Taxing Capital is Indeed Not a Bad Idea: The Role of Human Capital and Labor-Market Frictions

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Abstract: In a second-best optimal growth setup with only factor taxes as available instruments, is it optimal to *fully* replace capital by labor income taxation in the long run? The answer is generally positive based on Chamley, Judd, Lucas, and many follow-up studies. In the present paper, we revisit this important tax reform-related issue by developing a human capital-based endogenous growth framework with frictional labor search and matching. We allow each firm to create multiple vacancies and each worker to determine labor market participation endogenously. We consider a benevolent fiscal authority to finance direct transfers to households and unemployment compensation only by factor taxes. We find that Hosios' rule is a necessary but *not sufficient* condition for efficiency. We then conduct dynamic tax incidence exercises along the balanced growth path using a model calibrated to the U.S. economy with a pre-existing 20% flat tax rate on both the capital and labor income. Our numerical results suggest that, due to a dominant channel via the interactions between the firm's vacancy creation and the worker's market participation, it is optimal to switch only partially from capital to labor taxation in a benchmark economy where human capital formation depends on both physical and human capital stocks. When we completely remove the extensive margin of the labor-leisure trade-off or when the human capital accumulation process is independent of physical capital, the optimal tax mix features a larger shift from capital to labor taxation; but, the optimal capital tax rate in either case is still *positive*. When we return to a Walrasian economy without labor-market frictions, it is optimal to fully eliminate capital taxation.

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1 Introduction

Since the pivotal work by Chamley (1985a, 1985b, 1986) and Judd (1985, 1987), a large body of literature has addressed the issue of dynamic factor tax incidence in optimal growth models in order to identify the optimal factor tax mix in a second-best world where full access to the lump-sum tax is unavailable. Because labor endowment is fixed but capital can be accumulated over time, Chamley and Judd recommended that the optimal flat factor tax scheme be implemented to fully eliminate the tax on the more elastic capital and to impose a tax only on the inelastic labor in the long run. This Chamley-Judd proposition has been revisited and extended to various economic environments and the general conclusion has been fairly robust under a benevolent nonproductive central planner using flat-rate factor taxes without other meanings of financing.

About two decades ago, the celebrated work by Lucas (1988) provided a compelling argument that human capital is a primary engine of the endogenously determined economy-wide growth rate. Because human capital augments labor, an immediate question arises: Would it be welfare-reducing to tax labor in a human capital-based endogenous growth framework? Two years later, Lucas (1990) himself addressed this question based on dynamic factor tax incidence exercises and provided a policy recommendation that neither physical nor human capital should be taxed and that only raw labor should be taxed. His policy recommendation has hardly been challenged within the canonical balanced-growth framework with a benevolent nonproductive central planner using only flat-rate factor taxes to finance.

In this paper, we follow this convention by reexamining the validity of the Lucasian policy recommendation in a generalized human capital-based endogenous growth economy with individuals endogenously participating in the frictional labor market. It was well-documented in the celebrated work on labor search pioneered by Diamond (1982a), Mortensen (1982) and Pissarides (1984) that informational and institutional barriers to job search, employee recruiting, and vacancy creation were substantial. Such frictions have been shown to generate important impacts on macroeconomic performance over the business cycle (cf. Mortensen and Pissarides 1994; Merz 1995; Andolfatto 1996) as well as in the long run (cf. Aghion and Howitt 1994; Laing, Palivos and Wang 1995; Chen, Chen and Wang 2011).[1] It is therefore natural to inquire whether such frictions may influence individual decisions to generate sufficient "responsiveness" in the long run to a tax on labor income such that labor taxation becomes too distortionary to be used to fully replace capital taxation.

Our paper attempts to address this important issue that has practically valuable implications for tax reform considerations. Specifically, we construct a two-sector human capital-based endogenous growth framework with labor market search and matching frictions in which the worker's market participation is tied to the household's valuation of leisure. We assume that vacancy creation and

[1] See Rogerson, Shimer and Wright (2005) for a comprehensive survey.

maintenance as well as job search are all costly and that unfilled vacancies and active job seekers are brought together by a matching technology exhibiting constant returns. We consider "large" firms and "large" households where each firm creates and maintains multiple vacancies and each household contains a continuum of members comprising employed and nonemployed workers. The wage rate (in efficiency units) is determined based on a cooperative bargain between the matched firm and household pair. A benevolent fiscal authority finances direct transfers to households and unemployment compensation only by way of taxing factor incomes at flat rates, and maximizes social welfare given labor-market frictions. That is, by following the conventional tax incidence literature cited above, we do not consider other tax alternatives or time-varying tax schedules and assume that education or learning is completely tax-exempt. In the benchmark setup, we consider a general two-sector framework as proposed by Bond, Wang and Yip (1996) in which the accumulation of physical and human capital are both driven by physical and human capital stocks. We also consider an alternative setup with a Lucasian human capital accumulation process which is independent of physical capital. We calibrate our economy to fit observations in the U.S. over the post-WWII period, with a pre-existing 20% flat tax rate being levied on both capital and labor income. This enables us to conduct dynamic factor tax incidence exercises along the balanced growth path, and to draw policy recommendations based on a revenue-neutral welfare comparison of factor taxes.

Our main findings can be summarized as follows. We show that while the capital tax lowers the bargained wage rate (in efficiency units), the labor tax increases it. However, these factor taxes can generate very different effects on the wage discount that measures how much our equilibrium wage in the presence of labor market search and matching frictions is below the competitive counterpart in a frictionless Walrasian setup. Specifically, if the capital tax rate is initially too low (lower than its optimum), then an increase in the capital income tax rate accompanied by a revenue-neutral reduction in the labor tax turns out to raise the wage discount and to encourage firms to create more vacancies. This in turn raises the job finding rate and hence induces workers to more actively participate in the labor market to seek employment. Because this leads to positive effects on employment and output growth, a shift from a zero to a positive capital tax rate becomes welfare-improving, thereby yielding a policy recommendation different from that of Chamley-Judd-Lucas. Moreover, we show that Hosios' rule is a necessary but not sufficient condition for efficiency. In addition to Hosios' rule and conventional restrictions on firms discounting at the market rate and valuing capital the same as households, efficiency requires that distorted pre-existing taxes and subsidies be removed and that the wage discount be at an optimal level aligning labor-leisure-consumption trade-off atemporally and intertemporally in our endogenously growing economy with labor-market search and vacancy creation frictions.

By conducting factor tax incidence exercises in our benchmark economy calibrated to U.S. data, we find that, in the benchmark case with factor taxes at pre-existing rates of $(20\%, 20\%)$, it is optimal to only partially replace the capital tax by the labor tax: the optimal flat tax rates on capital and labor income are 16.11% and 24.09%, respectively. Since the above-mentioned vacancy creation-market participation channel in the presence of labor market frictions is quantitatively significant, the optimal capital tax rate is significantly greater than zero. As a consequence, such a reform induces a 0.0389% welfare gain in consumption equivalence whereas setting the capital tax rate to zero would lead to a large welfare loss of 0.6490% in consumption equivalence. Upon various sensitivity and robustness checks, we find that it is hardly optimal to fully replace capital by labor taxation within all reasonable ranges of parameterization so long as labor-search and vacancy-creation frictions are present. The conclusion remains even when we remove the extensive margin of the labor-leisure trade-off – in this case, taxing labor becomes quantitatively less harmful so the optimal tax rates on capital and labor income are 9.05% and 31.33%, respectively, featuring a larger shift from capital to labor taxation. When the human capital accumulation process is independent of physical capital as in Lucas (1988, 1990), the optimal tax rates on capital and labor income become 4.99% and 46.68%, respectively, also featuring a larger shift from capital to labor taxation. In either cases, the optimal capital tax rate is still significantly positive. Finally, in a Walrasian economy with a frictionless labor market, it is always optimal to fully eliminate capital taxation by taxing only labor income.

In sum, our results suggest that while labor search frictions and costly vacancy creation are crucial for dynamic factor tax incidence to feature a positive optimal capital tax rate in the long run, the presence of labor-leisure trade-off and the form of human capital accumulation is not. Although our tax incidence analysis is conducted with flat tax rates along a balanced growth path, our conclusions yield more general policy implications. In particular, since it is optimal to tax capital income in the long run, one would expect it must be so in the short run as physical capital is less elastic. Thus, our tax incidence exercises along the balanced growth path can be viewed as a conservative benchmark in favor of taxing labor income and our positive optimal capital tax result is expected to remain valid even by evaluating welfare in transition or by adopting time-varying Ramsey tax schedules.

Related Literature

Our paper is related to the discrete-time, real-business-cycle (RBC) search literature pioneered by Merz (1995) and Andolfatto (1996). In contrast with theirs, our model considers sustained economic growth with endogenous human capital accumulation. Previously, Laing, Palivos and Wang (1995) incorporated human capital-based endogenous growth into the Mortensen-Pissarides search framework, whereas Chen, Chen and Wang (2011) introduced human capital growth into the Andolfatto-Merz RBC search framework using a pseudo central planning setup. We follow the latter strategy, allowing comprehensive labor-leisure-learning-search trade-offs. Differing from their work, our paper performs dynamic factor tax incidence analysis in a fully decentralized setup with a more general human capital accumulation process.

Over the past two decades, several studies have investigated the long-run growth effects of factor taxes, including King and Rebelo (1990), Stokey and Rebelo (1995), Bond, Wang and Yip (1996), and Mino (1996), under perfectly competitive setups without externalities. This literature has been extended to incorporate positive externalities, productive public capital or market imperfections, such as Guo and Lansing (1999), Cassou and Lansing (2006) and Chen (2007). This strand of the literature, however, focuses exclusively on long-run growth or welfare effects of factor taxation rather than on factor tax incidence.

The closely related literature was initiated by Lucas (1990) who reexamined the Chamley-Judd proposition of dynamic factor tax incidence in a human capital-based endogenous growth framework.[2] His primary conclusion was that the government should not tax either physical or human capital but rather tax raw labor only. This Lucasian policy recommendation was reconfirmed by Jones, Manuelli and Rossi (1993) in which only investment goods enter physical and human capital accumulation (i.e., there is no trade-off between education time and work hours). Even in a more general setup by Jones, Manuelli and Rossi (1997) that allowed both investment goods and education time to enter human capital accumulation, the Lucasian policy recommendation still remains valid under constant-returns technologies in the absence of an alternative tax on consumption.

In a recent paper, Conesa, Kitao and Krueger (2009) pointed out clearly that a call for taxing capital may be due to borrowing constraints, uninsurable idiosyncratic income risk, and/or life-cycle settings where the tax code cannot be age-dependent. In their paper, these aforementioned features as well as the progressiveness of labor income taxes are all allowed. Yet, in their calibrated model M1 where borrowing constraints, idiosyncratic risk and progressive tax features are turned off, the optimal factor tax mix is to levy 10% and 19% tax rates, respectively, on capital and labor income (with consumption being taxed at 5%).[3] The positive capital tax prescription in this simple setup is purely a consequence of the life-cycle setting: assuming a period-by-period balanced government budget, capital taxation serves to equate aggregate private asset demand with the society's capital stock in the presence of life-cycle competitive inefficiency. Capital income becomes essentially tax-exempt when life-cycle competitive inefficiency is largely mitigated by a nontrivial age-dependent wage earning profile and a pay-as-you-go social security system. In our infinite lifetime model, such life-cycle competitive inefficiency is absent. Yet, by incorporating endogenous human capital accumulation together with endogenous participation in a frictional labor market, we still establish that based on very different underlying economic channels from those in Conesa, Kitao and Krueger "taxing capital is not a bad idea after all" .

[2] There is a recent strand in the literature on optimal taxation which does not incorporate human capital, but instead considers nonlinear labor taxation, alternative non-factor taxes, incentive problems and/or political economy. Its focus is very different from ours.

[3] There are several other papers under stochastic settings. Because our model is deterministic, we shall not intensively discuss this remotely related literature.

2 The Model

We consider a discrete-time model with a continuum of identical infinitely-lived large firms (of measure one), a continuum of identical infinitely-lived large households (of measure one) and a fiscal authority determining the factor tax mix.

The adoption of the large household setup is to ease unnecessary complexity involved in tracking the distribution of the employed and the unemployed, so as to eliminate the possibility of an endogenous distribution of human and physical capital stocks as a result of idiosyncratic search and matching risk in the frictional labor market. The large household consists of a continuum of members (of measure one), who are either (i) employed, by engaging in production or on-the-job learning activity, or (ii) nonemployed, by engaging in job seeking or leisure activity. We assume that households own both productive factors, capital and labor.

While the goods market is Walrasian and the capital market is perfect, the labor market exhibits search and matching frictions. In particular, a firm can create and maintain (multiple) vacancies only upon paying a vacancy creation and maintenance cost in the form of labor inputs. The household's (endogenously determined) search activity is also costly with a foregone earning cost. Unfilled vacancies and active job seekers are brought together through a Diamond (1982b) type matching technology, where each vacancy can be filled by exactly one searching worker. In our model, the flow matching rates (job finding and employee recruitment rates) are both endogenous, depending on the masses of both matching parties. In every period, filled vacancies and employed workers are separated at an exogenous rate.

The benevolent fiscal authority's behavior is simple: it taxes factor incomes at flat rates to finance direct transfers to households and unemployment compensation, given labor-market frictions. The optimal factor tax mix is to maximize social welfare by maintaining a periodically balanced budget.

