Robust Incentive Contract with Disagreement about Performance Evaluation and Compensation

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Abstract

This paper examines a principal-agent model with moral hazard, where the agent can disagree with the principal about how he should be evaluated and paid, and creates organizational frictions when the actual outcome falls short of what he thinks he deserves. The main result shows that seemingly rigid policies, such as long-term performance appraisals, compressed compensation schemes and seniority-based promotions, are robust tools for providing incentives and mitigating disagreements, especially when the exact cause of the disagreement is not commonly known between contracting parties.

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1 Introduction

Disagreements about employee performance evaluation and compensation abound in everyday workplace. For example, Babcock et al. (1996) documents the difficulty in reaching pay settlement between workers whose self-serving bias attributes successes to themselves but failures to external factors; Bracken et al. (2001) studies the job dissatisfaction that arises from the dispute over subtle aspects of employee performance; Frank (1984) and Frank (1985) suggest the use of flat wage contracts to mitigate the interpersonal comparison between employees with similar task assignment; and Milgrom and Roberts (1992) details how changes in the incentive system can lead to agony and mistrust after mergers and acquisitions.

While modern management has developed various ways to handle the disagreement about employee evaluation and compensation, HR managers still face challenges, mainly because what triggers disagreements — may it be the employee’s psychological bias, subtle information or prior experience — varies greatly across employees and over time, and is not commonly known between contracting parties (Bracken et al. (2001), Schuler and Jackson (2007), Guffey and Loewy (2012)). Thus, it is imperative to develop organizational solutions that achieve desirable outcomes regardless of the exact cause of the disagreement.

The current paper takes a step in this direction. Specifically, I develop a framework for representing the various kinds of disagreements and for investigating their robust implications for firms’ internal organizations. The main result shows that seemingly rigid policies, such as long-term performance appraisal, compressed compensation scheme and seniority-based promotion, are robust tools for achieving incentive provision and disagreement management simultaneously, especially when the exact cause of the disagreement is not commonly known between contracting parties.

To begin with, I consider an otherwise standard principal-agent model with moral hazard, where I use non-contractible signals to represent the performance evaluation that the agent thinks he deserves (henceforth referred to as disagreement points). For most part, I remain agnostic about the stochastic process that drives the formation of the disagreement. Instead, I only assume that the agent compares the incentive pay based on the actual performance evaluation with what he could have earned according

\(^1\)All above described studies cite workforce diversity and the lack of mutual understanding of what triggers disagreements as main challenges for conflict resolution.
to the disagreement point. If the actual incentive pay falls short of the perceived incentive pay, then the agent feels shortchanged. As a result, conflict arises, and each party bears a deadweight loss that increases with the difference between the perceived and the actual incentive pays.

In this setting, successful contract design provides incentives to the agent on the one hand and mitigates the disagreement between contracting parties on the other hand. In one-shot agency relationships, these goals are typically conflicting, as the first one requires the use of high-powered incentive schemes, whereas the second one stipulates the use of flat payment schedules. The main research question of the current paper concerns how we can relax this tension in a robust manner. In a multi-period setting, I propose a test contract, which inflicts a reasonably large penalty on the agent if his long-term performance falls short of a carefully chosen threshold. Under mild regularity conditions, I derive an analytical lower bound for the profitability of the test contract, and demonstrate that this lower bound holds uniformly over all disagreement point processes and converges to the full surplus as the horizon grows to infinity.

This result can be understood in two steps. First, I use the gain-loss asymmetry to derive a robust upper bound for the disagreement cost. Specifically, since disagreement arises from the agent’s feeling of being underpaid, it follows that under the test contract, disagreement arises if the agent fails the test but thinks that he deserves to pass, an event that happens only if the agent fails the test in the first place. As a result, the probability of having disagreements is bounded above by the probability that the agent fails the test, and this is true for all disagreement point processes. Second, by exploiting basic properties of the monitoring technology, I argue that when the horizon is reasonably long, the agent exerts the efficient effort level most of the time and fails the test with a small probability. The overall efficiency loss per period is small, as long as the disagreement cost is not increasing too fast in how underpaid that the agent feels.

The main features of the test contract, namely long-term performance appraisal and payment compression, emerge in contracting situations where the principal does not fully know the disagreement point process and is concerned about the profitability in the worst-case scenario. Formally, I consider a max-min game between the principal and an adversary, where the principal commits to an incentive contract before the adversary specifies the disagreement point process that minimizes the principal’s
expected profit. When the adversary is limited to choosing specific kinds of disagreement point processes, the max-min contract, which coincides with the optimal Bayesian contract, exhibits “local payment compression,” meaning that the per-period expected difference between the incentive pay at and below a certain percentile of the pay distribution is vanishing in the horizon length. When the adversary’s choice set is rich, the max-min contract exhibits “global payment compression,” meaning that the per-period difference between any two percentiles of the pay distribution is vanishing in the horizon length. Together, these results suggest that payment compression will become more salient as we diversify the employees’ socio-economic backgrounds and prior employment histories, or as we disclose information that facilitates the formation of disagreements (e.g., pay ranking). In controlled environments, these predictions can be tested apart from alternative explanations for payment compression such as the agent’s standard attitude towards risks.

The baseline model can be extended to study anticipatory conflicts and promotion system design in the shadow of interpersonal comparisons. One result suggests that if employees envy the achievement of colleagues who work on the same organizational hierarchy, then any merit-based promotion policy is strictly inefficient, whereas a carefully designed seniority-based promotion system is near-efficient. In the online appendix, I extend the baseline model to encompass risk aversion, discounting, as well as general production and monitoring technologies. There I use numerical analyses to assess the performance of the test contract and to evaluate the validity of maintained assumptions.

1.1 Related Literature

Disagreement in organizations Several authors have examined how specific kinds of disagreements can affect the internal organization of firms. For example, Moldovanu et al. (2007) characterizes the optimal contest between agents with social preferences. Eliaz and Spiegler (2013) and Eliaz and Spiegler (2014) attribute pay rigidity and labor market fluctuation to the worker’s lack of motivation when the actual earning falls below his expectation. Herweg et al. (2010) characterizes the optimal static contract when the agent has reference-dependence preference. These authors either abstract away from the incentive problem that arises from imperfect monitoring (e.g., Eliaz and Spiegler, 2014), or work with specific kinds of disagreements in static envi-
In contrast, I allow for a variety of disagreements and demonstrate how carefully designed long-term contracts can provide near-efficient incentives and mitigate disagreements in a robust manner.

**Modeling disagreement** The current analysis is inspired by Hart and Moore (2008), which uses incomplete contract to mitigate players’ disappointment about the outcome of bilateral trade. It also builds upon the vast literature on reference-dependent preference and social preference (see, among many others, Kahneman and Tversky (1979) and Koszegi and Rabin (2006, 2007); Rabin (1993), Fehr and Schmidt (1999), Bolton and Ockenfels (2000)). In contrast to most existing studies, I remain agnostic about the exact cause of the disagreement in order to derive robust implications for the internal organization of firms.

**Efficiency in dynamic agency models** The current result exploits two basic properties of the monitoring technology: *measure concentration* and *informativeness*. As my companion work Li (2014) shows, these conditions are essential for the efficiency property of dynamic agency models with moral hazard. Roughly speaking, measure concentration says that we can *uniformly* bound the probability that a well-behaved long-term test statistic is concentrated around its mean. Meanwhile, informativeness requires that the agent’s long-term effort is near-efficient when the mean of a long-term test statistic is close to its efficient counterpart. Based on these conditions, I derive a robust lower bound for the test contract’s profitability, which holds even if the disagreement point process that affects the agent’s effort choice is non-stationary and exhibits arbitrary history dependence.

