Fragment multiplicity dependent charge distributions in heavy ion collisions

National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824

A. Botvina**
Institute for Nuclear Research, 117312 Moscow, Russia

M-C. Lemaire and S. R. Souza
Laboratoire National SATURNE, CEN Saclay, 91191 Gif-sur-Yvette, France

G. Van Buren, R. J. Charity, and L. G. Sobotka
Department of Chemistry, Washington University, St. Louis, Missouri 63130

C. Schwarz, U. Lynen, J. Pochodzalla, H. Sann, and W. Trautmann
Gesellschaft für Schwerionenforschung, D-64220 Darmstadt 11, Germany

D. Fox§§ and R. T. de Souza
Department of Chemistry and IUCF, Indiana University, Bloomington, Indiana 47405

N. Carlin
Instituto de Fisica, Universidade de Sao Paulo, CEP 01498, Sao Paulo, Brazil

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The dependence of intermediate-mass-fragment (IMF) element distributions on the multiplicity, \( N_{\text{IMF}} \), of detected fragments has been measured for \(^{84}\text{Kr} + ^{197}\text{Au}\) collisions at \( E/A = 35, 55, 70, 100, 200, \) and \( 400 \) MeV. The observed dependence can be parametrized as \( P(N_{\text{IMF}}|Z) \propto P(Z) \exp(-c \cdot N_{\text{IMF}} \cdot Z) \), where \( c \) is an expansion- and excitation-energy dependent parameter. Previous work indicated this parameter is zero in the liquid-gas coexistence region and positive in the gaseous phase. In contrast, we observe both negative and positive values for \( c \), revealing the meaning of this parameter to be less straightforward than previously assumed. The magnitude of \( c \) appears nonetheless to provide a nontrivial test of multifragmentation models.

Fragment size distributions contain significant information about fragmentation processes [1] and have provided important tests of both dynamical [2, 3] and statistical models [4–7]. In the limit of infinite equilibrated systems, they play a central role in the determination of critical exponents [8–10]. For systems produced in nuclear collision experiments, the distribution of intermediate mass fragments (IMF: \( 3 \leq Z_{\text{IMF}} \leq 20 \)) may, however, be significantly altered by the breakup geometry [11, 12], Coulomb interaction [13], collective expansion [14], and conservation laws [15–17].

Recently, charge conservation effects have attracted considerable interest [15–17] when evidence was found that fragment \( Z \) distributions may display qualitatively different dependences on fragment multiplicity for final states which contain a heavy residue (resembling a mixture of bulk liquid and vapor) as compared to final states which do not contain a heavy residue (resembling a vapor phase). Specifically, fragment \( Z \) distributions were found to exhibit a simple exponential relationship on fragment multiplicity, \( N_{\text{IMF}} \), and element number, \( Z \),

\[
P(N_{\text{IMF}}|Z) \propto P(Z) \exp(-c \cdot N_{\text{IMF}} \cdot Z) \approx \exp[-(\alpha_0 + c \cdot N_{\text{IMF}} \cdot Z)],
\]

where the parameters \( \alpha_0 \) and \( c \) depend on beam and excitation energy. For \( c > 0 \), the \( Z \) distributions become steeper (i.e., the average fragment size decreases) with increasing
values of $N_{\text{IMF}}$; hence the parameter $c$ has been associated [15–17] with the constraints imposed by charge conservation.

The dependence of $c$ on excitation energy $E^*$ has been explored [15–17] by assuming it to be proportional to the detected charged particle multiplicity $N_C$ [16] or, alternatively, the total transverse energy of the detected charged particles, $E_t = \sum E_i \sin \theta_i$ [15,17]. Typically, $c$ was found to be close to zero (but positive) for small $E_t$ and to increase for larger $E_t$ until reaching a saturation value. Values of $c \approx 0$ at low $E_t$ were interpreted [17] as due to the existence of one or more heavy residues in the exit channel (reminiscent of bulk liquid in the coexistence region of the liquid-gas phase diagram) which reduces the importance of charge conservation effects on the fragment $Z$ distribution. Large positive values of $c$ at high $E_t$, on the other hand, were interpreted [17] as due to collisions with no heavy residue in the exit channel (reminiscent of a pure vapor phase) for which charge conservation effects are more important. These qualitative expectations were supported [17] by percolation model simulations which, in addition, indicated that $c \approx 1/Z_0$, where $Z_0$ denotes the size of the emitting source.

In Refs. [15–17], the interplay between excitation energy deposition and charge conservation constraints was explored by cuts on $N_C$ or $E_t$, i.e., by varying the impact parameter at fixed bombarding energy [18,19]. Previous investigations could thus not disentangle an excitation energy dependence from a collision geometry dependence. Alternatively, one may vary the bombarding energy at fixed impact parameter. Both techniques have disadvantages: collision geometries and source sizes vary significantly with impact parameter [18,19], while flow and reaction time scales vary significantly with incident energy [20–24]. To compare the two approaches and separate the common statistical features from those that are peculiar to the specific method for varying the excitation energy, we investigated the beam energy dependence of the parameter $c$ at fixed impact parameter for $^{84}\text{Kr}+^{197}\text{Au}$ reactions at $E/A = 35, 55, 70, 100, 200, \text{and} 400$ MeV.

