable in the absence of occupational choice.

By allowing occupational choice in such a two-sector overlapping-generations setting, we add an additional choice variable that helps stabilize capital adjustments. More specifically, due to capital–skill complementarity, an increase in the proportion of the population becoming skilled raises the capital rental, a stabilizing force. Moreover, as the mass of the skilled increases, the aggregate costs of education increase and the loanable funds supply decreases. Both of these changes reduce net savings and capital investment, which again help stabilize capital adjustments. As a result, the conventional condition on the factor intensity rankings is no longer needed for local indeterminacy.

Finally, one may inquire whether the factor intensity rankings matter at all for dynamic adjustments in our model. Let us focus on the case with a sufficiently concentrated distribution of ability (ϵ small). In this case, the local dynamics feature a one-dimensional stable manifold over two jump variables (which can be expressed as $x_t = \phi(p_t)$). Consider that, in a particular period, agents expect the long-run relative price of consumption to be higher (p increases). When the consumption sector is more capital intensive, the Stolper–Samuelson theorem implies that the returns on capital must go up more than proportionately whereas the returns on labor must go down. As a result, the capital stock rises and the fraction of skilled workers drops (x decreases). Thus, along the transition path, we obtain downward adjustments in x associated with upward adjustments in p (i.e., $\phi' < 0$). When the consumption sector is more labor intensive, the transition path will feature upward adjustments in both x and p (i.e., $\phi' > 0$). In summary, although the factor intensity rankings do not affect the stability properties, they affect the configuration of the underlying dynamic adjustments. Moreover, the configuration of these dynamic adjustments in the relative price and occupational choice help explain why dynamic indeterminacy can arise, through self-fulfilling prophecies. More precisely, updating the *SS* locus by one period and applying the stable manifold relationship, we get:

$$[\eta + W(p_t)]R(p_{t+1}) - \delta W(p_{t+1}) = \epsilon(1 - 2\phi(p_t)).$$

When the consumption sector is more capital intensive ($\theta_1 < \theta_2$), an expected increase in p_{t+1} (sunspot driven) raises the LHS. Since $\phi' < 0$, an increase in p_t can change occupational choice to restore the equilibrium relationship. Moreover, since $dp_{t+1}/dp_t > 0$, such an increase in p_t leads to an increase in p_{t+1} and hence the expectations are self-fulfilled.

The case of $\theta_1 > \theta_2$ can be worked out by similar arguments, where $R' < 0$, $W' > 0$ and $\phi' > 0$.

5. Concluding remarks

This paper has presented a two-sector overlapping-generations model with endogenous occupational choice where borrowing is required for investment in education. We have demonstrated that the economy may exhibit indeterminacy of converging paths, implying that expectations-driven endogenous fluctuations can emerge. In our model, the utility function is linear in consumption in old age, the production technologies take the Cobb–Douglas forms with constant returns, and the credit market is perfect. As such, the dynamic indeterminacy result in our economy mainly stems from the presence of the occupational choice behavior of the agents, though the two-sector structure is also essential for the result.

Here we may naturally ask: what happens if we generalize the borrower-lender relationship? In our benchmark economy, we have assumed that the agents to be educated are borrowers and the agents who do not plan to be educated are lenders. This is because all agents are assumed to be two-period lived and to consume only in their old age. If we consider instead three-period lived agents who consume in both the second and third periods, there need not be a one-to-one relationship between educational decision (educated versus uneducated) and occupational choice (borrowers versus lenders). As such a generalization provides additional intertemporal reinforcing forces, one would expect that it may increase the likelihood of dynamic indeterminacy.

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