Several authors, including Radner (1981), Rubinstein and Yaari (1983) and Radner (1985), have used theories of large sample statistics to establish efficiency results in dynamic agency models. Radner (1981) and Rubinstein and Yaari (1983) examine a different question, that of whether exerting the efficient effort all the time is

2 Several authors, including Fehr and Schmidt (2007) and Cohn et al. (2012), have tested how social preference affects labor supply in labs or fields. However, these authors abstract away from the issue of imperfect monitoring in order to focus on the trade-off between social preference and monetary reward.

3 Wage disputes are also common in reality. See Bewley (1999), Card et al. (2012), Clark and Oswald (1996) and Mas (2006) for empirical evidence.

an $\epsilon$-best-response to long-term performance tests. Therefore, their results cannot be immediately used for equilibrium characterization or welfare analysis. Radner (1985) establishes a Folk Theorem in infinitely repeated agency games with high discount factors. However, his construction, which hinges on the recursiveness of the environment, cannot be immediately applied to the current setting, where the problem faced by the principal lacks recursive structures.

**Robust incentive contract** This paper adds to a growing literature on robust incentive contract, most of which concerns the contract’s worst-case performance when the principal does not fully know the probability distribution or the stochastic process of the agent’s output (see, for example, [Chassang (2013), Carroll (2015), Antic (2015)]). Chassang (2013) examines a dynamic model where the principal does not know the stochastic process of the agent’s output, and uses techniques in robust control to construct a limited-liability contract that approximates the performance of linear contracts. In contrast, I assume that the principal may not know the disagreement point process, and use the gain-loss asymmetry and tools in finite sample statistics to construct a near-efficient contract for all disagreement point processes.

**Contracting with risk-averse agent** The current analysis predicts that long-term performance appraisal and payment compression will become more salient as we diversify the employees’ socio-economic backgrounds and prior employment histories, or as we disclose information that facilitates the formation of disagreements. In controlled environments, we can distinguish these predictions from alternative explanations for long-term performance appraisal and payment compression such as the agent’s standard attitude towards risks (e.g., Spear and Srivastava (1987)). To facilitate comparisons, I will present my results first and then discuss the relationship between these papers in Section 3.

**Contracting with subjective performance evaluation** Several authors, including Levin (2003), Fuchs (2007), Chan and Zheng (2011) and Maestri (2012), study contracting models where the principal observes a private signal (or subjective evaluation) that is informative of the agent’s effort, with the aim of minimizing the deadweight loss that is needed for inducing truth-telling from the principal. Maestri (2012) allows the agent to observe a private signal that is potentially correlated with the principal’s signal. In this setting, he obtains a totally different result: if players’ private
signals are conditionally independent given the agent’s effort, then an efficiency wage contract that fires the agent for poor long-term performances is near-efficient (this is essentially [Fuchs (2007)](#)); however, if private signals are correlated given the agent’s effort, then the efficiency wage contract is strictly inefficient, whereas a bonus contract that periodically varies the incentive payment with players’ reports is near-efficient. In the current paper, deadweight loss arises when the agent’s self-perception differs from the objective performance evaluation; and the efficiency wage contract is shown to be asymptotically efficient, regardless of whether the disagreement point is public or private information, and whether it is conditionally independent or correlated with the objective performance evaluation.

The remainder of this paper proceeds as follows: Section 2 introduces the baseline model and establishes the main result; Section 3 examines the result’s robust optimal features and testable implications; Section 4 investigates several extensions of the baseline model; Section 5 concludes. See Appendix A for omitted proofs, as well as the online appendix for additional results.

2 Baseline Model

2.1 Setup

Players A principal (she) and an agent (he) are both risk-neutral and interact for finite $T$ periods without discounting. In each period $t = 1, \cdots, T$, the agent privately exerts one of the efforts in $A = \{0, 1\}$ at a cost $c(a_t)$ and yields an expected revenue $a_t$ to the principal. Throughout, assume that $1 > c(1) = c > c(0) = 0$, and hence that the high effort is efficient. At the end of period $t$, a contractible signal $X_t$ that evaluates the agent’s period-$t$ performance is publicly realized. $X_t$ takes values in $S = \{H, L\} \subset \mathbb{R}$ and depends only on $a_t$. The probability of obtaining a high performance evaluation conditional on $a_t$ is given by $p(a_t)$, where $1 > p(1) > p(0) > 0$.

Incentive Contract An incentive contract $\Psi_T = \{\psi_{t,T} : S^t \to \mathbb{R}\}_{t=1}^T$ consists of a sequence of contingent payment schemes. At the outset, the contract becomes binding

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6The baseline model is kept deliberately simple for illustrative purpose. The online appendix investigates an extension that encompasses risk aversion, discounting, as well as general production and monitoring technologies.
if the agent prefers to participate rather than to pursuing his outside option (normalize payoff to zero). At the end of each period $t = 1, \cdots, T$, the contract pays the agent $\psi_{t,T}(X^t)$ based on the history of performance evaluations $X^t = (X_1, \cdots, X_t)$.

**Disagreement** At the end of period $t$, players also observe the realization of a non-contractible signal $Y_t \in S$ that represents the period-$t$ performance evaluation that the agent thinks he deserves.\(^7\) Henceforth I shall refer to $Y_t$ as the period-$t$ disagreement point, and use $I(T) = \{ \{ X_t, Y_t : a^T \} : a^T \in A^T \}$ to denote the collection of signal processes that are generated by all feasible $T$-period effort profiles. Assume for now that $I(T)$ is commonly known between contracting parties.

In reality, $Y_t$ differs from $X_t$ for various reasons:

**Example 1 (Self-Serving Bias).** If the agent suffers from self-serving biases (Babcock et al. (1996)), then in the most extreme case he always thinks highly of himself, i.e., $Y_t = H, t = 1, \cdots, T$.

**Example 2 (Subtle (private) Information).** Sometimes the principal ignores hard-to-verify information that affects the agent’s performance (e.g., whether a customer is difficult to deal with), resulting in disputes about the appropriateness of the performance evaluation (Bracken et al. (2001)). In case the subtle information is privately observed by the agent, $Y_t$ represents a (truthful or non-truthful) public report of the agent’s private observation.

**Example 3 (Interpersonal Comparison).** If agent $i$ compares his evaluation with that of colleague $-i$ who has a similar task assignment (Frank (1984) and Frank (1985)), then $Y_{i,t}$ can depend on $a_{-i,t}, X_{-i,t}, a_{i,t}$ and $X_{i,t}$.

**Example 4 (Inter-temporal Comparison).** If the agent cares about how he is evaluated now relative to the past, then $Y_t$ can depend on $a_t, a_s, Y_s, s < t$.

**Example 5 (Expectation).** If the agent aims to be above the $100q^{th}$ percentile of the pay distribution, where $q \in [0, 1]$ depends on his expectation or prior experience (Milgrom and Roberts (1992)), then $Y_t$ equals the signal realization that attains this payment level.\(^8\)

\(^7\) Under the assumptions that $Y_t$ is non-contractible and parties have commitment power (these assumptions will be further discussed at the end of this section), assuming that $Y_t$ is publicly observable is without loss of generality.
**Disagreement cost**  At the end of period $t = 1, \ldots, T$, the agent compares his actual incentive pay $\psi_{t,T} (X^t)$ with what he could have earned $\psi_{t,T} (Y^t)$ if he were evaluated at $Y^t$ instead. If $\psi_{t,T} (X^t) < \psi_{t,T} (Y^t)$, then the agent feels shortchanged. Therefore conflict arises, resulting in deadweight losses that take the form of excessive disputes, low morale, prolonged agony and undermined trust (see Section 1 for concrete examples).