2.1 Households

The economy is populated with a continuum of large households of mass one, each consisting of a continuum of members of unit mass. In addition to the labor endowment and human capital h_t, households are assumed to own the entire stock of physical capital k_t, where the initial stocks of human and physical capital are given by, $h_0 > 0$ and $k_0 > 0$. A representative large household with a unified preference pools all resources and enjoyment from its members. In period t, a fraction n_t of the members are employed and $1 - n_t$ are nonemployed. In this setup, the unemployment rate is simply $u_t = 1 - n_t$. To simplify the analysis, let each member be endowed with one unit of "productive" time that may be used for production (work effort, denoted ℓ_t) or learning activity (learning effort, denoted $1 - \ell_t$). Further assume that a nonemployed member allocates the entire time to leisure (in this way, job search does not consume time but is costly because of foregone earnings). Thus, while leisure is inelastic on the "intensive" margin, it is elastic on the "extensive" margin as a result of the large household's allocation of its members to employed and nonemployed (i.e., whether or not to take leisure rather than how much it is taken). The allocation of labor of a large household is delineated in Figure 1.

Since job matches are not instantaneous, the level of employment from the household's perspective is given by the following birth-death process,

$$n_{t+1} = (1-\psi)n_t + \mu_t(1-n_t) \tag{1}$$

where ψ denotes the (exogenous) job separation rate and μ_t is the (endogenous) job finding rate. That is, the change in employment $(n_{t+1} - n_t)$ is equal to the inflow of workers into the employment pool $(\mu_t(1-n_t))$ net of the outflow as a result of separation (ψn_t).

We consider a general human capital accumulation technology proposed by Bond, Wang and Yip (1996) in which the production of incremental human capital requires both human and physical capital inputs. Denote the fraction of physical capital devoted to goods production as s_t and that to human capital accumulation as $1 - s_t$. The aggregate human capital of the household can thus be accumulated via learning by the employed and inputs of the market good – physical capital:

$$h_{t+1} - h_t = Dn_t(1-\ell_t)h_t + \widetilde{D}[(1-s_t)k_t]^\gamma [n_t(1-\ell_t)h_t]^{1-\gamma} \tag{2}$$

where $h_0 > 0$ is exogenously given, $\gamma \in (0,1)$ and $D, \widetilde{D} > 0$. When $\widetilde{D} = 0$ (and $s = 1$), human capital accumulation is independent of market goods. This linear human capital evolution process resembles that proposed by Lucas (1988): it reduces to the Lucasian setup when $n_t = 1$. Since the accumulation of human capital depends on the employment rate n_t, it gives the flavor of on-the-job learning. The above setup implies that the unemployed cannot accumulate human capital, or, more generally, their human capital accumulation is completely offset by their human capital depreciation.[4] In general, $\widetilde{D} > 0$ and the accumulation of human capital requires inputs of market goods. The functional form given above implies that physical capital is not necessary for human capital accumulation as long as $D > 0$. We follow Lucas (1990) assuming that education or learning activities are completely tax-exempt.

Denote the effective wage and the capital rental rates by w_t and r_t , respectively. The labor and capital income tax rates are constant over time, denoted by τ_L and τ_K, respectively. Let c_t be household consumption and δ_k be the physical capital depreciation rate. In addition, denote the ratio of unemployment compensation to the market wage by $\bar{b}$, the per household lump-sum profit distribution by π_t (to be specified below) and the per household lump-sum transfer from the government by T_t. The household faces the following budget constraint:

$$k_{t+1} = (1-\tau_L)w_t h_t \left[n_t\ell_t + (1-n_t)\bar{b}\right] + [(1-\delta_k) + (1-\tau_K)r_t s_t]k_t - c_t + \pi_t + T_t \tag{3}$$

[4] See Jacobson, LaLonde and Sullivan (1993) and Laing, Palivos and Wang (2003) for a further discussion of the human capital depreciation of displaced workers.

That is, the household allocates the total wage from employed members, total unemployment compensations from unemployed members, total rentals from market capital devoted to production ($s_t k_t$), total profits and total transfers, to consumption and gross investment.

Let $\rho > 0$ be the subjective rate of time preference. The representative household's preference takes a standard time-additive form:

$$\Omega = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \tilde{U}\left(c_t, n_t\right)$$

The periodic utility function is given by $\tilde{U}\left(c_t, n_t\right) = U\left(c_t\right) + \tilde{m}\left(1-n_t\right) G(z)$, where G is a function of leisure z facing each unemployed, taking a standard form with constant elasticity of intertemporal substitution $\epsilon \in (0,1)$: $G(z) = \frac{z^{1-\epsilon^{-1}}}{1-\epsilon^{-1}}$ (e.g., see Andolfatto 1995 and many others in the macro labor literature). In this setup, what is emphasized is the *extensive* margin: the unemployed takes leisure and workers participate in the labor market at the expenses of leisure. Although for simplicity we do not consider the intensive margin of leisure (i.e., z is exogenously given in our economy), the large household's leisure is endogenous due to the extensive margin related to worker's market participation n_t. We can thus rewrite the periodic utility function in a more convenient form:

$$\tilde{U}\left(c_t, n_t\right) = U\left(c_t\right) + m\left(1-n_t\right)$$

where $m \equiv \tilde{m}G(z)$ is a combination of the preference parameter $\tilde{m}$ and the intensity of enjoyment $G(z)$. With $\epsilon \in (0,1)$, we have $G(z) < 0$, so m is expected to be negative.

Let $\mathcal{H} = (k, h, n)$ denote the vector of current period state variables and $\mathcal{H}'$ denote that of the next period state variables. Then, the household's optimization problem can be expressed in a Bellman equation form as:

$$\Omega(k_t, h_t, n_t) = \max_{c_t,\, \ell_t,\, s_t} U\left(c_t\right) + m\left(1-n_t\right) + \frac{1}{1+\rho}\Omega(k_{t+1}, h_{t+1}, n_{t+1}) \tag{4}$$

subject to constraints (1), (2), and (3).

Define conveniently effective capital-labor ratios in the nonmarket and market sectors as $q^H = \dfrac{(1-s)k}{n(1-\ell)h}$ and $q^F = \dfrac{sk}{n\ell h}$, respectively. Then the household's optimizing decisions can be summarized as follows:[5]

Lemma 1. (Household's Optimization) *The representative large household's optimization satisfies the following first-order conditions (with respect to $\{c_t, \ell_t, s_t\}$),*

$$U_c = \frac{1}{1+\rho}\Omega_k(\mathcal{H}') \tag{5}$$

$$\Omega_k(\mathcal{H}')(1-\tau_L)w = \Omega_h(\mathcal{H}')\left[D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right] \tag{6}$$

$$\Omega_k(\mathcal{H}')(1-\tau_K)r = \Omega_h(\mathcal{H}')\widetilde{D}\gamma\left(q^H\right)^{\gamma-1} \tag{7}$$

[5]We note that the second-order conditions and the concavity property of the value functions for the household's and firm's optimization are rather complex, which are relegated to the Appendix.

together with the respective Benveniste-Scheinkman conditions (associated with $\{k_t, h_t, n_t\}$):

$$\Omega_k(\mathcal{H}) = \frac{1}{1+\rho}\Omega_k(\mathcal{H}')[(1-\delta_k)+(1-\tau_K)r] \tag{8}$$

$$\Omega_h(\mathcal{H}) = \frac{1}{1+\rho}(\Omega_k(\mathcal{H}')(1\text{-}\tau_L)w[n\ell+(1\text{-}n)\bar{b}]+\Omega_h(\mathcal{H}')\{1+n(1\text{-}\ell)[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^\gamma]\}) \tag{9}$$

$$\Omega_n(\mathcal{H}) = -m+\frac{\Omega_k(\mathcal{H}')(1\text{-}\tau_L)wh(\ell\text{-}\bar{b})+\Omega_h(\mathcal{H}')(1\text{-}\ell)h[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^\gamma]+\Omega_n(\mathcal{H}')(1\text{-}\psi\text{-}\mu)}{1+\rho} \tag{10}$$

From (6) and (7), we can solve the nonmarket effective capital-labor ratio q^H as a function of the after-tax wage-rental ratio alone:

$$\left(q^H\right)^{1-\gamma}\left[D+\widetilde{D}(1-\gamma)\left(q^H\right)^\gamma\right] = \widetilde{D}\gamma\frac{(1-\tau_L)w}{(1-\tau_K)r} \tag{11}$$

This positive relationship may be thought of as the relative factor input schedule to nonmarket activity: the higher the after-tax wage-rental ratio is, the greater the nonmarket effective capital-labor ratio will be. How sensitive the nonmarket effective capital-labor ratio q^H is to changes in the after-tax wage-rental ratio depends on technology parameter $\widetilde{D}$.

2.2 Firms

The economy is populated by a continuum of large firms of mass one, each creating and maintaining multiple job vacancies. A representative firm produces a single final good y_t by renting capital k_t from households and employing labor of mass n_t under a constant-returns-to-scale Cobb-Douglas technology,

$$y_t = A\left(s_t k_t\right)^\alpha\left(n_t \ell_t h_t\right)^{1-\alpha} \tag{12}$$

where $A > 0$ and $\alpha \in (0, 1)$.

Denoting η_t as the (endogenous) recruitment rate and v_t as (endogenous) vacancies created, we can specify the level of employment from the firm's perspective as follows:

$$n_{t+1} = (1-\psi)n_t + \eta_t v_t \tag{13}$$

where the change in employment is equal to the inflow of employees ($\eta_t v_t$) net of the outflow (ψn_t).

To be consistent with a balanced growth equilibrium, we assume that the unit cost of creating and maintaining a vacancy is proportional to the average firm output $\bar{y}_t$. This setup is natural – the more production the economy has, the more firms will compete for resources and the greater the vacancy creation cost will be. Moreover, it is parsimonious – the optimization is simple because $\bar{y}_t$ is regarded as given to each individual firm. Furthermore, it is neutral – the base in which vacancy costs grow is not biased toward one of the two production factor inputs. Thus, the resource cost for vacancy creation and maintenance is given by $\Phi(v_t) = \phi\bar{y}_t v_t$, where $\phi > 0$. The level of employment is the only state variable in the representative firm's optimization problem. Each unit of employment is augmented by the multiple of both work effort and human capital, $x_t = \ell_t h_t$. In this endogenous growth framework, both capital stocks grow unboundedly. To ensure the stationarity of the optimization problem (i.e., bounded firm value), we consider the firm's flow profit $\pi_t = A(s_t k_t)^\alpha (n_t x_t)^{1-\alpha} - w_t n_t x_t - r_t s_t k_t - \phi \bar{y}_t v_t$ in effective units by dividing it by the "effective productivity" (x_t) of the state variable n_t, where x_t is taken as given by the representative firm (see Chen-Chen-Wang 2011). Given the discount rate R_t, the associated Bellman equation can then be written as:

$$\Gamma(n_t) = \max_{v_t,\, k_t} \left\{ A(s_t k_t)^\alpha (n_t x_t)^{1-\alpha} - w_t n_t x_t - r_t s_t k_t - \phi \bar{y}_t v_t \right\} / x_t + \frac{1}{1+R_t} \Gamma(n_{t+1}) \tag{14}$$

subject to constraint (13).

Lemma 2. (Firm's Optimization) *The representative firm's optimization satisfies the following first-order conditions (with respect to $\{v_t, k_t\}$) and the Benveniste-Scheinkman condition (associated with $\{n_t\}$):*

$$\frac{\eta}{1+R} \Gamma_n(n') = \phi A \left(q^F\right)^\alpha n \tag{15}$$

$$\alpha A \left(q^F\right)^{\alpha-1} = r \tag{16}$$

$$\Gamma_n(n) = (1-\alpha) A \left(q^F\right)^\alpha - w + \frac{1-\psi}{1+R} \Gamma_n(n') \tag{17}$$

Moreover, in the interest of the owner's of the firm, the discount rate is equal to the market rental rate, i.e., $R = r$.

From (16), we can derive the market effective capital-labor ratio q^F as a function of the capital rental rate alone:

$$q^F = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}} \tag{18}$$

which is downward-sloping as expected.

2.3 Labor Matching and Bargaining

While the capital market is assumed to be perfect, the labor market exhibits search frictions. Following Diamond (1982), we assume pair-wise random matching in which the matching technology takes the following constant-returns form:

$$M_t = B(1-n_t)^\beta (v_t)^{1-\beta} \tag{19}$$

where $B > 0$ measures the degree of matching efficacy and $\beta \in (0,1)$.

In our model economy, the household's surplus accrued from a successful match is measured by its incremental value of supplying an additional worker (Ω_n) whereas the firm's surplus is measured by its incremental value of hiring an additional employee (Γ_n). The representative household and the representative firm determine the effective wage rate through cooperative bargaining to maximize their joint surplus:

$$\max_{w_t} (\Omega_{n_t})^{\zeta} (\Gamma_{n_t})^{1-\zeta}$$

where $\zeta \in (0,1)$ denotes the bargaining share to the household. In solving this wage bargaining problem, the household-firm pair treats matching rates (μ_t and η_t), the beginning-of-period level of employment (n_t), and the market rental rate (r_t) as given.[6]

Lemma 3. (Wage Bargain) *The wage bargaining problem satisfies the following first-order condition:*

$$\frac{\zeta}{w_t}\left(\frac{w_t}{\Omega_{n_t}}\frac{d\Omega_{n_t}}{dw_t}\right) = \frac{1-\zeta}{w_t}\left(-\frac{w_t}{\Gamma_{n_t}}\frac{d\Gamma_{n_t}}{dw_t}\right) \tag{20}$$

With a frictional labor market and cooperative bargaining, firms will have none zero flow profit, which will be redistributed in a lump-sum fashion to households as given in (3).