The measurements at $E/A = 35, 55,$ and 70 MeV were performed at the National Superconducting Cyclotron Laboratory at Michigan State University; those at $E/A = 100, 200,$ and 400 MeV were performed at the Laboratoire National SATURNE at Saclay. Charged particles and IMF’s were detected with the MSU/Washington University Miniball/Miniwall array [25], consisting of 276 low-threshold plastic-scintillator-CsI(Tl) phoswich detectors and covering polar angles of $\theta_{\text{lab}} = 5.4^\circ - 160^\circ$ and a total solid angle of approximately 90% of $4\pi$. The energy thresholds in the wall detectors ($\theta_{\text{lab}} = 5.4^\circ - 25^\circ$) were set to $E_{\text{th}}/A = 7$ MeV (7.5 MeV) for $Z = 3(10)$ particles; corresponding thresholds of the ball detectors ($\theta_{\text{lab}} = 25^\circ - 160^\circ$) were $E_{\text{th}}/A = 2(4)$ MeV. To reduce triggers from low-energy electrons, hardware discriminator thresholds were set at about 20 MeV proton energy for the Miniball and 5 MeV for the Miniball. Unit charge resolution up to $Z = 10$ was routinely achieved for particles that traversed the fast plastic scintillator. More details about the experiment can be found in Ref. [26].

Figure 1 shows the extracted values of $\alpha_0$ of Eq. (1) as a function of $E_i$. For peripheral collisions (small $E_i$), $\alpha_0$ decreases rapidly as $E_i$ increases. This decrease becomes less rapid for more central collisions (large $E_i$). These qualitative trends are similar to those reported in previous work [27,28]. Only the data at $E/A = 400$ MeV deviate from this general behavior and show a local minimum for $\alpha_0$ as a function of $E_i$.

Figure 2 shows the extracted values of $c$ as a function of $E_i$. Except at $E/A = 400$ MeV, $c$ assumes negative values at small $E_i$, passes through zero, and continues to increase rapidly to a gradual plateau at $c \approx 0.02 - 0.03$. Negative values of $c$ are unexpected and have not been previously reported. They are inconsistent with a simple charge conservation argument, since $c < 0$ means that the fragment distributions become flatter with increasing fragment multiplicity. Further, the steady rise from negative to positive values of $c$ makes it necessary for $c$ to pass through zero; this trend, by itself,
does not mandate that values of $c \approx 0$ unambiguously signal the existence of a heavy residue in the exit channel or a mixture of liquid and vapor. Indeed, heavy residues should survive over a broader range of excitation energies; one would therefore expect values $c \approx 0$ over a finite interval of $E_t$. This is, in fact, a prediction of the percolation simulations of Ref. [17]. In contrast, the present data show a rapid transition from $c < 0$ to $c > 0$, indicating that the previous interpretation of the parameter $c$ may be too simplistic [17].

The observed negative values of $c$ may be an artifact of poor event characterization. For peripheral collisions (i.e., small $E_t$), charged particle emission is less important than neutron evaporation in removing excitation energy [29]. Thus a narrow cut on small $E_t$ may not adequately constrain the excitation energy deposition. Since $N_{\text{IMF}}$ grows rapidly as a function of excitation energy, the requirement for larger values of $N_{\text{IMF}}$ needed to determine the parameter $c$, shifts the average excitation energy towards higher values despite the loose constraint imposed by the cut on small $E_t$. Moreover, the rapid decrease of $\alpha_0$ with increasing $E_t$ shown in Fig. 1 means that the $Z$ distributions become flatter with increasing excitation energy. Therefore, flatter $Z$ distributions at higher values of $N_{\text{IMF}}$ (corresponding to $c < 0$) are the expected consequence of any bias towards higher average excitation energy imposed by the gates on larger values of $N_{\text{IMF}}$ required to extract $c$. As the accuracy of event characterization increases and the sensitivity of $\alpha_0$ decreases for larger $E_t$, this bias on excitation energy imposed by selecting on $N_{\text{IMF}}$ should become less important. Thus at larger values of $E_t$, charge conservation constraints should become predominant, producing values $c > 0$. In this scenario, values of $c \approx 0$ have no particular significance.

This possible impact-parameter bias from additional $N_{\text{IMF}}$ constraints at low $E_t$ can be explored by studying azimuthal $\alpha-\alpha$ correlations which are known to become more isotropic with increasing excitation energy [30,31] and/or decreasing impact parameter [19]. Figure 3 shows azimuthal $\alpha-\alpha$ correlations for the $^{84}\text{Kr}+^{197}\text{Au}$ reaction at $E/A = 35$ MeV selected by $E_t < 100$ MeV and the additional cuts $N_{\text{IMF}} = 1, \ldots, 6$. Consistent with previous work [19,30,31], the azimuthal $\alpha-\alpha$ correlations exhibit “V” shapes with enhanced emission at $\Delta \phi = 0^\circ$ and $180^\circ$, reflecting preferential emission in the reaction plane. With increasing $N_{\text{IMF}}$, the azimuthal $\alpha-\alpha$ correlations become more isotropic, corroborating the existence of the additional bias towards smaller average impact parameters and/or larger average excitation energy deposition imposed by gates on higher values of $N_{\text{IMF}}$.