Formally, let $\Lambda_i (\chi_t (X^t, Y^t))$ be the deadweight loss that player $i = p, a$ bears, where

$$\chi_t (X^t, Y^t) = [\psi_{t,T} (Y^t) - \psi_{t,T} (X^t)]^+$$

measures how underpaid that the agent feels, whereas $\Lambda_i : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $\Lambda_i(0) = 0$ and is increasing in its argument. The *social disagreement cost* is defined by the sum of deadweight losses across agents, i.e.,

$$\Lambda (\chi_t (X^t, Y^t)) = \sum_{i=p,a} \Lambda_i (\chi_t (X^t, Y^t)) .$$

**Strategy and payoff**  Under an incentive contract, the agent’s decision rule $\sigma_T = \{\sigma_{t,T} : A^{t-1} \times S^{2(t-1)} \to \Delta(A)\}$ maps the history of efforts, performance evaluations and disagreement points to the current effort choice. The optimal decision rule is defined by

$$\sigma_T^* \in \arg \max_{\sigma_T} \sum_{t=1}^T \mathbb{E} [\psi_{t,T} (X^t) - c(a_t) - \Lambda_a (\chi_t (X^t, Y^t)) \mid \sigma_T; I(T)] .$$

A contract satisfies the agent’s participation constraint if

$$\sum_{t=1}^T \mathbb{E} [\psi_{t,T} (X^t) - c(a_t) - \Lambda_a (\chi_t (X^t, Y^t)) \mid \sigma_T^*; I(T)] \geq 0 .$$

And it yields the following expected profit to the principal:

$$\sum_{t=1}^T \mathbb{E} [a_t - \psi_{t,T} (X^t) - \Lambda_p (\chi_t (X^t, Y^t)) \mid \sigma_T^*; I(T)] .$$

**Key assumptions**  The current analysis makes the following assumptions:
The principal plays an indispensable role in assessing the agent’s performance. This assumption holds in large companies where performance appraisal demands resources and expertise, whereby the commonly believed trick, that of selling the company to the agent, cannot be used to eliminate the disagreement cost.

Disagreement points are non-contractible. According to numerous studies on human resource management (see, for example, Baron and Kreps (1999), Bracken et al. (2001), Schuler and Jackson (2007) and Guffey and Loewy (2012)), while modern management has opening channels for employees to express disagreements and frustrations, these sentiments are seldom reflected in the ultimate pay, mainly because the exact cause of the disagreement — may it be complex combinations of psychological biases (Examples 1 and 3), non-verifiable information (Example 2) and prior experience/expectation (Example 5) — varies greatly across employees and over time, and cannot be easily assessed by the manager throughout the duration of employment. Formally, this means that the principal lacks precise knowledge about the disagreement point process, let alone to contract effectively on disagreement points.

For this reason, I shall henceforth impose no assumption on the disagreement point process, and focus instead on those contract that yield robust profit guarantees against all disagreement point processes.

Gain-loss asymmetry. This assumption is supported by the vast literature on loss aversion and social preference. For simplicity and without loss of generality, I ignore the agent’s emotional gain from feeling overpaid.

For simplicity, I assume for now that the disagreement about the period-t outcome occurs only in period t. This assumption is supported by the aforementioned HR studies, which find that most disagreements occur shortly after the outcome of performance appraisal is released. Section 4.1 relaxes this assumption by introducing a random variable $Z_t \in \{0, 1\}$ that indicates whether the agent is in a conflict mood or not. Such generalization enables the investigation of interesting extensions, such as anticipatory and reactive conflicts.

The reduced-form disagreement cost best represents the friction between members of the organization. Section 2.3.2 discusses how we can analyze other kinds of deadweight losses (e.g., early quitting) within the current framework.
• Players have commitment power. Section 2.3.2 examines the impact of early termination and contract renegotiation within the current framework.

2.2 Benchmark: Static Case

This section illustrates the tension between incentive provision and disagreement management in static environments. Specifically, let $T = 1$ and drop the notation for $t$. For each $a \in A$, let

$$\rho(a) = \mathbb{P}(X = L, Y = H \mid a; I(1))$$

denote the probability that the agent perceives himself to be under-evaluated, given his effort $a$. The next assumption is maintained throughout this section. Roughly speaking, it says that the probability of having disagreements varies moderately with changes in the agent’s effort, and that the disagreement cost borne by the agent is not increasing too fast in how underpaid he feels.

Assumption 1. There exists $z^* > 0$ such that $z - \frac{\rho(1) - \rho(0)}{p(1) - p(0)} \Lambda_a(z) \geq \frac{c}{p(1) - p(0)}$ if and only if $z \geq z^*$.

Under Assumption 1, I characterize the optimal incentive contract in two steps. First, notice that the optimal contract that elicits the low effort pays zero wage and triggers no disagreement. Second, I characterize the optimal contract that elicits the high effort, and prescribe conditions for the high effort to be more profitable than the low effort. The next lemma formalizes the discussion so far.

Lemma 1. Under Assumption 1, the optimal contract that elicits the high effort satisfies $\psi(H) - \psi(L) = z^*$ and yields an expected profit $1 - c - \rho(1) \Lambda(z^*)$. Thus eliciting the high effort is optimal if and only if $1 - c \geq \rho(1) \Lambda(z^*)$.

Lemma 1 illustrates the tension between incentive provision and disagreement management in static settings. On the one hand, incentive provision is achieved by rewarding the agent a bonus $z^*$ for delivering a high performance. On the other hand, this creates disagreements whenever the agent delivers a low performance but thinks that he deserves a high evaluation. When the expected disagreement cost $\rho(1) \Lambda(z^*)$ exceeds the productivity gain from eliciting the high effort, the principal prefers to elicit the low effort using a flat-wage contract instead.
2.3 Dynamic Case

This section demonstrates how we can relax the tension between incentive provision and disagreement management in dynamic settings. Specifically, fix any $T \in \mathbb{N}$ and consider the test contract $(\varphi_T, b_T, \overline{\psi}_T, \underline{\psi}_T)$ which, at the end of period $T$, tests if the sample test statistic

$$\varphi_T(X^T) = \frac{1}{T} \sum_{t=1}^{T} X_t$$

exceeds the following threshold:

$$\mathbb{E}[X|a = 1] - b_T,$$

where

$$b_T = T^{-\frac{1}{2}+\epsilon} \text{ for some arbitrary } \epsilon \in \left(0, \frac{1}{2}\right).$$

If the answer is affirmative, then the agent passes the test and earns a high incentive pay $\overline{\psi}_T$. Otherwise he fails the test and earns a low incentive pay $\underline{\psi}_T$. The difference between $\overline{\psi}_T$ and $\underline{\psi}_T$, denoted by $B_T$, is given by

$$B_T = \alpha cT \text{ for some arbitrary } \alpha > 1.$$

In reality, depending on whether players have (unmodeled) future interactions or not, the period-$T$ penalty can represent either a reduction in the agent’s immediate consumption or a decrease in his share of the future surplus. The magnitude of this penalty depends on how fast the test statistic converges to its mean. The online appendix conducts numerical analyses that address this issue.

For each $i = p, a$, let

$$\lambda_{i,T} = \frac{\Lambda_i(B_T)}{T}$$

denote the disagreement cost that player $i$ bears per period. Define

$$\kappa = \frac{1}{(p(1) - p(0))(H - L)}$$

and

$$\mu_T = \exp \left( - \frac{2Tb_T^2}{(H - L)^2} \right),$$
and notice that $\mu_T \sim o(b_T)$ as $T \to \infty$.\footnote{Throughout, the Landau notations $O(\cdot)$, $o(\cdot)$ and $\Theta(\cdot)$ are used to denote “at most the order of,” “smaller than the order of” and “exactly the order of,” respectively.} The next assumption, which holds if $\Lambda_a(\alpha cT)$ is not increasing exponentially fast in $T$ and if $\Lambda_p(\alpha cT) \sim o\left(T^{\frac{1}{2} - \epsilon}\right)$, is maintained throughout this section.\footnote{It is easy to check that Assumptions 1 and 2 are compatible in many situations.}

**Assumption 2.** $\lim_{T \to \infty} \max\{\lambda_{a,T}\mu_T, b_T\} \cdot (1 + \lambda_{p,T}) = 0$.

The main result of this paper prescribes a robust lower bound for the per-period expected profit for the test contract against all disagreement point processes, and shows that this lower bound converges to the full surplus as $T \to \infty$. Formally, let $\mathcal{F}$ denote the event where the agent fails the test, and we have the following result.