2.4 The Government

The purpose of this paper is to study tax incidence in an economy with labor search frictions and costly vacancy creation. In order for better comparisons with the conventional tax incidence studies under canonical Walrasian settings, we regard the government as a pure tax authority, which *cannot* coordinate labor matching/wage bargain and *cannot* internalize the externality from vacancy creation via $\{\bar{y}_t\}$. Moreover, as in the conventional tax incidence, the government size is taken as given which is determined by revenues from the pre-existing distortionary taxes. That is, we are solving for a *third best* solution.[7] The government's objective is to maximize the social welfare under a balanced budget taking matching rates as given (i.e., regard matching rates $\{\mu, \eta\}$ as given). Its budget constraint is given by,

$$T_t + w_t h_t (1-n_t)\bar{b} = \tau_L w_t h_t \left[n_t \ell_t + (1-n_t)\bar{b}\right] + \tau_K r_t s_t k_t \tag{21}$$

That is, the government receives wage and capital income taxes to spend on direct transfers to households and unemployment compensation. Of particular note is that the inclusion of transfers is to ensure that the government's budget is balanced in the presence of pre-existing factor taxes and unemployment compensation that fits the data observations.

Since firms' profits are redistributed to households, the social welfare is measured simply by the household's lifetime utility Ω. Thus, our dynamic tax incidence problem is to determine optimal flat tax rates (τ_K^*, τ_L^*) by evaluating the long-run welfare outcome measured by Ω, subject to all the

[6] Again, we relegate the second-order condition of the wage bargaining problem to the Appendix.

[7] It should be noted that even if the government can coordinate to reach the efficient wage bargain frontier, the conventional Hosios' rule, $\beta = \zeta$, would not hold true under our endogenous growth setting (see the Appendix).

policy functions obtained from the household's and the firm's optimization problems, the bargaining problem, and the government's budget constraint (21).

3 Equilibrium

A *dynamic search equilibrium* is a tuple of individual quantity variables, $\{c_t,\ \ell_t,\ v_t,\ k_t,\ h_t,\ n_t,\ y_t,\ q_t\}_{t=0}^{\infty}$, a pair of aggregate quantities $\{M_t,\ T_t\}_{t=0}^{\infty}$, a pair of matching rates $\{\mu_t,\ \eta_t\}_{t=0}^{\infty}$, and a pair of prices, $\{w_t,\ r_t\}_{t=0}^{\infty}$, such that: (i) all households and firms optimize; (ii) human capital and employment evolution hold, (iii) labor-market matching and wage bargaining conditions are met; (iv) the government budget is balanced; and (v) the goods market clears. A *balanced growth path* (BGP) is a dynamic search equilibrium along which consumption, physical and human capital, and output all grow at positive constant rates. In our model, both the market goods and the human capital investment production technologies are homogeneous of degree one in reproducible factors (k and h). Thus, all endogenously growing quantities (c, k, h and y) must grow at a common rate, g, on a BGP, whereas employment (n), vacancies (v) and equilibrium matches (M) must all be stationary. Given the common growth property, we can divide all the perpetually growing variables by h to obtain stationary ratios, $\frac{k}{h}$, $\frac{c}{h}$, and $\frac{y}{h}$, where the latter two ratios measure effective consumption and effective output, respectively.

Along a BGP, the labor market must satisfy the steady-state matching (Beveridge curve) relationships given by,

$$\psi n = \mu(1-n) = \eta v = B(1-n)^{\beta}\,(v)^{1-\beta} \tag{22}$$

An additional condition to the previously defined employment evolution and labor-market matching equations, (1), (13) and (19), is to require the equilibrium employment inflows from the household side ($\mu(1-n)$) to be equal to those from the firm side (ηv). The above relationships enable us to obtain:

Lemma 4. (Steady-State Matching) *Under steady-state matching, the job finding rate (μ), the employee recruitment rate (η) and equilibrium vacancies (v) can all be solved as functions of employment (n) only:*

$$\mu(n) = \frac{\psi n}{1-n} \tag{23}$$

$$\eta(n) = B^{\frac{1}{1-\beta}}\mu(n)^{\frac{-\beta}{1-\beta}} \tag{24}$$

$$v(n) = B^{\frac{-1}{1-\beta}}\mu(n)^{\frac{\beta}{1-\beta}}\psi n \tag{25}$$

While the job finding rate and equilibrium vacancies are positively related to equilibrium employment, the employee recruitment rate is negatively related to it.

Accordingly, we can also derive the labor-market tightness measure (from the firm's point of view), $\theta = v/(1-n)$, as:

$$\theta(n) = \left[\frac{\mu(n)}{B}\right]^{\frac{1}{1-\beta}} \tag{26}$$

which is positively related to the job finding rate and hence equilibrium employment.

In order to generate a BGP equilibrium, we must impose a logarithmic utility function: $U(c) = \ln c$.[8] Under this preference specification, households' lifetime utility is always bounded along a BGP. Moreover, $\Gamma_n(n')$ and $\Omega_n(\mathcal{H}')$ are constant along a BGP, whereas $\Omega_k(\mathcal{H}')$ and $\Omega_h(\mathcal{H}')$ are decreasing at rate g. Using (6), (8), and (9), we obtain:

$$g = \frac{(1-\tau_K)r - \delta_k - \rho}{1+\rho} \tag{27}$$

$$\rho + (1+\rho)g = [D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}][n + (1-n)\bar{b}] \tag{28}$$

While (27) is a standard Keynes-Ramsey relationship governing consumption growth, (28) is an intertemporal optimization condition governing human capital accumulation. Following the argument in Bond, Wang and Yip (1996), we assume throughout the paper the following condition to ensure nondegenerate balanced growth:

Condition G. (Nondegenerate Growth) $\inf\{r\} > \frac{\delta_k+\rho}{1-\tau_K}$.

This condition basically limits the range of factor price frontier.[9]

Moreover, we can apply the human capital evolution equation (2) to relate learning effort to the nonmarket effective capital-labor ratio, employment and the balanced growth rate:

$$1 - \ell = \frac{g}{n\left[D + \widetilde{D}\left(q^H\right)^{\gamma}\right]} \tag{29}$$

Further define the unit wage income as $S_w = (1-\tau_L)\left[1 + \frac{(1-n)\bar{b}}{n\ell}\right]$ and the unit rental income as $S_r = \left[(1-\tau_K)r - \frac{g+\delta_k}{s}\right]$. From the definition of π and (16), the flow profit redistribution to each household in effective units is specified as:

$$\frac{\pi}{h} = n\ell\left\{A\left(q^F\right)^{\alpha}[(1-\alpha) - \phi v] - w\right\} \tag{30}$$

From (3), the definition of q^F and the flow profit redistribution given above, we can derive effective consumption along a BGP as:

$$\begin{aligned}\frac{c}{h} &= \left(S_w w + S_r q^F\right)n\ell + \frac{\pi}{h} + \frac{T}{h} \\ &= \left\{A\left(q^F\right)^{\alpha}[(1-\alpha) - \phi v] + S_r q^F - (1-S_w)\,w\right\}n\ell + \frac{T}{h}\end{aligned} \tag{31}$$

[8] Suppose the utility function takes a constant elasticity of intertemporal substitution form. It can be easily verified that, should this elasticity be different from one, (10) would violate the BGP requirements.

[9] Since the wage herein is determined by cooperative bargaining, it is not easy to derive a clean condition as in the Walrasian framework of Bond, Wang and Yip (1996).

where T is regarded as given by individuals with its equilibrium value being pinned down by the government budget constraint (21).

To solve the wage bargaining problem, we first note that the household-firm pair in the bargaining game must take $\{\mu, \eta, n, r\}$ as given. From (18), q^F must also be regarded as predetermined. Using (11) and (28), we can express both the nonmarket effective capital-labor ratio and the balanced growth rate as increasing functions of the bargained wage only: $q^H = q^H(w)$ and $g = g(w)$. Intuitively, while it is clear that a higher wage and hence a higher wage-rental ratio (given r) leads to a higher non-market effective capital-output ratio, the latter in turn raises the BGP human capital accumulation rate. Combining (28) and (29) yields $\ell(w)$ as follows:

$$\ell(w) = 1 - \frac{(1-\gamma)g(w)}{n}\left[\frac{(1+\rho)\,g(w)+\rho}{n+(1-n)\bar{b}} - \gamma D\right]^{-1} \tag{32}$$

The bargained wage serves as an incentive to encourage households, on the one hand, to devote more effort to market activity, while, on the other hand, accumulating more human capital. When the long-run human capital accumulation effect dominates (as it will in the calibrated economy), it is expected that an increase in the bargained wage will reduce work effort. By the definitions of q^F and q^H, we have:

$$\frac{q^F}{q^H(w)} = \frac{s}{1-s}\frac{1-\ell(w)}{\ell(w)} \tag{33}$$

which can then be used to derive $s = s(w)$ as a decreasing function of the bargained wage. Intuitively, a higher bargained wage raises the learning effort $(1-\ell)$ and, by capital-labor complementarity, results in a larger fraction of capital being devoted to human capital accumulation (i.e., a higher $1-s$).

Endowed with the functions $q^H(w)$, $g(w)$, $\ell(w)$ and $s(w)$ given above, we are now ready to determine the equilibrium wage. Substituting (5) and (7) into (10), we can write the household's surplus accrued from a successful match as follows:

$$\Omega_n = \frac{1+\rho}{\rho+\psi+\mu}\left[(1-\tau_L)(1-\bar{b})\frac{w}{c/h} - m\right] \tag{34}$$

where from (31) $\frac{c}{h}$ is increasing in w but less than proportionally, implying that the household's surplus is increasing in w.

It is informative to compute the wage discount that measures how much the bargained wage is below its competitive counterpart (i.e., the marginal product of labor, MPL):

$$\Delta \equiv \frac{MPL-w}{MPL} = 1 - \frac{w}{(1-\alpha)A\left(q^F\right)^{\alpha}} \tag{35}$$

Straightforward differentiation of the surplus accrued by each party leads to $-\frac{w}{\Gamma_n}\frac{d\Gamma_n}{dw} = \frac{1-\Delta}{\Delta}$ and $\frac{w}{\Omega_n}\frac{d\Omega_n}{dw} = \frac{S_r q^F + (\pi+T)/(n\ell h)}{S_w w + S_r q^F + (\pi+T)/(n\ell h)}$. While the former is decreasing in the wage discount Δ and hence increasing in w, the latter is decreasing in w. Thus, we can manipulate (20) to obtain:[10]

Lemma 5. (Wage Determination) *The bargained wage is characterized by,*

$$MB_w = \frac{\zeta}{w - m\frac{(S_w w + S_r q^F)n\ell + (\pi+T)/h}{(1-\tau_L)(1-\bar{b})}} \frac{S_r q^F n\ell + \frac{\pi+T}{h}}{(S_w w + S_r q^F)\, n\ell + \frac{\pi+T}{h}} = \frac{1-\zeta}{(1-\alpha)A\left(q^F\right)^{\alpha} - w} = MC_w \tag{36}$$

where MB_w is decreasing but MC_w is increasing in w.

The determination of bargained wage is illustrated in the top panel of Figure 2, when the marginal benefit from the household's point of view (MB_w) equals the marginal cost from the firm's point of view (MC_w).

We are now prepared to characterize the effects of factor taxes on bargained wages, given $\{\mu, \eta, n, r\}$ and hence the effective capital-labor ratio q^F (refer to (18)). An increase in τ_K has a direct negative effect via the after-tax rental on the unit rental income (S_r), which decreases the household's marginal benefit and leads to a downward shift in the MB_w locus. There is also an indirect effect via the extensive margin of leisure (which would have vanished if $m = 0$), which tends to shift the MB_w locus upward (recall that $m < 0$). Similarly, there is a direct effect via the after-tax wage of higher labor taxation τ_L: in this case, by reducing S_w, it shifts the MB_w locus up. The indirect effect via the extensive margin of leisure is generally ambiguous (one via S_w and another via the net opportunity cost of staying unemployed, $(1-\tau_L)(1-\bar{b})$). Since q^F is taken as given for a particular value of r, it is clear that the MC_w locus will not respond to changes in factor tax rates. As a result, the marginal benefit from the household's point of view is decreasing in the capital tax rate, whereas it is increasing in the labor tax rate when the marginal valuation of leisure is sufficiently low (such that the magnitude of m is sufficiently small). Moreover, by similar arguments, we can show that the marginal benefit from the household's point of view is increasing in employment when the marginal valuation of leisure is sufficiently low. We thus arrive at:

Proposition 1. (Wage Offer) *There is a unique bargained wage $w\,(n; \tau_K, \tau_L)$ solving (36), which possesses the following properties:*

(i) *it is decreasing in the capital tax rate (τ_K) unambiguously, but increasing in the labor tax rate (τ_L) if the marginal valuation of leisure is sufficiently low;*

(ii) *it is increasing in employment (n) if the marginal valuation of leisure is sufficiently low (m sufficiently small in magnitude).*

[10] The bargained wage rate and the equilibrium wage can be derived by solving the following quadratic equation:

$$S_w(1\text{-}\zeta)F_1 w^2 + \{S_r q[(1\text{-}\zeta)F_1 + \zeta]\text{-}S_w F_2(1\text{-}\zeta)\}w - S_r q[(1\text{-}\zeta)F_2 + \zeta(1\text{-}\alpha)Aq^{\alpha}] = 0$$

where $F_1 = \frac{(1-\tau_L)(1-\bar{b}) + mn\ell S_w}{(1-\tau_L)(1-\bar{b})} > 0$ and $F_2 = \frac{-m[S_r qn\ell + (\pi+T)/h]}{(1-\tau_L)(1-\bar{b})} < 0$.

