1 Algebra: Solutions to the Problems

(A.) Equations. Solve the following equations for x, or answer the question given:

1. $3x + 7 = 13$ solution: $x = 2$
2. $(3x - 5x + 8x)/2 = 24$ $x = 8$
3. $3x - 5x + 8x/2 = 24$ $x = 12$
4. $ax - bx + xy/c = z$ $x = \frac{z}{a-b+y/c} = \frac{cz}{(a-b)c+y}$
5. $\frac{5x+6}{2} = \frac{9+x}{5}$ $x = -12/23$
6. $\frac{5x+6}{2} + \frac{3x-1}{7} = \frac{9+x}{5}$ $x = -74/191$
7. $3 \cdot \sqrt{x+1} + 3 = 5$ $x = -5/9$
8. $(x+3)^2 = 36$ $x = 3$ or $x = -9$
9. $\sqrt[3]{4x-9} = 3$ $x = 9$
10. $|x+4| = 5$ $x = 1$ or $x = -9$
11. $(x-1)(x-2) = 0$ $x = 1$ or $x = 2$
12. $x^2 - 3x + 2 = 0$ $x = 1$ or $x = 2$
13. $2x^2 + 2x - 24 = 0$ $x = -\frac{1}{2} \pm \frac{7}{2} = -4$ or 3
14. $\frac{x^2}{2} = -8 - 4x$ $x = -4$
15. Does $x^2 - 4x + 5 = 0$ have any solutions in real numbers? If so, give one. If not, explain why.
    - Answer: there is no solution in real numbers because the radicand in the quadratic formula, $b^2 - 4ac = 4^2 - 4(1)(5)$, is negative.
16. Solve for y: $x^2 + y^2 = r^2$ $y = \pm\sqrt{r^2 - x^2}$
17. $10^x \cdot 10^{2x} = 1$ million $x = 2$
18. $\log_{10} x = 7$ $x = 10^7$
19. $\log_{10}(xy) = a$ $x = 10^a/y$
20. $\log_x(y) = a$ where $x > 0$ $x = \sqrt[a]{y}$
21. $y = A^x B^{1-x}$ $x = \frac{\log y - \log B}{\log A - \log B} = \frac{\log(y/B)}{\log(A/B)}$

22. Write each of the following expressions in the simplest possible form:

(a) $(-x^4y^2)^2$ x^8y^4 or $(x^2y)^4$

(b) $\frac{x^4}{x^3}$ x

(c) $y^7y^6y^5y^4$ y^{22}

(d) $\frac{2a/7b}{11b/5a}$ $\frac{10a^2}{77b^2}$

(e) x^0 1

(f) $\frac{n!}{(n-1)!}$ n

23. (Exercise 1.3.1 in P&R) Solve simultaneously for x and y:

$$\begin{array}{rcrcr} x & + & 2y & = & 3 \\ 2x & - & 3y & = & 13 \end{array}$$

- $x = 5$, $y = -1$. From the first equation, $x = 3 - 2y$; so substitute $3 - 2y$ for x in the second equation, and solve it to get $y = -1$. Then substitute -1 for y back in the first equation to get $x = 5$.

24. (Exercise 1.3.3 in P&R) Solve simultaneously for x and y:

$$\begin{array}{rcrcrcr} x & + & 2y & + & 2z & = & 1 \\ 2x & - & 2y & + & z & = & 2 \\ x & - & y & + & 3z & = & 3 \end{array}$$

- $x = 1/5, y = -2/5, z = 4/5$.

25. (Exercise 1.3.2 in P&R) Show that the following system of equations in x and y, where c is a constant, has no solution unless $c = 1/2$. For the case $c = 1/2$, describe what values of x and y solve the system.

$$\begin{array}{rcrcr} 2x & - & 5y & = & c \\ 4x & - & 10y & = & 1 \end{array}$$

- Solving the first equation for x gives $x = c/2 + 5y/2$; substituting this for x in the second equation and solving, the y disappears from the equation, leaving just $c = 1/2$. This indicates that the two equations cannot both be satisfied simultaneously if $c \neq 1/2$.
- If $c = 1/2$, the first equation is satisfied provided, for any given y, that $x = \frac{1+10y}{4}$. Likewise, the second equation is satisfied for any y provided $x = \frac{1+10y}{4}$.

(B.) Inequalities. Transform each of the following inequalities into an equivalent inequality, or combination of inequalities, having x alone on one side:

1. $\frac{x}{-y^2} \geq 1$ — $x \leq -y^2$
2. $ax + b \geq c$ — $x \geq \frac{c-b}{a}$ if $a > 0$; $x \leq \frac{c-b}{a}$ if $a < 0$;
   any x if $a = 0$ and $b \geq c$;
   no such x if $a = 0$ and $b < c$
3. $x^2 > 4$ — $x > 2$ or $x < -2$
4. $x^2 \leq 0$ — $x = 0$ [or x imaginary, that is, $x = a + b\sqrt{-1}$, with $a = 0, b \neq 0$]
5. $xy \leq y$ — $x \leq 1$ if $y > 0$; $x \geq 1$ if $y < 0$; any x if $y = 0$
6. $\frac{1}{x^2} < 100$ — $|x| > 1/10$, that is, $x > 1/10$ or $x < -1/10$
7. $\frac{1}{-x^2} < -y^2$ — $|x| < 1/|y|$
8. $x^2 + y^2 \leq 4$ — $|x| \leq \sqrt{4 - y^2}$ if $|y| \leq 2$; otherwise never
9. $2x^2 + 2x - 24 < 0$ — $-4 < x < 3$