**Theorem 1.** Take any $\epsilon \in (0, \frac{1}{2})$ and $\alpha > 1$. For each $T \in \mathbb{N}$ and $I(T)$, the optimal decision rule $\sigma_T^*$ induced by $\left(\varphi_T, b_T, \overline{\psi}_T, \underline{\psi}_T\right)$ yields the following outcomes:

(i) $\mathbb{P}(\mathcal{F} | \sigma_T^*) \leq 2\mu_T + \pi_T$, where

$$\pi_T \leq \frac{(\alpha + 2 + \lambda_{a,T}/c)\mu_T + 2\kappa b_T(1 - 2\mu_T)}{\alpha - 1 + 2\kappa b_T}.$$

(ii) The per-period expected profit is bounded below by

$$(1 - \pi_T - 2\mu_T)(1 - 2\kappa b_T) - [c + (\alpha c + \lambda_{a,T})\mu_T] - \lambda_{p,T}\mathbb{P}(\mathcal{F} | \sigma_T^*; I(T)).$$

Take an arbitrary sequence $\{I(T)\}_{T=1}^{\infty}$ that satisfies Assumption 2. As $T \to \infty$,

(a) $\mathbb{P}(\mathcal{F} | \sigma_T^*; I(T)) \sim O(\max\{\lambda_{a,T}\mu_T, b_T\})$;

(b) The per-period expected profit is $1 - c - O(\max\{\lambda_{a,T}\mu_T, b_T\} \cdot (1 + \lambda_{p,T}))$.

Parts (i) and (ii) of Theorem prescribe a lower bound for the principal’s per-period expected profit under the test contract. In particular, $(1 - \pi_T - 2\mu_T)(1 - 2\kappa b_T)$ is a lower bound for the per-period expected revenue, $c + (\alpha c + \lambda_{a,T})\mu_T$ is an upper bound for the per-period expected payment to the agent, and $\lambda_{p,T}\mathbb{P}(\mathcal{F} | \sigma_T^*; I(T))$ is an upper bound for the principal’s share of the disagreement cost. In the online appendix, I derive similar bounds in a richer model and use numerical analyses to
assess the test contract’s performance under reasonable assumptions about model parameters.

The test contract conducts long-term performance appraisals and exhibits payment compression. In the next section, I argue that these features should emerge in optimal contracting situations where the principal does not fully know the disagreement point process and is concerned about the profitability in the worst-case scenario. This result, together with Theorem 1, allows us to bound the principal’s loss from not knowing the exact cause of the disagreement, and to compare the gain from writing a detailed contract versus the (potential) loss from model misspecification.

2.3.1 Proof Sketch

Theorem 1 can be shown in two steps. First, I use the gain-loss asymmetry to bound the principal’s share of the disagreement cost. Specifically, since disagreement is driven by the feeling of being underpaid, it follows that under the test contract, disagreement arises only if the agent fails the test but thinks that he deserves to pass, an event that happens only if the agent fails the test in the first place. Therefore, the per-period disagreement cost borne by the principal is at most $\lambda_{p,T} \mathbb{P}(\mathcal{F}|\sigma^*_T; I(T))$, and this is true for all disagreement point processes.

I next bound the probability that the agent fails the test. At first glance, this seems like a daunting task, because the agent may be able to pass the test by sheer luck or by fine-tuning his effort choice with past effort and signal realizations. Fortunately, two basic properties of the monitoring technology, namely measure concentration and informativeness, imply that the first kind of deviation is rare, whereas the gain from the second kind of deviation is limited. Together, these conditions imply that the agent will work hard and pass the test most of the time when the penalty for failure is reasonably large.

**Measure concentration** For any test statistic $\varphi : S^T \to \mathbb{R}$ and $\epsilon > 0$, we say that $\varphi$ is $\epsilon$-concentrated around its mean if $|\varphi - \mathbb{E}[\varphi]| < \epsilon$, and that $\varphi$ is $\epsilon$-semi-concentrated around its mean if $\varphi - \mathbb{E}[\varphi] > -\epsilon$. The next lemma, proven by Hoeffding (1963), shows that for any independent (but not necessarily identical) random variables $Z_1, \cdots, Z_T$ taking values in $S$, the probability that the sample test statistic $\varphi_T(Z^T)$ is $b_T$-concentrated (resp. $b_T$-semi-concentrated) around its mean is bounded below by $1 - 2\mu_T$ (resp. $1 - \mu_T$).
Lemma 2 (Hoeffding (1963)). Fix any $T \in \mathbb{N}$ and any independent random variables $Z_1, \cdots, Z_T$ that are defined on $S$. Then $\mathbb{P} \left( \left| \varphi_T (Z^T) - \mathbb{E} \left[ \varphi_T (Z^T) \right] \right| > b_T \right) > 1 - 2\mu_T$ and $\mathbb{P} \left( \left| \varphi_T (Z^T) - \mathbb{E} \left[ \varphi_T (Z^T) \right] \right| > -b_T \right) > 1 - \mu_T$.

It is worth noting that the probability bounds prescribed by Lemma 2 exhibit universality, meaning that they hold for all independent random variables defined on $S$. This property is crucial for the analysis below.

The implication of Lemma 2 is twofold. First, if the agent exerts the high effort all the time, then the probability that he fails the test is at most $\mu_T$, and the per-period expected loss that he bears (including the effort cost, the disagreement cost and the penalty for failure) is at most $c + \mu_T \cdot (\alpha c + \lambda_{a,T})$. This observation imposes a tight lower bound on the agent’s payoff under the optimal decision rule, since exerting the high effort all the time is a feasible decision rule.

Second, from the assumption that $X_t$ depends only on $a_t$, it follows that under the agent’s optimal decision rule, past and future performance evaluations are correlated only through future effort choices. Hence for any given profile of realized effort choices $a^T, X_1, \cdots, X_T$ are conditionally independent and bounded, and therefore satisfy the inequalities prescribed by Lemma 2. Now since these inequalities hold independently of the signal-generating effort profile, I deduce, without fully characterizing the optimal decision rule, that under this decision rule, the probability that $\varphi_T$ is $b_T$-concentrated around its mean at least $1 - 2\mu_T$. Therefore when $T$ is large, we can focus on the high probability event where $\varphi_T$ is $b_T$-concentrated around its mean without worrying about the exact details of the optimal decision rule.

Informativeness The next lemma shows that the agent’s long-term effort is near-efficient whenever he passes the test in the high probability event where $\varphi_T$ is $b_T$-concentrated around its mean.

Lemma 3. Take any $T \in \mathbb{N}$ and $a^T \in A^T$. If $|\varphi_T (X^T) - \mathbb{E} \left[ \varphi_T (X^T) \right] | < b_T$ and $\varphi_T (X^T) \geq \mathbb{E}[X|a = 1] - b_T$, then $\frac{1}{T} \sum_{t=1}^{T} a_t \geq 1 - 2\kappa b_T$ and $\frac{1}{T} \sum_{t=1}^{T} c(a_t) \geq c \cdot (1 - 2\kappa b_T)$.