Intuitively, a higher capital tax discourages capital accumulation, thus lowering the marginal product of labor and the bargained wage. On the contrary, a higher labor tax discourages household's participation in the labor market, thereby requiring a high wage to induce the participation. We can also plot the relationship between the wage discount and the wage rate, which is downward-sloping based on the expression in (35) above (see the bottom panel of Figure 2).

Once the bargained wage is determined (see w_0 in Figure 2), we can then solve the associated wage discount using (35). Notice that this wage discount schedule only depends on the market effective capital-labor ratio q^F. From (11), (18), (27) and (33), we can see that for each w, the pre-tax real rental rate r is increasing in the capital tax rate but decreasing in the labor tax rate as long as the labor cost share in the goods sector $(1-\alpha)$ is sufficiently high:

Condition LC. (Sufficiently High Goods-Sector Labor Cost Share) $1-\alpha > \sup_w \{(1-s(w))s(w)\}$.

Under this (sufficient but not necessary) condition on labor cost shares, by raising the pre-tax real rental rate and hence reducing q^F, an increase in τ_K shifts the wage discount schedule down; on the contrary, an increase in τ_L raises q^F and shifts the wage discount schedule up.

We then obtain the following:

Proposition 2. (Wage Discount Function) *Under Condition LC, the wage discount function possesses the following properties:*

(i) *its schedule $\Delta(w)$ is a decreasing function of the bargained wage (w), shifting down in response to a higher capital tax rate (τ_K) and up in response to a higher labor tax rate (τ_L);*

(iii) *its value Δ is increasing in the capital tax rate and decreasing in the labor tax rate when the bargained wage is sufficiently responsive to changes in factor tax rates.*

Basically, in response to a higher capital tax, the bargained wage is lower, the pre-tax rental is higher and the wage discount schedule shifts downward. When the bargained wage is sufficiently responsive, the wage discount is higher. By similar arguments, with a sufficiently responsive bargained wage, the wage discount is lower in response to a higher labor tax. Such negative relationships between the bargained wage and the wage discount are intuitive and natural, which are supported by our calibration analysis.

Furthermore, we can manipulate (15), (16), (17) and (24) to obtain an expression that relates employment and capital rental to the wage rate:

$$w = \frac{rq^F}{\alpha}\left[(1-\alpha) - \frac{(r+\psi)\phi}{\eta(n)}n\right] \tag{37}$$

Using the capital rental function derived above $r(g;\tau_K,\tau_L)$ and the wage function $w(n;\tau_K,\tau_L)$ given in Proposition 1, we can then express (37) as a relationship in (n,g). This relationship summarizes a firm's efficiency conditions that govern capital demand, labor demand and vacancy creation, with steady-state matching and bargained wage conditions embedded, which is referred to as the *equilibrium firm efficiency* (FE) relationship. Note that rq^F is decreasing in r whereas $\eta(n)$ is decreasing in n, so the righthand side of (37) is decreasing in both r and n. From (27), r is increasing in g, whereas from Proposition 1, w is increasing in n when the marginal valuation of leisure is sufficiently low. Thus it is clear that the FE locus is downward sloping. Moreover, a higher tax on capital income unambiguously raises the pre-tax capital rental and reduces the barged wage whereas a higher tax on labor income generates opposite effects. Thus, when the own price effect via the after-tax wage-rental ratio dominates, an increase in either tax rate shifts the FE locus downward (see Figure 3).

Similarly, we can substitute the capital rental function $r(g;\tau_K,\tau_L)$ and the wage function $w\,(n;\tau_K,\tau_L)$ into (11) and use it with (28) to obtain another balanced growth relationship in (n,g), which is referred to as the *optimized human capital accumulation* (HA) relationship. It is obvious that, with r increasing in g and w increasing in n, the HA locus is upward sloping. Recall that a higher capital tax or a lower labor tax increases the pre-tax capital rental and reduces the bargained wage. Notably, (28) indicates that factor taxes only affect this HA locus via the nonmarket effective capital-labor ratio q^H that is an increasing function of the after-tax wage-rental ratio. Because an increase in the capital tax rate reduces the after-tax rental $(1-\tau_K)r$ and an increase in the labor tax rate decreases the after-tax wage $(1-\tau_L)w$, it is easily seen that, when the own price effect dominates, a higher capital tax tends to raise q^H whereas a higher labor tax tends to lower it. Thus, while a higher capital tax shifts the HA locus upward, a higher labor tax shifts the locus downward (see Figure 3). We should note that the nonmarket effective capital-labor ratio will not be responsive to changes in the after-tax wage-rental ratio when the technology parameter $\widetilde{D}$ is small. This implies that the factor tax effects on the HA locus become negligible as $\widetilde{D}$ becomes sufficiently small.

To characterize the effects of factor income taxes on employment and growth, we further impose a condition on factor price responses:

Condition FP. (Dominant Own Price Effects of Factor Taxes) Each factor price and after-tax factor price are more responsive to its own factor tax rate.

From (11), it is clearly seen that Condition FP holds true if the technology parameter $\widetilde{D}$ is sufficiently small. We then have:

Proposition 3. (Employment and Growth) *Under Conditions LC and FP, the balanced growth equilibrium possesses the following properties:*

(i) *an increase in either the capital tax rate (τ_K) or the labor tax rate (τ_L) shifts the FE locus down, but an increase in the capital tax rate (τ_K) shifts the HA locus up whereas an increase* *in the labor tax rate (τ_L) shifts the HA locus down;*

(ii) *when the technology parameter $\widetilde{D}$ is sufficiently small such that the HA locus is not too responsive to changes in the factor taxes, an increase in either factor tax reduces employment and growth.*

The results depicted in Figure 3 are under own price dominance (Condition FP) and sufficiently small $\widetilde{D}$.

Finally, using the discussion above, we can conclude:

Proposition 4. (Factor Allocation) *Under Conditions LC and FP, in the balanced growth equilibrium, an increase in the capital tax rate (τ_K) reduces the effective capital-labor ratio in the market sector (q^F) whereas an increase in the labor tax rate (τ_L) raises the effective capital-labor ratio in the nonmarket sector (q^H).*

The result is intuitive: while, under dominance of the own price effect, a higher capital tax induces physical capital to be allocated away from the market sector (thus decreasing the effective capital-labor ratio in the market sector), a higher labor tax encourages human capital to be allocated to nonmarket activity (thus lowering the effective capital-labor ratio in the nonmarket sector). Just how capital taxation may affect the effective capital-labor ratio in the nonmarket sector or how labor taxation may affect the effective capital-labor ratio in the market sector will depend on both factor substitution and other indirect effects, which cannot be pinned down analytically in a clear-cut manner.

4 Efficiency

In this section, we turn to efficiency issues by considering a quasi-social planner's problem where the central planner takes $\bar{y}_t$ as given (as the vacancy-creation externality is purely for providing unbiased support for endogenous growth, which is not present in the standard efficient bargain literature).

The quasi-social planner can allocate consumption, labor, capital and vacancy as well as coordinate labor matching to achieve efficiency (in the second best sense due to its ignorance of the vacancy-creation externality). The problem is given by,

$$\Lambda(k_t, h_t, n_t) = \max_{c_t,\, \ell_t,\, s_t,\, v_t} U(c_t) + m(1 - n_t) + \frac{1}{1+\rho}\Lambda(k_{t+1}, h_{t+1}, n_{t+1}) \tag{38}$$

subject to:

$$k_{t+1} = A(s_t k_t)^{\alpha}(n_t \ell_t h_t)^{1-\alpha} - \phi \bar{y}_t v_t - c_t \tag{39}$$

$$h_{t+1} - h_t = D n_t (1 - \ell_t) h_t + \widetilde{D}[(1 - s_t)k_t]^{\gamma}[n_t(1 - \ell_t)h_t]^{1-\gamma} \tag{40}$$

$$n_{t+1} = (1-\psi)n_t + B(1-n_t)^{\beta}(v_t)^{1-\beta} \tag{41}$$

It is noted that while the resource constraint is straightforward by replacing income with net output and the human capital accumulation equation is identical to the decentralized problem, the evolution equations of employment differs from the decentralized program now with coordinated labor matches.

Lemma 6. (Quasi-Social Planner's Optimization) *The quasi-social planner's optimization satisfies the following first-order conditions (with respect to $\{c_t, \ell_t, s_t, v_t\}$),*

$$U_c = \frac{1}{1+\rho}\Lambda_k(\mathcal{H}') \tag{42}$$

$$\Lambda_k(\mathcal{H}')A(1-\alpha)(q^F)^{\alpha} = \Lambda_h(\mathcal{H}')\left[D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right] \tag{43}$$

$$\Lambda_k(\mathcal{H}')A\alpha(q^F)^{\alpha-1} = \Lambda_h(\mathcal{H}')\widetilde{D}\gamma\left(q^H\right)^{\gamma-1} \tag{44}$$

$$\Lambda_k(\mathcal{H}')\phi A(q^F)^{\alpha}n\ell h = \Lambda_n(\mathcal{H}')(1-\beta)B(1-n)^{\beta}(v)^{-\beta} \tag{45}$$

together with the respective Benveniste-Scheinkman conditions (associated with $\{k_t, h_t, n_t\}$):

$$\Lambda_k(\mathcal{H}) = \frac{1}{1+\rho}\Lambda_k(\mathcal{H}')[1-\delta_k + A\alpha(q^F)^{\alpha-1}] \tag{46}$$

$$\Lambda_h(\mathcal{H}) = \frac{1}{1+\rho}(\Lambda_k(\mathcal{H}')A(1-\alpha)(q^F)^{\alpha}n\ell + \Lambda_h(\mathcal{H}')\{1+n(1-\ell)[D+\widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}]\}) \tag{47}$$

$$\begin{aligned}\Lambda_n(\mathcal{H}) = {} & -m + \frac{1}{1+\rho}\{\Lambda_k(\mathcal{H}')A(1-\alpha)(q^F)^{\alpha}\ell h + \Lambda_h(\mathcal{H}')(1-\ell)h[D+\widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}] \\ & +\Lambda_n(\mathcal{H}')(1-\psi-\beta B(1-n)^{\beta-1}(v)^{1-\beta})\}\end{aligned} \tag{48}$$

The main task here is to derive conditions for efficiency bargaining by setting the decentralized solution to be the same as the centralized solution. Recall that in order to generate a BGP equilibrium, we impose $U(c) = \ln c$ and, along a BGP, $\Gamma_n(n')$ and $\Omega_n(\mathcal{H}')$ are constant whereas $\Omega_k(\mathcal{H}')$ and $\Omega_h(\mathcal{H}')$ are decreasing at rate g. We shall use these properties of the value functions in our analysis below.

4.1 Decentralized Solution

For the purpose of comparison, it is convenient to rewrite the conditions in Lemmas 1 and 2 in the decentralized problem, along the BGP, as:

$$\begin{aligned}\Omega_k(\mathcal{H}') &= (1+\rho)U_c \\ \Omega_k(\mathcal{H}')(1-\tau_L)w &= \Omega_h(\mathcal{H}')\left[D+\widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right] \\ \Omega_k(\mathcal{H}')(1-\tau_K)r &= \Omega_h(\mathcal{H}')\widetilde{D}\gamma\left(q^H\right)^{\gamma-1} \\ (1+\rho)(1+g) &= [1-\delta_k+(1-\tau_K)r] \\ (1+\rho)(1+g)-1 &= \left[D+\widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right][n+(1-n)\bar{b}]\end{aligned} \tag{49}$$

$$\Omega_n = \frac{1+\rho}{\rho+\psi+\mu}\left[\left(1-\bar{b}\right)(1-\tau_L)whU_c - m\right] \tag{50}$$

$$\Gamma_n = \frac{1+R}{\psi+R}\left[(1-\alpha)A\left(q^F\right)^\alpha - w\right] \tag{51}$$

By combining (50) with the cooperative Nash wage bargain expression, we obtain:

$$\left(1-\bar{b}\right)(1-\tau_L)whU_c - (1-\zeta)\,m = \zeta\left(1-\bar{b}\right)(1-\tau_L)h(1-\alpha)A\left(q^F\right)^\alpha U_c \tag{52}$$

4.2 Centralized Solution

Rewriting the conditions in Lemma 5 along the BGP, we have:

$$\begin{gathered}
\Lambda_k(\mathcal{H}') = (1+\rho)\,U_c \\
\Lambda_k(\mathcal{H}')A(1-\alpha)(q^F)^\alpha = \Lambda_h(\mathcal{H}')\left[D+\widetilde{D}(1-\gamma)\left(q^H\right)^\gamma\right] \\
\Lambda_k(\mathcal{H}')A\alpha(q^F)^{\alpha-1} = \Lambda_h(\mathcal{H}')\widetilde{D}\gamma\left(q^H\right)^{\gamma-1} \\
\Lambda_k(\mathcal{H}')\phi A(q^F)^\alpha n\ell h = \Lambda_n(\mathcal{H}')(1-\beta)B(1-n)^\beta\,(v)^{-\beta} \\
(1+\rho)\,(1+g) = \left[1-\delta_k + A\alpha(q^F)^{\alpha-1}\right] \\
(1+\rho)\,(1+g) - 1 = \left[D+\widetilde{D}(1-\gamma)\left(q^H\right)^\gamma\right]n
\end{gathered} \tag{53}$$

$$\left[\rho+\psi+\beta B(1-n)^{\beta-1}\,(v)^{1-\beta}\right]\Lambda_n = (1+\rho)\left[A(1-\alpha)(q^F)^\alpha hU_c - m\right] \tag{54}$$

$$(1-\beta)B(1-n)^\beta\,(v)^{-\beta}\,\Lambda_n = \phi A(q^F)^\alpha n\ell h\,(1+\rho)\,U_c \tag{55}$$

where the last two expressions can be combined with $\psi n = \mu(1-n) = \eta v = B(1-n)^\beta\,(v)^{1-\beta}$ to yield:

$$\begin{aligned}
A(1-\alpha)(q^F)^\alpha hU_c - m &= \frac{\rho+\psi+\beta B(1-n)^{\beta-1}\,(v)^{1-\beta}}{(1-\beta)B(1-n)^\beta\,(v)^{-\beta}}\phi A(q^F)^\alpha n\ell hU_c \\
&= \frac{\rho+\psi+\beta\mu}{(1-\beta)\eta}\phi A(q^F)^\alpha n\ell hU_c
\end{aligned}$$

or,

$$(1-\alpha)A(q^F)^\alpha hU_c - \frac{\rho+\psi+\beta\mu}{\eta v}\Phi U_c - (1-\beta)m = \beta(1-\alpha)A(q^F)^\alpha hU_c \tag{56}$$

where $\Phi = \phi v\bar{y} = \phi vA(q^F)^\alpha n\ell h$ is the vacancy creation cost.

