

Exercises on sequences and convergence

For each of the sequences below, write the first four terms of the sequence, and explain whether the sequence converges to a limit, diverges but approaches ∞ or $-\infty$ in the limit, or simply diverges.

If the sequence converges to a limit, give a formula to prove it: that is, derive for any $\epsilon > 0$ a value N_ϵ such that $n > N_\epsilon$ guarantees that corresponding elements of the sequence are all within ϵ of the limit.

For a sequence that diverges to infinity, derive the analogous formula: for any $B > 0$ find N_B so that $n > N_B$ guarantees that those elements of the sequence are above B or below $-B$, as appropriate.

1. $u_n = (-1)^{3n}$
2. $x_n = 2^{-n}$
3. $y_n = \frac{1-n}{1+n}$
4. $z_n = \log_{10} n$