Lemma 3 follows from the very definition of efficiency. Specifically, whenever $|\varphi_T (X^T) - \mathbb{E} \left[ \varphi_T (X^T) \right] | < b_T$ and $\varphi_T (X^T) \geq \mathbb{E}[X|a = 1] - b_T$, we have $\mathbb{E}[\varphi_T (X^T) | a^T] \geq \mathbb{E}[X|a = 1] - 2b_T$, or equivalently $\frac{1}{T} \sum_{t=1}^{T} a_t \geq 1 - 2\kappa b_T$. Now since $a = 1$ is efficient, it follows that $\frac{1}{T} \sum_{t=1}^{T} c(a_t) \geq c \cdot (1 - 2\kappa b_T)$, because otherwise
there is a way to generate more surplus than by exerting the efficient level of effort all the time, a contradiction.

Exploiting these uniform probability and payoff bounds yields sharp characterizations for the outcome that the optimal decision rule induces. Under the test contract, there are two kinds of events, depending on whether $\varphi_T$ is $b_T$-concentrated around its mean or not. Lemma 2 says that the first kind of event is rare when $T$ is large. Meanwhile, if the agent fails the test frequently in the second kind of event, then the only way to balance this out is to gain significantly from fine-tuning the effort choice with past effort and signal realizations, a possibility that is ruled out by Lemma 3. Thus when the horizon is reasonably long, it is optimal for the agent to exert the high effort and to pass the test most of the time, from which the other results follow.

2.3.2 Discussions

General model The online appendix investigates a general model where the monitoring technology satisfies measure concentration and informativeness. As my companion paper Li (2014) shows, these conditions are essential for the attainability of asymptotic efficiency in dynamic agency models with moral hazard. Roughly speaking, measure concentration says that we can uniformly bound the probability that a well-behaved sample test statistic is concentrated around its mean, whereas informativeness says that the agent’s long-term effort is near-efficient whenever the mean of the test statistic under the true effort profile is close to its efficient counterpart. Based on the uniform probability and payoff bounds that these conditions prescribe, we can bound the profitability of the test contract, even if the disagreement point process that affects the agent’s effort choice is non-stationary and exhibits arbitrary history dependence.

Two things are noteworthy. First, measure concentration and informativeness are compatible with more sophisticated preferences over risk and time. Second, many, but not all monitoring technologies satisfy both measure concentration and informativeness. In the current setting, these conditions are both satisfied because under the assumption that $\mathcal{S} = \{H, L\}$, there exists a sequence $\{b_T\}_{T=1}^\infty$ such that when $T$ is large, $\varphi_T$ is both $b_T$-concentrated around its mean and reveals the agent’s long-term effort with almost certainty. Li (2014) shows that in agency models with frequent actions, similar results hold for signals that follow Poisson processes, Gamma processes,
etc., but not for signals that follow Brownian motions with a drift. Interested readers should consult the online appendix and Li (2014) for further details.

**Limited liability** In case the agent is protected by limited liability, the profitability of the test contract depends on the magnitude of the penalty for failure. In case the probability and payoff bounds prescribed by Lemmas 2 and 3 converge fast, the test contract attains near-efficiency over a short horizon, suggesting that the penalty for failure — which is proportional to the horizon length — is only moderately large. The online appendix evaluates this statement in two ways. First, I use back-of-the-envelope calculations to estimate how long it takes for the test contract to attain near-efficiency. Second, I examine an infinite-horizon model with frequent actions and Poisson signals, and use the test contract as a building block to construct a near-efficient equilibrium where the agent is penalized by probabilistic relationship severance.

**Early termination and contract renegotiation** In reality, firms may fire the disgruntled employee or renegotiate the contract in order to save the disagreement cost. However, this solution requires that firms keep close track of the employee’s emotional status, which itself is resource consuming. Theorem I enables us to evaluate whether such spending is worthwhile or not.

**Quitting** In reality, the agent may express disagreements through quitting (e.g., at any history $h^t$, the agent considers quitting with a probability that increases with $\mathbb{E} \left[ \chi_{T,T} (X^T, Y^T) | h^t \right]$). But even if this option is available, it is easy to show, based on the uniform probability and payoff bounds prescribed by Lemmas 2 and 3, that under the test contract, the probability of quitting is small as long as the cost of job change is reasonably large.

## 3 Robust Optimal Features

In this section, I argue that the main features of the test contract, namely long-term performance appraisal and payment compression, should emerge in contracting situations where the principal does not fully know the disagreement point process and is concerned about the worst-case profitability. Throughout this section, assume for simplicity that $\Lambda_a(\chi) = 0$ for all $\chi \geq 0$. Consider the following game between the principal, an adversary and the agent:
1. The principal commits to a review contract $\Psi_T$, which specifies (i) the dates $T \subset \{1,2,\cdots,T\}$ for performance reviews and (ii) a collection of contingent payment schemes $\left\{\psi_{t,T} : S^t \to \mathbb{R}\right\}_{t \in T}$, where $t^- = \max\{s \in T \cup \{0\}, s < t\}$ denotes the most recent date for performance review before date $t$.

2. The adversary, whose goal is to minimize the principal’s expected profit, specifies the agent’s disagreement point process $\{Y_t\}$, where each $Y_t$ takes value in $S$ and can depend arbitrarily on $(a_t, X_t, Y_t^-)$.

3. The agent exerts efforts and generates signals throughout $t = 1, \cdots, T$. At each date $t \in T$, the principal pays $\psi_{t,T}(X_t^-)$ based on the performance evaluations within the current review cycle $X_t^- = (X_{t+1}^- , \cdots, X_t)$. The agent compares $\psi_{t,T}(X_t^-)$ with $\psi_{t,T}(Y_t^-)$ and inflict a cost $\Lambda_p(\chi_{t,T}(X_t^-,Y_t^-))$ on the principal.

Suppose that the agent’s strategy is independent of the disagreement point (because $\Lambda_a(\chi) = 0$), and that both the agent and the adversary adopt a deterministic tie-breaking rule in case of indifference. Under these assumptions, write the principal’s problem as follows:

$$\max_{\Psi_T} \min_{\{Y_t\}} \mathbb{E} \left[ \sum_{t=1}^{t=T} a_t - \left[ \psi_{t,T}(X_t^-) + \Lambda_p(\chi_{t,T}(X_t^-,Y_t^-)) \right] \cdot 1_{t \in T} \right],$$

s.t. $\sigma_T$ satisfies (IC) and (IR) with respect to $\Psi_T$.

Denote the solution to this problem by $\{\Psi^m_T, \{Y_t\}^m_T, \sigma^m_T\}$ (see Proposition 1 for existence). For each $q \in [0,1]$ and $t \in T$, let $\psi_{t,m}^m(q; \sigma_T^m)$ denote the 100$q$th percentile of the pay distribution at date $t$.

$$L_T = 2\kappa b_T + (\alpha c + \lambda_{a,T})\mu_T + (\pi_T + 2\mu_T)(1 - 2\kappa b_T + \lambda_{p,T}).$$

denote the upper bound for the per-period efficiency loss that Theorem 1 prescribes for the test contract. The next proposition characterizes the max-min review contract.

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10 Review contracts are commonly used in practice. Limiting attention to this class of contracts greatly simplifies the analysis. The test contract is a special kind of review contract.

11 The gain from learning and contract revision is limited if the adversary’s strategy space is sufficiently rich. For example, if the disagreement point process can vary across review cycles, then what can be learned from the past does not help predict the future.

12 Formally, $\psi_{t,m}(q; \sigma_T^m) \triangleq \max \left\{ x : \mathbb{P}(\psi_{t,T}(X_t^-) \leq x | \sigma_T^m) \geq q, \mathbb{P}(\psi_{t,T}(X_t^-) < x | \sigma_T^m) < q \right\}.$
Proposition 1. For each $T \in \mathbb{N}$, $\{\Psi^m_T, \{Y_t\}^m, \sigma^m_T\}$ exists and satisfies

$$
\frac{1}{T} \mathbb{E} \left[ \sum_{t \in T^m} \Lambda_p \left( \left[ \psi^m_{t,T}(q; \sigma^m_T) - \psi^m_{t,T}(X^t_{t-}) \right]^+ \right) \right] \leq L_T, \quad \forall q \in [0, 1].
$$

Under Assumption 2, the right-hand side of this inequality is vanishing in $T$.