4.3 Conditions for Efficiency

By comparing the decentralized and centralized solutions, (49) and (53), we can identify four conditions in a more straightforward manner:

$$R = r$$

$$\begin{aligned} \Omega_k(\mathcal{H}') &= \Lambda_k(\mathcal{H}') \\ \tau_K &= 0 \\ \bar{b} &= 0 \end{aligned}$$

To ensure the labor-leisure-consumption trade-off under decentralization and centralization to be identical, we need to establish equivalence between (52) and (56), which holds true under the following conditions:

$$\begin{aligned} \zeta &= \beta \\ \tau_L &= 0 \\ w &= (1-\Delta^*)\,(1-\alpha)A(q^F)^\alpha \end{aligned}$$

where efficient wage discount Δ^* is given by,

$$\Delta^* = \frac{(\rho+\psi+\beta\mu)\,\phi v\ell}{\psi(1-\alpha)} = \frac{(\rho+\psi+\beta\mu)\,\phi n\ell}{(1-\alpha)\eta}$$

We are therefore arrived at:

Proposition 5. (Efficiency) *By taking the vacancy creation externality ($\bar{y}$) as given in both decentralized and centralized problems, the decentralized dynamic search equilibrium along the balanced growth path achieves second-best, solving the quasi-social planner problem, under the following conditions:*

(i) *(discounting and valuation of capital) firms discount at the market rental rate ($R = r$) and value capital in the same manner as the quasi-social planner ($\Omega_k(\mathcal{H}') = \Lambda_k(\mathcal{H}')$);*

(ii) *(removal of distortionary factor taxes and subsidies) $\tau_K = \tau_L = \bar{b} = 0$;*

(iii) *(Hosios' rule) $\zeta = \beta$;*

(iv) *(efficient wage discount): wage discount is set at $\Delta = \Delta^* = \frac{(\rho+\psi+\beta\mu)\phi n\ell}{(1-\alpha)\eta}$.*

That is, Hosios' rule is a necessary but *not sufficient* condition for efficiency. This is not surprising because Hosios' rule is derived based purely on labor-market matching and bargain efficiency. In our model, with endogenous growth and endogenous labor-leisure-consumption trade-off atemporally and intertemporally, we need additional conditions to achieve efficiency. Notably, both the first set of conditions regarding discounting and valuation of capital and the third entailing the Hosios' rule are standard in the literature, which will be imposed throughout the remainder of the paper. Unfortunately, the second set of conditions involves removal of distortionary factor taxes and subsidies, which cannot be imposed in our analysis because the tax incidence is the primary purpose of our paper. As a consequence, we shall not impose an efficient wage discount as it is a property derived in the absence of any pre-existing distortionary factor taxes or subsidies. Thus, in the calibrated economy below, the optimality is more precisely referred to as to achieve the third best (by not correcting the vacancy creation externality and by allowing for pre-existing distortionary factor taxes and subsidies).

5 Numerical Analysis

We now turn to calibrating our benchmark model. We then conduct comparative-static exercises quantitatively, particularly focusing on the balanced growth effects of the two factor tax rates. We then perform tax incidence exercises and derive the optimal factor tax mix numerically. Finally, we perform sensitivity analysis to examine the robustness of our numerical results.

5.1 Calibration

We calibrate parameter values to match the U.S. quarterly data during the post-WWII period. We set the quarterly per capita real GDP growth rate to $g = 0.45\%$ and the quarterly depreciation rate of capital to 0.01 to match the annual per capita real GDP growth rate of 1.8% and the annual depreciation rate of capital in the range of $3 - 8\%$, respectively. With an annual time preference rate of 5%, we set our quarterly rate of time preference to 0.0125. The output elasticity of capital is set at the average capital income share $\alpha = 0.28$. Based on the observation and the factor tax incidence exercises conducted by Judd (1985) and many others, we set the pre-existing flat tax rates: $\tau_K = 0.2$ and $\tau_L = 0.2$. The capital rental rate can then be calibrated by using (27): $r = 0.03382$, which implies a capital-output ratio $k/y = \frac{\alpha}{r} = 8.279$, close to the observed value. As argued by Kendrick (1976), human capital is as large as physical capital. We thus set the benchmark value of the physical to human capital ratio at $k/h = 1$.

The ratio of unemployment compensation to the market wage ($\bar{b}$) in the benchmark case is set to 0.42, in line with Shimer (2005) and Hall (2005). Also based on Shimer (2005), the monthly separation rate is given as 0.034 and the monthly job finding rate as 0.45. These enable us to compute the quarterly separation rate $\psi = 1 - (1 - 0.034)^3 = 0.0986$ and the quarterly job finding rate $\mu = 1 - (1 - 0.45)^3 = 0.8336$. From (22), we can compute: $n = \frac{\mu}{\mu+\psi} = 0.8943$. By following Shimer (2005) to normalize the vacancy-searching worker ratio $(\frac{v}{u})$ as one, we can utilize (22) and (23) to calibrate $\eta = B = 0.834$ and use (25) to obtain $v = \frac{\psi n}{\eta} = 0.1057$. Following Blanchard and Diamond (1990), we set the benchmark value of the worker elasticity of matching as $\beta = 0.4$. Because the Hosios' rule is a necessary condition for efficient bargains, we impose $\zeta = \beta = 0.4$.

Next, we follow Andolfatto (1995), setting $\epsilon = 0.5$. In Andolfatto, the marginal utility from leisure accrued by the unemployed is $\tilde{m} = 1.37$. In addition, we can have a quick accounting of households' time use to obtain a reasonable allocation of time for work, learning and leisure at 20%, 8% and 72%, respectively. These together with the calibrated value of n yield total units of time facing the large household $N = n\ell + n(1-\ell) + (1-n)z = 3.167$, equilibrium work effort $\ell = 0.725$ and equilibrium leisure $z = 21.5$ (i.e., at the household level, the fractions of work, learning and leisure time are $\frac{n\ell}{N}$, $\frac{n(1-\ell)}{N}$ and $\frac{(1-n)z}{N}$, respectively, which match the respective targets). Thus, $m = -1.37 \cdot (21.5)^{-1} = -0.064$.

Moreover, from the definitions of q^F and q^H, we can write:

$$\begin{aligned} q^F &= \frac{sk}{n\ell h} = \frac{s}{0.8943 \cdot 0.725} = q^F(s) \\ q^H &= \frac{(1-s)\,k}{n\,(1-\ell)\,h} = \frac{1-s}{0.8943 \cdot 0.275} = q^H(s) \end{aligned}$$

which can be substituted into (18) to yield:

$$A(s) = \frac{r}{\alpha} q^F(s)^{1-\alpha}$$

While $S_w = (1-\tau_L)\left[1 + \frac{(1-n)\bar{b}}{n\ell}\right]$ is a given number, $S_r(s) = \left[(1-\tau_K)r - \frac{g+\delta_k}{s}\right]$ is a function of s alone. Since human capital investment is expected to be more human capital-intensive than goods production (i.e., $\gamma < \alpha = 0.28$), we set the benchmark value of $\gamma = 0.25$. From (28) and (29), we have:

$$D + \widetilde{D}\left(q^H(s)\right)^{\gamma} = \frac{g}{n\,(1-\ell)}$$

$$\begin{aligned} \rho + (1+\rho)g &= \{\gamma D + (1-\gamma)[D + \widetilde{D}\left(q^H\right)^{\gamma}]\}[n + (1-n)\bar{b}] \\ &= \{\gamma D + (1-\gamma)\frac{g}{n\,(1-\ell)}\}[n + (1-n)\bar{b}] \end{aligned}$$

From the latter expression, we solve $D(s)$, which can then be plugged into the former to derive $\widetilde{D}(s)$. These can then be substituted into (11) and (37) to obtain, respectively:

$$w(s) = \frac{(1-\tau_K)r}{(1-\tau_L)\gamma\widetilde{D}(s)}\left(q^H(s)\right)^{1-\gamma}\left[D(s) + \widetilde{D}(s)(1-\gamma)\left(q^H(s)\right)^{\gamma}\right]$$

$$\phi(s) = \frac{\eta}{(r+\psi)n}\left[(1-\alpha) - \frac{\alpha w(s)}{r q^F(s)}\right]$$

By writing (21), (30) and (31), we now get, respectively:

$$\frac{T}{h}(s) = w\left[\tau_L n\ell - (1-\tau_L)\,(1-n)\bar{b}\right] + \tau_K r s \frac{k}{h}$$

$$\frac{\pi}{h}(s) = n\ell\left\{A\left(q^F(s)\right)^{\alpha}\left[(1-\alpha) - \phi v\right] - w(s)\right\}$$

$$\frac{c}{h}(s) = \left(S_w w(s) + S_r(s) q^F(s)\right) n\ell + \frac{\pi}{h}(s) + \frac{T}{h}(s)$$

The above expressions can then be substituted into (36) to compute $s = 0.9981$. Thus, in this calibrated economy, most of the physical capital inputs are used for goods production. By plugging the calibrated value of s into the above functions of s, we can then compute: $q^F = 1.539$, $q^H = 0.007908$, $A = 0.1648$, $D = 0.01779$, $\widetilde{D} = 0.001715$, $\phi = 3.631$, $\frac{T}{h} = 0.01033$, $\frac{\pi}{h} = 0.01588$, and $\frac{c}{h} = 0.05977$. Thus, the lump-sum government and firm profit redistributions and household consumption are about 8.6%, 12.3%, and 50.0%, respectively, in our benchmark economy. We can further plug in the value of w into (35) to compute $\Delta = 0.7161$.

We summarize the observables, benchmark parameter values and calibrated values of key endogenous variables in Table 1.

5.2 Numerical Results

We next simulate the benchmark model to examine quantitatively the effects of two factor tax rates (τ_K and τ_L) on an array of endogenous variables of interest, including the balanced growth rate (g), effective consumption (c/h), the physical-human capital ratio (k/h), effective output (y/h), employment (n), work effort (ℓ), the wage (w), the wage discount (Δ), the workers' job finding rate (μ), the firms' employee recruitment rate (η), and firms' vacancies (v). The results obtained based on the responses of these endogenous variables around the balanced growth equilibrium to a 10% increase in each of the factor tax rates are reported in Table 2.

In our calibrated economy, we can now quantify the effects of the two factor tax rates (τ_K and τ_L) on the bargained wage and the wage discount in our calibrated economy. A higher capital tax is found to lower the bargained wage and to raise the wage discount slightly, whereas a higher labor tax raises the bargained (pre-tax) wage rate but lowers the wage discount. While both factor taxes discourage vacancy creation and suppress employment, the negative effects of capital taxation are much stronger than those of labor taxation. In response to either capital or labor taxation, the market becomes tighter to workers (i.e., $\theta = \frac{v}{u}$ is lower) and hence it is easier for firms to recruit but harder for workers to locate jobs. Either tax suppresses learning effort and the balanced growth rate, as well as the after-tax capital rental rate and the after-tax effective wage rate. Since factor taxation has a stronger negative effect on the taxed factor, the physical-human capital ratio falls in response to higher capital taxation, but rises in response to higher labor taxation. Our numerical results also suggest that a higher capital tax rate reduces output and consumption more than proportionately than human capital, whereas labor taxation suppresses human capital more than proportionately than output.

We turn next to examining the growth effects of the factor taxes based on the calibrated benchmark economy. Since factor taxation encourages a shift from market to tax-exempt nonmarket activity, it partly offsets the distortion on households' incentives to accumulate human capital. This, together with a small calibrated value of the technology parameter $\widetilde{D}$, implies that the HA locus is not too responsive to changes in the factor taxes. On the contrary, either tax increase reduces firm efficiency, thus implying a sizable downward shift in the FE locus. Our numerical results suggest that capital taxation induces a larger shift in the FE locus. As a result, capital taxation causes a larger drop in employment and balanced growth compared to labor taxation.

5.3 Factor Tax Incidence

We are now prepared to conduct tax incidence analysis in our endogenously growing economy. In particular, we change the composition of the two factor tax rates by keeping the government revenue unchanged. Under the pre-existing rates $(\tau_K, \tau_L) = (20\%, 20\%)$, the effective lump-sum tax is computed as $(T/h)^* = 0.0103$. This benchmark value will be kept constant and the government budget constraint (21) will remain balanced in our revenue-neutral tax-incidence exercises.

