# 2 Sets: Solutions to the Problems

1. Use set-theoretic notation to describe each of the following:

   (a) Some non-numerical sets:

   - the Three Stooges
     {Larry, Moe, Curly}
   - the Baltic States
     {Estonia, Latvia, Lithuania}
   - U.S. Presidents elected in the 1800s after Reconstruction
     {Garfield, Cleveland, Harrison, Cleveland, McKinley}
     = {Garfield, Cleveland, Harrison, McKinley}

   (b) possible totals from the roll of two dice
   $\{2, \ldots, 12\}$ or $\{n : n = x + y \text{ where } x, y \in \{1, 2, 3, 4, 5, 6\}\}$

   (c) possible outcomes of the roll of two dice

   $$\{(m, n) : m, n \in \{1, 2, 3, 4, 5, 6\}\} = \{(1, 1), (1, 2), \ldots, (6, 6)\}$$

   or, if you don't distinguish the two dice,

   $$\{(m, n) : m, n \in \{1, 2, 3, 4, 5, 6\} \text{ with } m \leq n\}$$

   (d) all positive even numbers
   $\{2, 4, 6, \ldots\}$ or $\{n = 2k : k \in \{1, 2, \ldots\}\}$

   (e) all positive proper fractions in simplest terms

   $$\{m/n \;:\; m, n \in \{1, 2, \ldots\}; \text{ and } m < n; \text{ and}$$
   $$\text{there are no positive integers } k, r, s \text{ such that } m = rk \text{ and } n = sk\}$$

   (f) the real numbers between $-1$ and 1, inclusive
   $[-1, 1]$ or $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$

   (g) the positive real numbers no greater than 7
   $(0, 7]$ or $\{x \in \mathbb{R} : 0 < x \leq 7\}$

   (h) the set of all subsets of $\{1, 2, 3\}$
   $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

2. For each of the pairs of sets $A$ and $B$ defined below, describe $B \setminus A$:

   (a) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $B = \{6, 7, 9, 11, 12\}$
   $B \setminus A = \{11, 12\}$

   (b) $A = \{1, 2, 3, \ldots, 100\}$, $B = \{1, 3, 5, \ldots, 99\}$
   $B \setminus A = \emptyset$

   (c) $A = \{x \in \mathbb{R} : x > 4\}$, $B = \{x \in \mathbb{R} : x < 10\}$
   $B \setminus A = \{x \in \mathbb{R} : x \leq 4\}$

(d) $A = \{x \in \mathbb{R} : x > 4\}, \quad B = \{x \in \mathbb{R} : x > 10\}$
$B \setminus A = \emptyset$

(e) $A = [0, 2]$, $B = [1, 2]$
$B \setminus A = \emptyset$

(f) $A = (0, 1)$, $B = [0, 1]$
$B \setminus A = \{0, 1\}$

3. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{0, 1, 4, 8\}$, and $C = \{2, 5, 7, 9, 11, 13, 17\}$. Compute each of the following.

   (a) $A \cup B = \{0, 1, 2, 3, 4, 8\}$

   (b) $A \cap B = \{1, 4\}$

   (c) $A \setminus (B \cup C) = \{3\}$

   (d) $A \cap (B \cap C) = \emptyset$

   (e) $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5\}$

   (f) $(A \cap C) \setminus (B \cap C) = \{2, 5\}$

4. Find the cardinality of each set:

   (a) $\#\{1, 2, 3, 4, \ldots, n\} = n$

   (b) $\#\{1, 1, 2, 2, 3, 3, 2, 2, 1, 1\} = 3$

   (c) $\#[0, 1] = \infty$

   (d) $\#[0, 1/2] = \infty$

   (e) the cardinality of the set of all subsets of $\{1, 2, 3\}$ is 8

5. For each $n \in \{1, 2, 3, \ldots\}$, define $A_n = (0, 1 + 1/n)$. Give the simplest description you can of each of the following sets:

   (a) $A_2 = (0, \frac{3}{2})$

   (b) $A_1 \cap A_2 = A_2 = (0, \frac{3}{2})$

   (c) $A_1 \cap A_2 \cap A_3 = A_3 = (0, \frac{4}{3})$

   (d) $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n = A_n$

   (e) $A_1 \cap A_2 \cap A_3 \cap \ldots = (0, 1]$

6. Let A be the set of all integer multiples of 5 and B be the set of all even integers. Compute:

   (a) $A \cup B = \{\ldots, -10, -8, -6, -5, -4, -2, 0, 2, 4, 5, 6, 8, 10, \ldots\}$

   (b) $A \cap B = \{\ldots, -20, -10, 0, 10, 20, \ldots\}$

   (c) $A \setminus B = \{\ldots, -25, -15, -5, 5, 15, 25, \ldots\}$

   (d) $B \setminus A = \{\ldots, -12, -8, -6, -4, -2, 2, 4, 6, 8, 12, \ldots\}$

7. Explain why each of the following is true or false.

   (a) $[0, 100] = \{0, 1, 2, \ldots, 100\}$

      - False since the right-hand side omits all but the integers from the left.

   (b) $\{-1, -2, -3, 1, 2, 3, 0\} = \{-3, -2, -1, 0, 1, 2, 3\}$

      - True since the elements are just written in a different order.

   (c) $\{1, 1, 2, 2, 3, 3, 2, 2, 1, 1\} = \{1, 2, 3\}$

      - True since the left-hand set just redundantly restates some elements.

   (d) $\{7, 8, 9\} \subseteq \{0, 1, 2, \ldots, 100\}$

      - True since the elements on the left are just three members from the list on the right.

   (e) $[7, 9] \subseteq \{0, 1, 2, \ldots, 100\}$

      - False since the right doesn't include the non-integer real numbers between 7 and 9, which are included on the left.

   (f) $[1, 2] \subseteq [0, 2)$

      - false since 2 is included on the left but excluded on the right.