To better understand Proposition 1, notice that Theorem 1, which holds true for all disagreement point processes, remains valid in the current setting. This observation, albeit a simple one, implies that under the max-min review contract, the expected difference between the incentive pay at and below any $100q^{th}$ percentile of the pay distribution is vanishing in $T$. The reason is straightforward: if contrary is true, then the adversary can insist that the agent deserves the $100q^{th}$ percentile of the pay distribution and create a non-vanishing disagreement cost per period. But then the principal strictly prefers the test contract to the max-min review contract, a contradiction.

In the discussion below, suppose $\Lambda_p(\chi) = \chi$ for illustrative purpose, though the result remains true as long as $\Lambda_p(\chi)$ is not increasing too fast in $\chi$.

Implication for optimal Bayesian contract Proposition 1 suggests that when the adversary is limited to choosing specific kinds of disagreement point processes, the max-min contract, which coincides with the optimal Bayesian contract, exhibits “local payment compression,” meaning that the per-period expected difference between the incentive pays at and below a certain percentile of the pay distribution is vanishing in $T$. For example, if the agent demands the highest incentive pay (Example 1), then the optimal contract is compressed at the top of the pay distribution. Meanwhile, if the agent demands the average pay of many other agents (Example 3), or expects to be above $100q^{th}$ percentile of the pay distribution (Example 5), then the optimal contract exhibits compressions at the mean and the $100q^{th}$ percentile of the pay distribution, respectively.

Implication for worker heterogeneity Proposition 1 suggests that payment compression will become increasingly global if the principal, who is concerned about the worst-case profitability, faces an increasingly diverse pool of agents holding different views how they should be evaluated and paid. When the agent can demand
any percentile of the pay distribution, the max-min compensation scheme is “globally compressed,” meaning that the per-period difference between any two percentiles of the pay distribution is vanishing in $T$. Corollarily, the max-min contract must conduct long-term performance appraisals, because any sequence of static contracts that elicits the high effort most of the time (see Section 2.2) does not satisfy global payment compression.

**Testable implications** Results so far suggest that long-term performance appraisal and payment compression will become more salient when we diversify the employees’ socio-economic backgrounds and prior employment histories, or as we disclose information that facilitates the formation of disagreements (e.g., relative pay ranking). In controlled environments, these unique predictions can be distinguished from alternative explanations for long-term performance appraisals and payment compressions, including the agent’s risk aversion (e.g., Spear and Srivastava (1987)) and the subjectivity of the principal’s performance evaluation (e.g., Fuchs (2007)).

4 Extensions

This section investigates two extensions of the baseline model: Section 4.1 introduces anticipatory conflicts to the analysis; Section 4.2 studies promotion system design in the shadow of interpersonal comparisons.

4.1 Anticipatory Conflict

Fix the test contract and an arbitrary decision rule $\sigma_T$. At the end of period $t$, players observe the realizations of $X_t, Y_t$, as well as a state variable $Z_t \in \{0, 1\}$. If $Z_t = 1$, then conflict arises, and each player $i$ bears a cost $\Lambda_i(h^t; \sigma_T)$ that can depend on the entire $t$-period history $h^t = (a^t, X^t, Y^t, Z^{t-1})$ and the agent’s decision rule $\sigma_T$. By letting $Z_t = 1$ when the agent expects to fail the test but thinks that he deserves to pass, we allow for anticipatory conflicts and nest the baseline model as a special case.

The next assumption is maintained throughout this section.

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13 All aforementioned HR studies cite workforce diversity and the lack of mutual understanding between contracting parties as main challenges for conflict resolution. See Rebitzer and Taylor (2011) and the references therein for how pay secrecy helps mitigate interpersonal comparisons.

14 In a related study, Card et al. (2012) discloses the salary ranking among employees of the University of California system, and finds that people below the mean feel significantly worse-off.
Assumption 3. There exist a weakly concave function \( \hat{\Lambda} : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) and \( \beta \in (0, 1) \) such that

(i) \( \Lambda_a (h^t; \sigma_T) \leq \hat{\Lambda} \left( \mathbb{E} \left[ \chi_T \left( X^T, Y^T \right) | h^t; \sigma_T \right] \right) \) and \( \frac{\Lambda_p (h^t; \sigma_T)}{\Lambda_a (h^t; \sigma_T)} \leq \frac{\beta}{1-\beta} \) for all \( t, h^t \) and \( \sigma_T \).

(ii) As \( T \to \infty \),

\[
m_T \triangleq \hat{\Lambda} (B_T \cdot \mu_T) \mathbb{E} \left[ \frac{1}{T} \sum_{t=1}^{t=T} Z_t | a_t \equiv 1 \right] \to 0,
\]

\[
\rho_T \triangleq \frac{1}{T} \sum_{t=1}^{t=T} \text{Cov} \left( \hat{\Lambda} \left( \mathbb{E} \left[ \chi_T \left( X^T, Y^T \right) | h^t; a_t \equiv 1 \right] \right), Z_t | a_t \equiv 1 \right) \to 0.
\]

Part (i) of Assumption 3 is a technical condition that enables us to bound the equilibrium disagreement cost through Jensen’s inequality. Part (ii) of Assumption 3 summarizes the effect of anticipatory conflicts by two parameters (both evaluated at the efficient effort profile): (1) \( m_T \), which measures the average disagreement cost per period, and (2) \( \rho_T \), which measures the correlation between the agent’s emotional state and how underpaid he feels. Two things are noteworthy. First, a big \( \rho_T \) means that conflicts tend to arise when the agent expects himself to fail the test at date \( T \); second, the assumption \( \lim_{T \to \infty} \rho_T = 0 \) can be easily satisfied: specifically, since failure rarely occurs at the efficient effort profile, all we need is that at this effort profile, conflict arises only when the agent is very certain that he will fail the test at date \( T \).

In the current setting, let \( I(T) = \{ \{X_t, Y_t, Z_t : a^T \} : a^T \in A^T \} \) denote the collection of signal processes that are generated by all feasible effort profiles. The agent’s decision rule \( \sigma_T = \{ \sigma_{\ell,T} : A^{t-1} \times \mathcal{S}^{2(t-1)} \times \{0, 1\}^{t-1} \to \Delta(A) \} \) now maps the history of performances, disagreement points and emotional states to the current effort choice. The optimal decision rule is defined by

\[
\sigma^*_T \in \arg \max_{\sigma_T} \frac{1}{T} \mathbb{E} \left[ \psi_{T,T} \left( X^T \right) - \sum_{t=1}^{t=T} \left[ c(a_t) + \Lambda_a (h^t; \sigma_T) \cdot Z_t \right] | \sigma_T; I(T) \right]
\]

The next proposition quantifies the effect of anticipatory conflicts on the test contract.

Proposition 2. Take an arbitrary sequence \( \{I(T)\}_{T=1}^{\infty} \) that satisfies Assumption 3. As \( T \to \infty \), the outcomes induced by \( \left( \varphi_T, b_T, \overline{\psi}_T, \underline{\psi}_T \right) \) satisfy the following properties:
(i) \( \mathbb{P}(\mathcal{F} \mid \sigma^*_I; I(T)) \sim \mathcal{O}(\max\{b_T, m_T, \rho_T\}) \);

(ii) The per-period expected profit is \( 1 - c - \mathcal{O}(\max\{b_T, m_T, \rho_T\}) \).

### 4.2 Promotion System Design in the Shadow of Interpersonal Comparison

**Setup** An infinitely-lived principal owns two junior positions and one senior position. An agent can work at the junior position without pay for at most \( 2T \) periods. In each period \( t \), the agent in position \( i = 1, 2 \) exerts an effort \( a_{i,t} \in A \) and generates a public performance measure \( X_{i,t} \). A vacant junior position is filled immediately with new candidates.