We next compute the social welfare measure along the BGP. Setting $h_0 = k_0 = 1$, we can calculate the lifetime utility as follows:

$$\Omega(\tau_K, \tau_L) = \frac{1+\rho}{\rho}\left[\ln(\frac{c}{h}(\tau_K, \tau_L)) + m\,(1 - n(\tau_K, \tau_L)) + \frac{1}{\rho}\ln(1 + g(\tau_K, \tau_L))\right] \tag{57}$$

where effective consumption is given by (31) with $T/h = (T/h)^*$. In short, social welfare is mainly driven by three endogenous variables: effective consumption $(\frac{c}{h})$, leisure $(1-n)$ and the economy-wide balanced growth rate (g), all of which depend on factor tax rates (τ_K, τ_L).

Figure 4 plots the dynamic factor tax incidence results. From Table 2, an increase in either the capital tax or the labor tax rate from its benchmark value of 20% leads to higher effective consumption as a result of a larger reduction in human capital. The effect of a shift from labor to capital taxation on effective consumption turns out to be hump-shaped and peaked at around $\tau_K = 20.57\%$. In contrast, a shift from labor to capital taxation always reduces growth. Moreover, there is an effect via the extensive margin of leisure. Combining all together, we find that our welfare measure (the lifetime utility of a household) is hump-shaped and maximized at $(\tau_K^*, \tau_L^*) = (16.11\%, 24.09\%)$. That is, in the absence of other tax alternatives, the socially optimal factor tax mix requires a decrease in the capital tax rate in conjunction with an increase in the labor tax rate from their benchmark values. Such a tax reform will lead to a 0.203% increase in economic growth and a 0.016% increase in welfare, which is a 0.039% increase in consumption equivalence. Moreover, one may ask how much the welfare loss is if one would set the capital tax rate at zero. Our quantitative analysis shows such a loss is in the order of 0.649% in consumption equivalence. Our finding that the optimal capital tax rate is *significantly larger than zero* is in contrast to the conventional dynamic factor tax incidence literature within both the exogenous and endogenous growth frameworks.

Result 1. *For the tax incidence exercises in response to a shift from labor to capital taxation, effective consumption and welfare are both hump-shaped whereas economic growth and leisure are both decreasing. Under the benchmark parametrization, the optimal tax mix features a moderate shift from capital to labor taxation but the optimal capital tax rate is far above zero.*

It is important to understand the numerically dominant channel underlying this finding: the *vacancy creation-market participation* channel in a non-Walrasian economy with labor market frictions. Specifically, if initially the capital tax rate is too low, then a higher tax on capital income accompanied by a revenue-neutral reduction in the labor tax turns out to raise the (endogenous) wage discount and to encourage firms to create more vacancies. This in turn raises the (endogenous) job finding rate and hence induces workers to more actively participate in the labor market to seek employment. Because this leads to positive effects on employment and output growth, a shift from a zero to a positive capital tax rate becomes welfare-improving, thereby yielding a policy recommendation different from that of Chamley-Judd-Lucas.

5.4 Sensitivity Analysis

While our pre-set parameters in the calibration exercises are all justified basically, some of the calibration criteria may be open to discussion. We therefore perform sensitivity analysis to check the robustness of our results. In particular, we consider the following alternatives:

(i) We allow the worker elasticity of matching, β, to take alternative values used in the literature, including 0.235 (Hall 2005), 0.54 (Hall and Milgrom 2008) and 0.72 (Shimer 2005).

(ii) We allow the leisure parameter, m (which is a combination of the preference parameter $\tilde{m}$ and the intensity of enjoyment $G(z)$), to be 50% below and above its benchmark value.

(iii) We allow the labor-market tightness, $\theta = v/(1-n)$, the ratio of the unemployment compensation to the market wage, $\bar{b}$, and the capital share of human capital accumulation, γ, to be 10% below and above their respective benchmark values.

(iv) We allow the amount of physical capital to be half or twice as large as the amount of human capital, i.e., $k/h = 0.5, 2$.

(v) We allow the pre-existing tax rates to take alternative values used in previous studies, $(\tau_K, \tau_L) = (35\%, 20\%)$ (Judd 1987) and $(\tau_K, \tau_L) = (40\%, 36\%)$ (Lucas 1990).

The sensitivity analysis results are reported in Table 3.[11]

When we recalibrate the model with different capital shares of human capital accumulation, or different physical-human capital ratios, labor-market matching, bargaining and human capital accumulation are either unchanged or changed only negligibly. Thus, the wage discount effect and the vacancy creation-market participation channel are essentially identical to those in the benchmark case, thereby leaving the factor tax incidence result largely unaffected.

[11] In some cases, we do not report the welfare loss from setting $\tau_K = 0$, because there is no $\tau_L < 1$ to maintain revenue neutral.

When we vary the worker elasticity of matching to take alternative values $\beta = \{0.235, 0.54, 0.72\}$ used by Hall (2005), Hall and Milgrom (2008) and Shimer (2005), respectively, the optimal capital tax rate ranges from 10% to 23%, all significantly higher than zero. The higher the worker elasticity of matching is, the more important workers contribute to labor-market matching. As a consequence, labor taxation becomes more distortive and, eventually, when worker elasticity of matching is above a threshold level (β about 0.56), the optimal tax mix features a shift from labor to capital taxation.

Result 2. *Within a reasonable parameter range, the optimal tax mix always features a shift from capital to labor taxation compared to the pre-existing tax rates and the optimal capital tax rate is always positive, regardless of the relative magnitude of the bargaining share to the household* (ζ) *to the lab*or share in matching production (β).

When the leisure parameter m is 50% above its benchmark value, the optimal tax mix still features a shift from capital to labor taxation: $(\tau_K^*, \tau_L^*) = (21.10\%, 18.77\%)$, but such a shift is much smaller than the benchmark case. When the leisure parameter is 50% below its benchmark value, the optimal tax mix becomes: $(\tau_K^*, \tau_L^*) = (9.34\%, 30.47\%)$, featuring a larger shift from capital to labor taxation but still with a significantly positive tax on capital income. Notably, when m is sufficiently large in magnitude, for example twice as large as its benchmark value ($m = -0.064 \cdot 2 = -0.128$), the direct effect of labor taxation on leisure is so strong that the detrimental effect of a higher labor tax on the marginal benefit of the household in a wage bargain is larger than that of a higher capital tax. Due to its greater distortion on the wage discount, labor taxation becomes more harmful to welfare and the optimal tax mix in this case turns out to feature a shift from labor to capital taxation: $(\tau_K^*, \tau_L^*) = (25.01\%, 14.03\%)$.[12]

When the labor-market tightness measure θ is 10% higher than its benchmark value, the labor market is less tight to workers. As workers become less vulnerable to labor taxation, it is better to tax them. The optimal tax mix therefore features a larger shift from capital to labor taxation: $(\tau_K^*, \tau_L^*) = (2.25\%, 35.00\%)$. On the contrary, when θ is 10% lower, workers become more vulnerable to labor taxation and the optimal tax mix turns out to feature a shift from labor to capital taxation: $(\tau_K^*, \tau_L^*) = (32.78\%, 1.23\%)$. As one can see, when the labor-market tightness measure is further away from its benchmark value, the optimal tax mix will feature complete elimination of either capital taxation (with much less tightness to workers) or labor taxation (with much greater tightness to workers).

Our quantitative results are not too sensitive to either the unemployment compensation-market wage ratio $\bar{b}$ or the capital share of human capital accumulation γ. When $\bar{b}$ is 10% higher, it is required that the government raises both tax rates in order to maintain a balanced budget. Relatively speaking, however, the overall distortion of τ_L reduces because of better insurance provision against the unemployment state. Therefore, the optimal tax mix becomes: $(\tau_K^*, \tau_L^*) = (16.03\%, 24.12\%)$,

[12] We shall relegate the discussion of the inelastic leisure case ($m = 0$) to the next section.

which features a marginally larger shift from capital to labor taxation. When γ is 10% higher, more capital is required for human capital accumulation. Since education/learning is fully tax-exempt, the overall distortion of τ_K is lower. In this case, the optimal tax mix is: $(\tau_K^*, \tau_L^*) = (16.44\%, 23.75\%)$, featuring a marginally smaller shift from capital to labor taxation.

Finally, when pre-existing tax rates take the values used by Judd (1987) at $(\tau_K, \tau_L) = (35\%, 20\%)$ with a much higher capital tax rate initially, the optimal tax mix turns out to be very close to the pre-existing mix: $(\tau_K^*, \tau_L^*) = (34.97\%, 20.04\%)$, featuring a quantitatively negligible shift from capital to labor taxation. When both of the pre-existing tax rates take higher values $(\tau_K, \tau_L) = (40\%, 36\%)$ as used by Lucas (1990), the optimal factor tax mix becomes: $(\tau_K^*, \tau_L^*) = (42.75\%, 32.08\%)$, now featuring a small shift from labor to capital taxation. In both cases, it is still optimal to tax capital as in the benchmark economy and in the latter case replacing labor by capital taxation actually enhances welfare. Because the pre-existing factor tax distortions are almost optimal, the welfare gains from the respective tax reforms are very small.

6 Alternative Setups

In this section, we consider three alternative setups, labeled as Models I-III, that may potentially favor a higher tax imposed on labor income. The first two are on the household side while keeping the firm's optimization problem unchanged, whereas the last is a Walrasian model where there is no labor market friction. Table 4 summarizes the main tax incidence results.

6.1 Model I: Inelastic Leisure

In the benchmark model with endogenous labor-leisure choice, labor-related decisions become more elastic, implying that the tax on labor income is more distortionary than the case with inelastic leisure. While this *labor participation response* is tied to the extensive margin of leisure, just how important such a channel is to the optimal tax mix outcome is a quantitative matter.

With inelastic leisure, we take $m = 0$. By performing tax incidence analysis (see the results reported in Row 1, Table 4), we find that the optimal tax mix (τ_K^*, τ_L^*) is now at $(9.05\%, 31.33\%)$, featuring a sizable shift from capital to labor taxation (though the optimal capital tax rate is still far above zero). This suggests that, by removing the extensive margin of the labor-leisure trade-off, taxing labor becomes quantitatively much less harmful. In this case, a tax reform will lead to a nonnegligible welfare gain of 0.20% (in consumption equivalence).

Result 3. *The optimal capital tax is positive even by removing the extensive margin of the labor-leisure trade-off. By removing such a trade-off, however, it is optimal to shift more tax burden to labor income.*

6.2 Model II: Linear Human Capital Accumulation Function

In the benchmark case, we assume that human capital and physical capital are both required for human capital accumulation. Now we consider an alternative setup of human capital formation where only human capital is used as an input (the Lucasian human capital formation). One can think of this as a special case of (2) with $\widetilde{D} = 0$ and $s = 1$, that is,

$$h_{t+1} - h_t = Dn_t(1 - \ell_t)h_t$$

The modified optimization and BGP conditions are presented in the Appendix. In this case, the calibrated value of D is fairly close to the benchmark setup ($D = 0.0182$), whereas the calibrated bargaining share to household parameter is moderately higher ($\zeta = 0.3254$). Recall that human capital production is fully tax-exempt. When market goods (physical capital) are no longer inputs to human capital accumulation, the entirety of physical capital must be subject to taxation. As a consequence, the overall distortion of τ_K rises and the optimal tax mix now features a larger shift from capital to labor taxation: $(\tau_K^*, \tau_L^*) = (4.99\%, 46.68\%)$, which generates a larger welfare gain of 1.5407% (in consumption equivalence), compared to the benchmark case. Our results imply that elimination of the interactions between physical and human capital in the process of human capital accumulation tends to lower the distortion of labor taxation relative to capital taxation. Nonetheless, the optimal tax mix still features a positive capital tax rate even under this simple Lucasian form of human capital accumulation.

Result 4. *Under a simple Lucasian form of human capital accumulation, the optimal capital tax is still positive but at a lower rate than in the benchmark economy.*

6.3 Model III: Walrasian Economy

To highlight the role played by labor-market frictions, we investigate the tax incidence outcome in a frictionless Walrasian economy with full employment. By construction, $n = 1$ and hence there is no extensive margin of leisure (i.e., $m(1 - n) = 0$). The modified optimization and BGP conditions are presented in the Appendix. By comparing it with the optimal tax mix result in our benchmark case, the role of labor-market frictions can be identified. Specifically, we find that the optimal tax mix becomes: $(\tau_K^*, \tau_L^*) = (0\%, 27.51\%)$, which restores the Lucasian policy recommendation – the optimal tax mix in the Lucas (1990) case is $(\tau_K^*, \tau_L^*) = (0\%, 46\%)$ based on higher pre-existing tax rates $(\tau_K, \tau_L) = (40\%, 36\%)$. Thus, even in a human capital-based endogenous growth model, one should replace capital taxation fully by labor taxation *if the labor market is frictionless.* This verifies our intuition: it is the labor-market frictions that lead to a different dynamic factor tax incidence conclusion from previous studies.

Result 5. *In a model with a Walrasian frictionless labor market, it is optimal to fully eliminate capital taxation by taxing only labor income.*

7 Concluding Remarks

In this paper, we have developed a human capital-based endogenous growth framework with labor market search and matching frictions that permit individuals to participate in the labor force voluntarily. By conducting dynamic factor tax incidence exercises, we have found that it is never optimal to set the capital tax rate to zero when both physical and human capital are used as inputs of human capital accumulation. We have shown that, in the benchmark case with physical capital entering the human capital accumulation process and with a pre-existing flat rate of 20% on both capital and labor income, a partial shift from capital to labor taxation maximizes social welfare – this main finding is robust to different parameterization as well as to alternative setups with inelastic leisure or with a Lucasian human capital accumulation process that is independent of market goods (physical capital). Our results suggest that, in order to enhance social welfare, a proper tax reform must take into account labor market frictions. When such frictions are substantial, fully replacing capital with labor income taxation can be welfare-retarding.