The senior position becomes vacant once every \( T \) periods (henceforth *promotion cycle*). The principal fills this position with either a junior agent or an external candidate, where the second option costs \( K_T \sim \mathcal{O}(T) \) (e.g., search, training, or even nothing). From a junior agent’s point of view, the gain from promotion is given by \( B_T = 2\alpha c T \) for some \( \alpha > 1 \). Therefore, he is willing to work without pay for \( 2T \) periods in order to get promoted.

**Merit-based system** At the end of a promotion cycle, a merit-based system \( (\rho_1(\cdot), \rho_2(\cdot)) \) maps junior agents’ performance measures in the current promotion cycle to a promotion decision, subject to (1) the feasibility constraint, that at most one agent can be promoted, and (2) the anonymity constraint, that the promotion decision is independent of the agent’s identity, i.e.,

\[
(\rho_1, \rho_2) : \mathcal{S}^{2T} \to \{0, 1\}^2,
\text{s.t. } \sum_i \rho_i\left(x^T_i, x^T_j\right) \leq 1, \ \forall x^T_i, x^T_j \in \mathcal{S}^T, \quad \text{(Feasibility)}
\]

\[
\text{and } \rho_1\left(x^T, x'^T\right) = \rho_2\left(x'^T, x^T\right), \ \forall x^T, x'^T \in \mathcal{S}^T. \quad \text{(Anonymity)}
\]

Junior agents engage in interpersonal comparisons. For generality’s sake, I do not fully specify the game played by these agents, but assume instead that (1) disagreements and hence agents’ strategies are memoryless across promotion cycles, and (2) whenever promotion happens, the social disagreement cost is bounded below by \( \Lambda(B_T - cT) \), where \( B_T - cT \) is a lower bound for the payoff difference between the winner and the loser. The solution concept is Bayesian Nash equilibrium.
Lemma 4. Under any merit-based promotion system, the per-period expected efficiency loss incurred by any Bayesian Nash equilibrium $\sigma^m_T$ that is memoryless across promotion cycles is bounded below by

$$\frac{1}{T} \left\{ K_T \mathbb{P}(\rho_1 = \rho_2 = 0|\sigma^m_T) + \Lambda (B_T - c_T) \mathbb{P}(\exists i \in \{1, 2\} \text{ s.t. } \rho_i = 1|\sigma^m_T) + (1 - c) \sum_{t=1}^T \mathbb{E}[2 - a_{1,t} - a_{2,t}|\sigma^m_T] \right\}.$$ 

Notice that this efficiency loss consists of three parts: the cost of external hiring, the deadweight loss from interpersonal comparison, and the productivity loss due to shirking.

Seniority-based system The seniority-based promotion system labels the two junior positions as the “entry-level” and the “junior-level.” A newly-hired agent works at the entry level for $T$ periods and is promoted to the junior level if $\frac{1}{T} \sum_{t=1}^T X_t \geq \mathbb{E}[X|a = 1] - b_T$. He then works for another $T$ periods and is promoted to the senior position if $\frac{1}{T} \sum_{t=T+1}^{2T} X_t \geq \mathbb{E}[X|a = 1] - b_T$. A failed agent is replaced by a more junior person. In case both junior agents fail, he senior position is assumed by an external candidate.

Crucially, I assume that since agents at different levels of the hierarchy do not compete together, they do not engage in interpersonal comparisons and therefore create no organizational frictions.\footnote{By now it should be clear to the reader that even if we allow the failing agent to envy the passing agent, the equilibrium per-period expected disagreement cost is still small, because the probability of failure is small.}

Lemma 5. When $T$ is sufficiently large, there exists a Bayesian Nash equilibrium $\sigma^{s*}_{2T}$ of the seniority-based promotion system where the per-period expected efficiency loss is $O(b_T)$.

Merit-based system vs. seniority-based system The following assumption is maintained in the remainder of this section.

Assumption 4. $\Lambda (B_T - c_T) \gg \Theta \left( T^{\frac{1}{2} + \epsilon} \right)$ as $T \to \infty$.

Proposition 3. Under Assumption 4 when $T$ is large, there is a Bayesian Nash equilibrium of the seniority-based system which yields a higher per-period expected surplus than any memoryless Bayesian Nash equilibrium of any merit-based system.
Intuitively, the merit-based system provides incentives to agents by rewarding the winner and punishing the loser. However, this creates interpersonal comparisons whose damage can only be mitigated through limiting the chance of promotion. But then agents have little incentive to work, resulting in a non-trivial productivity loss per period. In contrast, the seniority-based system uses individual-based promotion to mitigate the tension between incentive and disagreement, yielding a near-efficient outcome when the promotion cycle is reasonably long.

5 Conclusion

I conclude by discussing related issues and suggesting potential avenues for future research. First, in the online appendix, I extend the baseline model to encompass risk aversion, discounting and general production and monitoring technologies. There I bound the profitability of the test contract, use back-of-the-envelope calculations to evaluate this contract’s performance, and demonstrate how we can endogenize the period-$T$ penalty through changes in players’ continuation values.

The analysis so far has focused on the worst-case scenario where the disagreement point process is difficult to learn. In average-case scenarios, characterizing mechanisms that facilitate learning and contract revision is an interesting avenue for future research.

A Mathematical Proofs

Proof of Lemma 1

Proof. Part (i): define $\chi = \psi(H) - \psi(L)$, and rewrite the optimal contract that elicits the high effort from the agent as the solution to the following problem:

$$\min_{\chi} \Lambda_p(\chi), \text{ s.t. } \chi - \frac{\rho(1) - \rho(0)}{p(1) - p(0)} \Lambda_a(\chi) \geq \frac{c}{p(1) - p(0)}.$$  

A casual inspection reveals that $\chi = \chi^*$.

Part (ii): the optimal contract that elicits the low effort pays zero wage and triggers no disagreement. This observation, together with Part (i), implies that high effort is more profitable than low effort if and only if $1 - c \geq \rho(1) \Lambda(\chi^*)$. 

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Proof of Lemma 3

Proof. \( \frac{1}{T} \sum_{t=1}^{T} X_t - \mathbb{E}[\frac{1}{T} \sum_{t=1}^{T} X_t | a^T] < b_T \) and \( \frac{1}{T} \sum_{t=1}^{T} X_t \geq \mathbb{E}[X | a = 1] - b_T \) imply that \( \mathbb{E}[\frac{1}{T} \sum_{t=1}^{T} X_t | a^T] \geq \mathbb{E}[X | a = 1] - 2b_T \), or equivalently that \( \frac{1}{T} \sum_{t=1}^{T} a_t > 1 - 2\kappa b_T \). Now since \( a = 1 \) is efficient, it follows that \( \frac{1}{T} \sum_{t=1}^{T} c(a_t) > c \cdot (1 - 2\kappa b_T) \), because otherwise there is a way to generate more surplus than by exerting the efficient level of effort all the time, a contradiction. \( \square \)

Proof of Theorem 1

Proof. Since \( X_t \) depends only on \( a_t \), it follows that under \( \sigma^*_T \), future performance evaluations depend on past efforts only through future effort choices. Thus for any given profile of realized effort choices \( a^T, X_1, \ldots, X_T \) are bounded and conditionally independent, and hence satisfy the probability bound prescribed by Lemma 2. Since this probability bound holds for all \( a^T \in A^T \), it follows that

\[
 r_T \triangleq \mathbb{P} \left\{ |\varphi_T - \mathbb{E} [\varphi_T | a^T] | < b_T | \sigma^*_T \right\} \geq 1 - 2\mu_T.
\]

Define

\[
 \pi_T = \mathbb{P} \left\{ |\varphi_T - \mathbb{E} [\varphi_T | a^T] | < b_T, \mathcal{F} | \sigma^*_T \right\}
\]

as the probability that the agent fails the test when \( \varphi_T \) is \( b_T \)-concentrated around its mean. Under \( \sigma^*_T \), the agent’s expected loss (i.e., effort cost, penalty, disagreement cost) is bounded below by

\[
 \pi_T \cdot \alpha c T + (r_T - \pi_T) \cdot c(1 - 2\kappa b_T) T + (1 - r_T) \cdot 0,
\]

where (1) is attained if the agent fails the test but exerts no effort when \( \varphi_T \) is \( b_T \)-concentrated around its mean, (2) is attained if the agent passes the test when \( \varphi_T \) is \( b_T \)-concentrated around its mean, and hence incurs at least \( c(1 - 2\kappa b_T) T \) per-period from exerting efforts (Lemma 3), and (3) is attained if the agent passes the test and exerts no effort when \( \varphi_T \) is not \( b_T \)-concentrated around its mean.

Second, notice that if the agent exerts the high effort all the time, then his probability of failure is at most \( \mu_T \), and his expected loss is bounded below by

\[
 c T + \mu_T \cdot (\alpha c T + \Lambda_a(B_T)).
\]
Since this expression weakly exceeds the previous one, it follows that

\[ \pi_T \leq \frac{(\alpha + 2 + \lambda_{a,T}/c) \mu_T + 2\kappa b_T(1 - 2\mu_T)}{\alpha - 1 + 2\kappa b_T}. \]

Part (i): \( \mathbb{P}(\mathcal{F}|\sigma^*_T; I(T)) \leq \pi_T + 2\mu_T. \)

Part (ii): under \( \sigma^*_T \), the principal’s expected revenue per period is bounded below by \( (1 - 2\mu_T - \pi_T)(1 - 2\kappa b_T) \). Meanwhile, she pays at most (i) \( c + \mu_T \cdot (\alpha c + \lambda_{a,T}) \) per period to the agent in order to induce the latter’s participation, as well as (ii) \( \lambda_{p,T} \mathbb{P}(\mathcal{F}|\sigma^*_T; I(T)) \) per period to the disagreement. Combining these bounds yields the result.

Proof of Proposition 1

Proof. Part (i): fix any \( T \in \mathbb{N} \) and drop the notation for \( T \). Let \( V(\Psi) \) denote the principal’s value function under \( \Psi \), given the agent’s optimal choice over finitely many decision rules, as well as the adversary’s optimal choice over finitely many disagreement point processes. Define \( V = \sup_{\Psi} V(\Psi) \) and notice that \( V \) is finite.

Now suppose, to the contrary, that no \( \Psi \) attains \( V \). By definition, there exists a sequence \( \{\Psi^n\}_{n=1}^\infty \) such that \( \lim_{n \to \infty} V(\Psi^n) = V \). Furthermore, since the agent’s choice set is finite, there exists \( n \) such that \( \Psi^n \) induces the same decision rule from the agent for all \( n > n \). But then \( \Psi \) is not attainable only if the disagreement point process is different for all \( n, n' > n \), which contradicts with the fact that the set of disagreement point processes is finite.

Part (ii): suppose, to the contrary, that there exists \( q \in [0, 1] \) such that

\[
\frac{1}{T} \mathbb{E} \left[ \sum_{t \in \mathbb{T}_m} \Lambda_p \left( \left[ \psi^m_{t,T}(q; \sigma^m_T) - \psi^m_{t,T}(X'_t) \right]^+ \right) | \sigma^m_T > L_T \right] > L_T.
\]

Then the adversary can choose this \( q \) and inflict a disagreement cost that equals the left-hand side of the above inequality. But then the principal strictly prefers the test contract to the max-min review contract, a contradiction.

Proof of Proposition 2

Proof. Part (i): let \( 1^T \) denote the \( T \)-period profile of high efforts. Under the optimal decision rule, the agent’s expected loss per period is bounded below by \( \pi_T \cdot \alpha c + (r_T - \pi_T) \cdot c(1 - 2\kappa b_T) \) as before. Under \( 1^T \), his expected loss per period is bounded above
by \( c + \alpha c \mu_T \) + disagreement cost at \( 1^T \), where the last term can be bounded above as follows:

\[
\frac{1}{T} \sum_{t=1}^{T} \sum_{h'} \mathbb{P}(h'|1) \hat{\Lambda}(\mathbb{E}[\chi_T (X^T, Y^T) | h'; 1^T]) \cdot Z_t
\]

\[
= \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{h'} \left[ \hat{\Lambda}(\mathbb{E}[\chi_T (X^T, Y^T) | h'; 1^T]) | 1^T \right] \mathbb{E}_{h'}[Z_t | 1^T] + \rho_T \quad \text{(defn of } \rho_T) \]

\[
\le \frac{1}{T} \sum_{t=1}^{T} \hat{\Lambda} \left( \mathbb{E}_{h'}[\chi_T (X^T, Y^T) | h'; 1^T] \right) \mathbb{E}_{h'}[Z_t | 1^T] + \rho_T \quad \text{(concavity of } \hat{\Lambda}(\cdot))
\]

\[
= \hat{\Lambda} \left( \mathbb{E}[\chi_T (X^T, Y^T) | 1^T] \right) \mathbb{E} \left[ \frac{1}{T} \sum_{t=1}^{T} Z_t | 1^T \right] + \rho_T \quad \text{(defn of conditional expectation)}
\]

\[
\le m_T + \rho_T \quad \text{(defn of } m_T) \]

Together, these results imply that

\[
\mathbb{P}(\mathcal{F} | \sigma^*_T; I(T)) \le \pi_T + 2\mu_T \le \frac{(\alpha + 2)\mu_T + 2\kappa b_T(1 - 2\mu_T) + (m_T + \rho_T)/c}{\alpha - 1 + 2\kappa b_T} + 2\mu_T.
\]

Part (ii): from the optimality of \( \sigma^*_T \) follows that

\[
c(1 - 2\kappa b_T)(1 - 2\mu_T - \pi_T) + \text{the agent’s disagreement cost under } \sigma^*_T \le c + \alpha c \mu_T + m_T + \rho_T.
\]

Under Assumption 3(a), this implies the following upper bound for the disagreement cost that the principal bears under \( \sigma^*_T \):

\[
\frac{\beta c}{1 - \beta} [(\alpha + 2)\mu_T + \pi_T + 2\kappa b_T(1 - \pi_T - 2\mu_T) + (m_T + \rho_T)/c].
\]

Plugging this expression into the per-period expected profit function yields the result.

\[ \square \]

Proof of Proposition 3

Proof. If \( \frac{\Lambda(B_T - cT)}{T} \mathbb{P} \left( \sum_i \rho_i = 1 | \sigma^*_T \right) \gg O(b_T) \), then the merit-based system is less profitable than the seniority-based system. Now suppose \( \frac{\Lambda(B_T - cT)}{T} \mathbb{P} \left( \sum_i \rho_i = 1 | \sigma^*_T \right) \sim \]

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\( O(b_T). \) Since

\[
\frac{1}{T} \sum_{t=1}^{t=T} \mathbb{E}[c(a_t)|\sigma^T_m] \leq \frac{B_T}{T} \mathbb{P}\left( \sum_i \rho_i = 1 | \sigma^*_{T} \right) \leq \frac{A(B_T - cT)}{T \Theta(b_T)} \mathbb{P}\left( \sum_i \rho_i = 1 | \sigma^*_{T} \right),
\]

where the first and second inequality follows from the agent’s participation constraint and Assumption 4, respectively, it follows that \( \frac{1}{T} \sum_{t=1}^{t=T} \mathbb{E}[c(a_t)|\sigma^T_m] \) is vanishing in \( T \), meaning that when \( T \) is large, the merit-based system yields a negligible per-period expected surplus and hence is less profitable than the seniority-based system.

\( \Box \)

**References**