For future research along these lines, it is perhaps most interesting to incorporate a pecuniary vacancy creation cost that requires capital financing. In the presence of credit market frictions as a result of private information, such a financing constraint is anticipated to increase the capital tax distortion. On the contrary, one may also extend the model to allow the separation rate to depend on on-the-job learning effort (as in Mortensen 1988). Since the labor income tax discourages on-the-job learning, it is anticipated that such a generalization may cause the labor tax to be more distorted. Thus, both extensions call for a revisit of dynamic factor tax incidence exercises: while the former may favor a shift from taxing capital to taxing labor income, the latter may yield opposite policy outcomes. Furthermore, since physical capital is less elastic in the short run, it is expected that by measuring welfare along the entire transition path, the optimal (flat) capital tax rate is even higher than the figure obtained in the long run. It may be interesting to examine whether the optimal tax mix might then feature a shift from labor to capital taxation. Along these lines, one may also compute optimal Ramsey taxation to investigate how fast the optimal capital tax rates would fall over time converging to the balanced-growth level.

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Appendix

(A Major Portion of the Appendix is Not Intended for Publication)

In the Appendix, we provide mathematical details of the second-order conditions of household/firm optimization and wage bargaining, the concavity of household/firm value functions, the centralized solution by coordinating labor matching and wage bargain, as well as the Alternative Models II (linear human capital accumulation) and III (Walrasian).

Second-Order Conditions

The second-order conditions of firm's optimization with respect to v and k are (to ease notation burden, we carry time subscript t only for perpetually growing variables):

$$\frac{\partial^2 \Gamma(n_t)}{\partial (v_t)^2} = \frac{\eta^2}{1+\bar{r}} \Gamma_{nn}\left(n'\right) < 0$$

$$\frac{\partial^2 \Gamma(n_t)}{\partial (k_t)^2} = \frac{s}{x_t} \alpha (\alpha - 1) A \left(\frac{s}{n_t x_t}\right)^{\alpha-1} (k_t)^{\alpha-2} < 0$$

which hold automatically under our functional form specifications.

The second-order conditions of household's optimization with respect to c, ℓ and s are:

$$\Omega_{cc}(\mathcal{H}) = U_{cc} + \frac{1}{1+\rho} \Omega_{kk}(\mathcal{H}') < 0$$

$$\begin{aligned}\Omega_{\ell\ell}(\mathcal{H}) = \frac{nh}{1+\rho} \{ &(1-\tau_L) w \left\{ \Omega_{kk}(\mathcal{H}')(1-\tau_L) wnh - \left[\Omega_{kh}(\mathcal{H}') + \Omega_{hk}(\mathcal{H}')\right] \left[D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right] nh \right\} \\ &- \Omega_h(\mathcal{H}') \widetilde{D} \gamma (1-\gamma) \left(q^H\right)^{\gamma-1} q^{H\prime}(\ell) + \Omega_{hh}(\mathcal{H}') \left[D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right]^2 nh \} < 0\end{aligned}$$

$$\begin{aligned}\Omega_{ss}(\mathcal{H}) = \frac{k}{1+\rho} \{ &(1-\tau_K) rk \left[\Omega_{kk}(\mathcal{H}')(1-\tau_K) r - \left[\Omega_{kh}(\mathcal{H}') + \Omega_{hk}(\mathcal{H}')\right] \widetilde{D} \gamma \left(q^H\right)^{\gamma-1}\right] \\ &- \Omega_h(\mathcal{H}') \widetilde{D} \gamma (\gamma - 1) \left(q^H\right)^{\gamma-2} q^{H\prime}(s) + \Omega_{hh}(\mathcal{H}') k \left[\widetilde{D} \gamma \left(q^H\right)^{\gamma-1}\right]^2 \} < 0\end{aligned}$$

which also hold under our functional form specifications and parameterization in the benchmark model.

Finally, we turn to the second-order condition of wage bargaining. From (36), it is easily see that $MB_{ww} < 0$ and $MC_{ww} > 0$, thus assuring the second-order condition: $d(MB_w - MC_w)/dw < 0$.

Concavity of Household and Firm Value Functions

The concavity of the value function $\Gamma(n_t)$ in firm's optimization is easily confirmed as:

$$\frac{\partial^2 \Gamma(n_t)}{\partial (n_t)^2} = \alpha(1-\alpha) A \left(q^F\right)^{\alpha-1} q^{F\prime}(n) + \frac{(1-\psi)^2}{1+\bar{r}} \Gamma_{nn}\left(n'\right) < 0$$

The concavity of the value function $\Omega(\mathcal{H})$ in household's optimization is not as trivial, as it requires the Hessian matrix of $\Omega(\mathcal{H})$

$$J^{\Omega} \equiv \begin{pmatrix} \Omega_{kk} & \Omega_{kh} & \Omega_{kn} \\ \Omega_{hk} & \Omega_{hh} & \Omega_{hn} \\ \Omega_{nk} & \Omega_{nh} & \Omega_{nn} \end{pmatrix}$$

to be negative semidefinite. We can easily show:

$$\Omega_{kk}(\mathcal{H}) = \frac{1-\delta_k+(1-\tau_K)r}{1+\rho}\{\Omega_{kk}(\mathcal{H}')[1-\delta_k+(1-\tau_K)rs]+\Omega_{kh}(\mathcal{H}')\gamma(1-s)\widetilde{D}\left(q^H\right)^{\gamma-1}\}$$

Under our parameterization in the benchmark model, $\Omega_{kk}(H) < 0$.

By exhaustive manipulations, we have:

$$\Omega_{hh}(\mathcal{H}) = \frac{1}{1+\rho}(TE1+TE2+TE3+TE4)$$

where

$$\begin{aligned}
TE1 &= \Omega_{kk}(\mathcal{H}')\left\{(1\text{-}\tau_L)w[n\ell+(1\text{-}n)\bar{b}]\right\}^2 < 0 \\
TE2 &= \Omega_{hh}(\mathcal{H}')\{1+n(1\text{-}\ell)[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}]\}^2 < 0 \\
TE3 &= \Omega_h(\mathcal{H}')n(1-\ell)\widetilde{D}(1\text{-}\gamma)\gamma\left(q^H\right)^{\gamma-1}q^{H\prime}(h) < 0 \\
TE4 &= [\Omega_{kh}(\mathcal{H}')+\Omega_{hk}(\mathcal{H}')](1\text{-}\tau_L)w[n\ell+(1\text{-}n)\bar{b}]\{1+n(1\text{-}\ell)[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}]\} > 0
\end{aligned}$$

Under our parameterization in the benchmark model, $\Omega_{hh}(\mathcal{H}) < 0$. Additional exhaustive manipulations yield:

$$\Omega_{nn}(\mathcal{H}) = \frac{1}{1+\rho}(TE5+TE6+TE7+TE8+TE9+TE10+TE11)$$

where

$$\begin{aligned}
TE5 &= \Omega_{kk}(\mathcal{H}')[(1\text{-}\tau_L)wh(\ell\text{-}\bar{b})]^2 < 0 \\
TE6 &= \Omega_{hh}(\mathcal{H}')\{(1\text{-}\ell)h[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}]\}^2 < 0 \\
TE7 &= \Omega_h(\mathcal{H}')(1\text{-}\ell)h\widetilde{D}(1\text{-}\gamma)\gamma\left(q^H\right)^{\gamma-1}q^{H\prime}(n) < 0 \\
TE8 &= \Omega_{nn}(\mathcal{H}')(1\text{-}\psi\text{-}\mu)^2 < 0 \\
TE9 &= [\Omega_{kh}(\mathcal{H}')+\Omega_{hk}(\mathcal{H}'](1\text{-}\tau_L)wh(\ell\text{-}\bar{b})(1\text{-}\ell)h[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}] > 0 \\
TE10 &= [\Omega_{kn}(\mathcal{H}')+\Omega_{nk}(\mathcal{H}')](1\text{-}\tau_L)wh(\ell\text{-}\bar{b})(1\text{-}\psi\text{-}\mu) > 0 \\
TE11 &= [\Omega_{hn}(\mathcal{H}')+\Omega_{nh}(\mathcal{H}')]1\text{-}\ell)h[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}](1\text{-}\psi\text{-}\mu) > 0
\end{aligned}$$

Under our parameterization in the benchmark model, $\Omega_{nn}(\mathcal{H}) < 0$. The 2×2 principal minors of J^{Ω} need be all positive and the determinant $\left|J^{\Omega}\right|$ need be negative, which are too complicated to identify clean sufficient conditions; nonetheless, they all hold true under our calibrated benchmark parametrization.

Efficiency

From the decentralized solution, we can differentiate (50) and (51) to obtain:

$$\begin{aligned}
d\Omega_n/dw &= \frac{1+\rho}{\rho+\psi+\mu}\left(1-\bar{b}\right)(1-\tau_L)hU_c \\
d\Gamma_n/dw &= -\frac{1+R}{\psi+R}
\end{aligned}$$

The cooperative Nash wage bargaining therefore implies:

$$\begin{aligned}
\Omega_n &= -\frac{\zeta}{1-\zeta}\frac{d\Omega_n/dw}{d\Gamma_n/dw}\Gamma_n \\
&= -\frac{\zeta}{1-\zeta}\frac{d\Omega_n/dw}{d\Gamma_n/dw}\frac{1+R}{\psi+R}\left[(1-\alpha)A\left(q^F\right)^{\alpha}-w\right]
\end{aligned}$$

The above expression can be combined with (50) to yield:

$$
\begin{aligned}
(1+\rho)\left[\left(1-\bar{b}\right)(1-\tau_L)whU_c - m\right] &= -\frac{\zeta}{1-\zeta}\frac{d\Omega_n/dw}{d\Gamma_n/dw}\frac{(1+R)(\rho+\psi+\mu)}{\psi+R}\left[(1-\alpha)A\left(q^F\right)^{\alpha}-w\right] \\
&= \frac{\zeta}{1-\zeta}(1+\rho)\left(1-\bar{b}\right)(1-\tau_L)hU_c\left[(1-\alpha)A\left(q^F\right)^{\alpha}-w\right]
\end{aligned}
$$

which can be simplified to (52).

From the centralized solution, we can substitute (22) into (54) and (55) to obtain:

$$
\begin{aligned}
A(1-\alpha)(q^F)^{\alpha}hU_c - m &= \frac{\rho+\psi+\beta B(1-n)^{\beta-1}(v)^{1-\beta}}{(1-\beta)B(1-n)^{\beta}(v)^{-\beta}}\phi A(q^F)^{\alpha}n\ell hU_c \\
&= \frac{\rho+\psi+\beta\mu}{(1-\beta)\eta}\phi A(q^F)^{\alpha}n\ell hU_c
\end{aligned}
$$

which can be simplified to (56).

Alternative Model II: Linear Human Capital Accumulation

In the case with a linear human capital accumulation process independent of market goods, the first-order condition of the household's optimization problem (5) is the same while (6) becomes:

$$
\Omega_k(\mathcal{H}')(1-\tau_L)w = \Omega_h(\mathcal{H}')D
$$

The Benveniste-Scheinkman conditions of the household's optimization problem are now:

$$
\Omega_k(\mathcal{H}) = \frac{1}{1+\rho}\Omega_k(\mathcal{H}')[(1-\delta_k)+(1-\tau_K)r]
$$

$$
\Omega_h(\mathcal{H}) = \frac{1}{1+\rho}\left\{\Omega_k(\mathcal{H}')(1-\tau_L)w[n\ell+(1-n)\bar{b}]+\Omega_h(\mathcal{H}')[1+Dn(1-\ell)]\right\}
$$

$$
\Omega_n(\mathcal{H}) = -m+\frac{1}{1+\rho}\left[\Omega_k(\mathcal{H}')\left(\ell\text{-}\bar{b}\right)(1\text{-}\tau_L)wh+\Omega_h(\mathcal{H}')D(1\text{-}\ell)h+\Omega_n(\mathcal{H}')(1\text{-}\psi\text{-}\mu)\right]
$$

The BGP equilibrium expressions follow by simply setting $\widetilde{D}=0$ and $s=1$.

Alternative Model III: Walrasian Model

We consider a Walrasian economy with $n=1$. Let $q_t^H = \dfrac{(1-s_t)k_t}{(1-\ell_t)h_t}$ and $q_t^F = \dfrac{s_t k_t}{\ell_t h_t}$. Then the firm's optimal decisions are:

$$
\begin{aligned}
\alpha A\left(q_t^F\right)^{\alpha-1} &= r_t \\
(1-\alpha)A\left(q_t^F\right)^{\alpha} &= w_t
\end{aligned}
$$

Combining these, we have:

$$
q_t^F = \frac{\alpha w_t}{(1-\alpha)r_t}
$$

The household faces the following budget constraint:

$$
k_{t+1} = (1-\tau_L)w_t\ell_t h_t + [(1-\delta_k+(1-\tau_K)r_t]k_t - c_t + T_t
$$

The main change is the Benveniste-Scheinkman condition with respect to h:

$$\Omega_h(\mathcal{H}) = \frac{1}{1+\rho}\left\{\Omega_k(\mathcal{H}')(1\text{-}\tau_L)w\ell+\Omega_h(\mathcal{H}')\left[1+(1\text{-}\ell)\left[D+\widetilde{D}(1\text{-}\gamma)\left(q^H\right)^{\gamma}\right]\right]\right\}$$

By imposing a log utility function $U(c) = \ln c$, we can derive the following equations along the BGP:

$$\rho + (1+\rho)g = \left[D + \widetilde{D}(1-\gamma)\left(q^H\right)^{\gamma}\right]$$

$$\ell = 1 - \frac{g}{D + \widetilde{D}\left(q^H\right)^{\gamma}}$$

The Keynes-Ramsey relationship (27) and (11) remain unchanged. The effective consumption along a BGP becomes:

$$\frac{c}{h} = (1-\tau_L)w\ell + \left[(1-\tau_K)r - \frac{g+\delta_k}{s}\right] q^F \ell + \frac{T}{h}$$

Table 1: Benchmark Parameter Values and Calibration

Benchmark Parameters and Observables		
per capita real economic growth rate	g	0.0045
physical capital's depreciation rate	δ_k	0.0100
time preference rate	ρ	0.0125
tax rate on capital	τ_K	0.2000
tax rate on income	τ_L	0.2000
unemployment insurance	$\bar{b}$	0.4200
capital's share	α	0.2800
physical capital-human capital ratio	k/h	1.0000
job separating rate	ψ	0.0986
job finding rate	μ	0.8336
vacancy-searching worker ratio	v/u	1.0000
labor searcher's share in matching production	β	0.4000
parameter of human capital accumulation	γ	0.2500
preference parameter of leisure	ε	0.5000
Calibration		
coefficient of goods technology	A	0.1648
coefficient of matching technology	B	0.8336
capital-output ratio	k/y	8.2790
consumption-human capital ratio	c/h	0.0598
effective flow profit redistribution	π/h	0.0159
transfer-human capital ratio	T/h	0.0103
fraction of physical capital devoted to goods production	s	0.9981
effective capital-labor ratio in the nonmarket sector	q^H	0.0079
effective capital-labor ratio in the market sector	q^F	1.5394
coefficient of the cost of vacancy creation and management	ϕ	3.6306
coefficient of human capital accumulation	D	0.0178
coefficient of human capital accumulation	$\tilde{D}$	0.0017
rate of return of capital	r	0.0338
fraction of time devoted to employment	n	0.8943
fraction of time devoted to work	ℓ	0.7250
leisure preference parameter augmented by the intensity of leisure enjoyment	m	-0.0640
bargaining share to the household	ζ	0.4000
vacancy creation	v	0.1057
employee recruitment rate	η	0.8336
wage	w	0.0380
wage discount	Δ	0.7161

Table 2: Numerical Results (τ_K=20%, τ_L=20%)

Key Variables	**Benchmark**	τ_K increases	τ_L increases
g	**0.004500**	-0.020797	-0.008461
c/h	**0.059771**	0.008626	0.008231
k/h	**1.000000**	-0.357480	0.002187
y/h	**0.120552**	-0.103532	0.001378
s	**0.998055**	-0.000407	0.000616
n	**0.894259**	-0.008813	-0.001134
$1-n$	**0.105741**	0.074383	0.009590
ℓ	**0.725000**	0.004200	0.001959
$(1-\ell)n$	**0.245921**	-0.019891	-0.006299
q^H	**0.007908**	-0.129685	-0.309929
q^F	**1.539406**	-0.353275	0.001979
r	**0.033820**	0.254361	-0.001425
$(1-\tau_K)r$	**0.027056**	-0.003501	-0.001425
w	**0.038001**	-0.101448	0.022356
$(1-\tau_L)w$	**0.030401**	-0.101448	-0.235506
Δ	**0.716148**	0.001002	-0.008648
μ	**0.833625**	-0.083196	-0.010724
η	**0.833625**	0.055464	0.007150
ν	**0.105741**	-0.064277	-0.008284

Note: Numbers reported are elasticities with respect to respective exogenous shift in tax rates.

Table 3: Sensitivity Analysis

	τ_K^*	τ_L^*	$(g^*-g)/g$	$(\Omega^* - \Omega)/\Omega$	Welfare gain in consumption equivalence	Welfare loss in consumption equivalence if $\tau_K = 0$
Benchmark	**16.11**	**24.09**	**0.2025**	**0.0158**	**0.0389**	**0.6490**
$\beta = 0.235$	10.12	30.28	1.0003	0.1166	0.2879	0.2881
$\beta = \zeta = 0.40$	16.11	24.09	0.2025	0.0158	0.0389	0.6490
$\beta = 0.54$	19.62	20.40	0.0102	0.0001	0.0003	0.8985
$\beta = 0.72$	22.93	16.88	-0.0145	0.0077	0.0190	1.1525
$m = -0.064*0.5$ (-0.032)	9.34	30.47	0.4542	0.0966	0.2367	0.1672
$m = -0.064*1.5$ (-0.096)	21.10	18.77	-0.0652	0.0015	0.0037	1.3947
$m = -0.064*2$ (-0.128)	25.01	14.03	-0.3255	0.0350	0.0876	2.4131
$\theta = 0.9$	32.78	1.23	-0.9782	0.2617	0.6717	5.6679
$\theta = 1.1$	2.25	35.00	0.7049	0.2927	0.7049	0.0109
$\bar{b} = 0.42*0.9$ (0.378)	16.21	24.04	0.2662	0.0148	0.0366	0.6497
$\bar{b} = 0.42*1.1$ (0.462)	16.03	24.12	0.1368	0.0166	0.0410	0.6511
$\gamma = 0.25*0.9$ (0.225)	15.74	24.48	0.2176	0.0186	0.0459	0.6109
$\gamma = 0.25*1.1$ (0.275)	16.44	23.75	0.1886	0.0134	0.0329	0.6857
$k/h = 0.5$	16.11	24.09	0.2025	0.0123	0.0389	0.6491
$k/h = 2$	16.11	24.09	0.2025	0.0220	0.0389	0.6490
$\tau_K = 35\%$, $\tau_L = 20\%$	34.97	20.04	0.0030	0.0000	0.0000	N/A
$\tau_K = 40\%$, $\tau_L = 36\%$	42.75	32.08	-0.3308	0.0254	0.0552	N/A

Note: Numbers reported are in percentage.

Table 4: Tax Incidence Analysis under Various Setups

	τ_K^*	τ_L^*	$(g^*-g)/g$	$(\Omega^* - \Omega)/\Omega$	Welfare gain in consumption equivalence	Welfare loss in consumption equivalence if $\tau_K = 0$
Benchmark	**16.11**	**24.09**	**0.2025**	**0.0158**	**0.0389**	**0.6490**
I. Inelastic leisure	9.05	31.33	0.39929	0.0833	0.2049	0.1194
II. Linear HCA	4.99	46.68	1.1434	0.5578	1.5407	0.3115
III. Walrasian	0.00	27.51	5.4285	4.5347	10.3581	0.0000

Note: Numbers reported are in percentage.

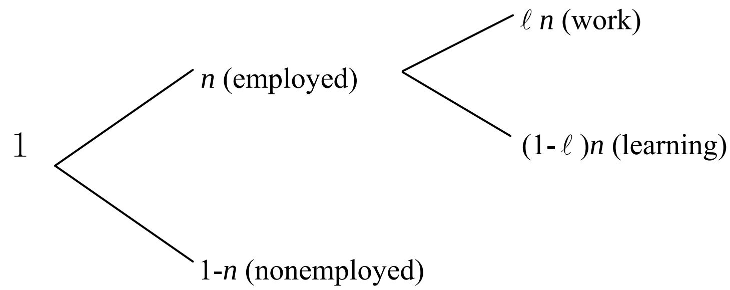

Figure 1: Labor Allocation by Households

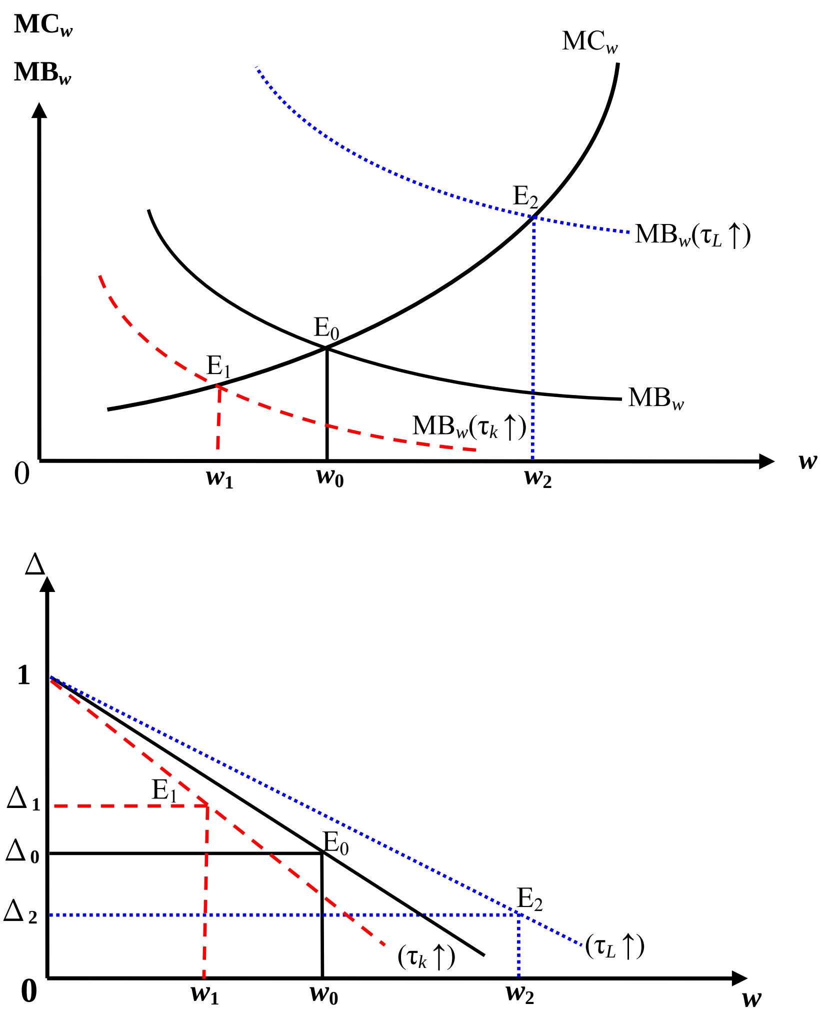

Figure 2: Effects of Factor Taxes on Wage Bargaining: Higher τ_K or τ_L

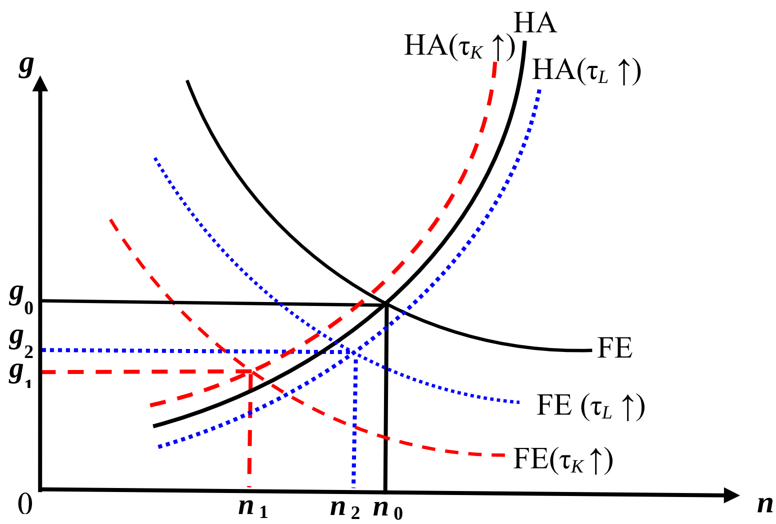

Figure 3: Growth Effects of Factor Taxes

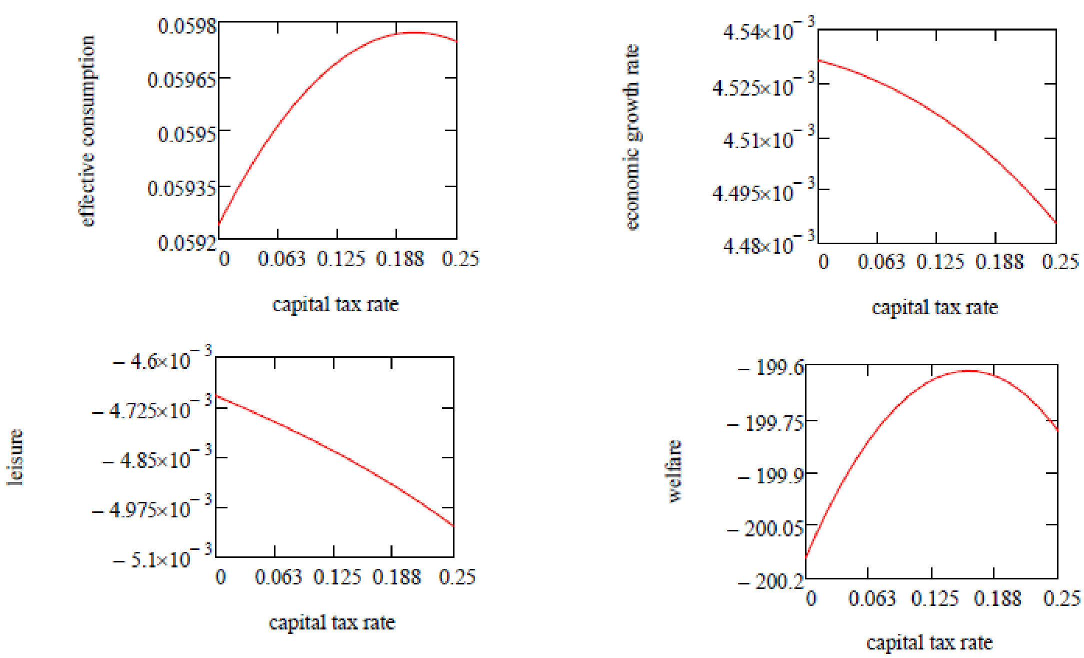

Figure 4: Dynamic Tax Incidence Results