Charge conservation effects may be better studied in central collisions where impact parameter fluctuations and differences in collision geometry may be reduced. For this purpose, we define a reduced impact parameter scale from the charged particle multiplicity [18,19,26] and select central collisions by means of the $b = b/b_{\text{max}} < 0.25$. The solid points in Fig. 4 show the parameters $c$ extracted for these collisions at different beam energies. At all energies, the parameter $c$ is positive, and its magnitude increases with incident energy. This trend is consistent with the simultaneous requirement of charge conservation accompanied by a general increase of the charged particle multiplicity, reflecting a monotonic increase in excitation energy deposition, with incident energy. It does not preclude the existence of smaller or vanishing values of $c$ at much lower energies, however.

The dependence of fragment $Z$ distributions on IMF multiplicity may provide meaningful constraints on statistical models which embody charge conservation. As an illustration, we have performed calculations with the statistical multifragmentation model (SMM) of Refs. [7] which assumes bulk fragmentation of an expanded, thermalized system. The model has been rather successful in describing multifragmentation data selected by small impact parameters [32–35].

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**FIG. 3.** Azimuthal $\alpha-\alpha$ correlation functions selected by cuts on $E_t < 100$ MeV and the additional constraints $N_{\text{IMF}} = 1, \ldots, 6$ for the reaction $^{84}\text{Kr}+^{197}\text{Au}$ at $E/A = 35$ MeV.

**FIG. 4.** The left panel shows the parameter $c$ as a function of incident energy. Data and SMM predictions are represented by solid and open points, respectively. The solid line is drawn to guide the eye. The right panel shows the predicted dependence of the parameter $c$ on source size, keeping the other parameters fixed to the values used for the SMM calculations at $E/A = 200$ MeV. The dashed horizontal lines indicate the experimental value of $c$. Details of the SMM calculations are given in Ref. [35] and are summarized in the text.
if one assumes that only a fraction of the composite system is thermalized and that the remainder of the system is emitted in a pre-equilibrium initial stage (not treated in the model). For central collisions, emission should predominantly occur from a single source [36], but some contribution in our data from other sources cannot be excluded. Since our SMM calculations are performed for a single source, they must be viewed as schematic.

Parameters $c$ predicted by SMM calculations are shown as open points in Fig. 4. Details of these calculations are given elsewhere [35]. Briefly, all calculations were filtered with the acceptance of the detector system. Source sizes, temperatures, and collective expansion energies were adjusted [35] to reproduce the average fragment and charged particle multiplicities and the average kinetic energies. The extracted source sizes ($A_{\text{source}} = 183, 167, 158, 143, 106$ for $E/A = 35, 55, 70, 100, 200$ MeV, respectively) decrease for higher beam energies, consistent with increased pre-equilibrium emission; they are much smaller than the combined $^{84}\text{Kr} + ^{197}\text{Au}$ system ($A_{\text{tot}} = 281$). For these sources, the predicted values of $c$ are much larger than observed experimentally, i.e., the model predicts too strong a charge conservation effect.

As illustrated in the right panel of Fig. 4, the predicted values of $c$ decrease for larger sources. For the data at $E/A = 200$ MeV, the experimental value of $c$ indicates a rather large source, $A_{\text{source}} = 240$ (compared to the value $A_{\text{source}} = 106$ from Ref. [35]). For such a large source, the experimental value of the average IMF multiplicity is overpredicted by a factor of 2. The reason for this internal inconsistency of the SMM calculations is not understood. Further theoretical work is needed to clarify whether contributions from other (smaller) projectile and target spectator sources could provide a more consistent description, whether the assumption of a spherical geometry is unrealistic [37–41], whether an explicit accounting for the emission time sequence of particle must be considered [42] or whether dynamical emission mechanisms [43] are important. In any case, the calculations illustrate that model predictions for the parameter $c$ provide additional constraints for models of multifragmentation.

In summary, the dependence of IMF $Z$ distributions on IMF multiplicity, $N_{\text{IMF}}$, has been studied for $^{84}\text{Kr} + ^{197}\text{Au}$ collisions over a broad range of incident energies, $E/A = 35–400$ MeV. Consistent with previous work, the observed $N_{\text{IMF}}$ dependence could be described in terms of Eq. 1. Different from previous work, negative values of the parameter $c$ are observed at small transverse energies. This observation casts doubt on a previous interpretation [17] that values $c = 0$ signal the existence of a large residue in analogy to a liquid-vapor mixture. Instead the low-$E_t$ behavior of the parameter $c$ may arise from the known fact that at low-excitation energies, the IMF $Z$ distributions become flatter as the excitation energy increases, from poor event characterization at low $E_t$, and from the bias caused by additional constraints on fragment multiplicity. Since an improved event characterization may be obtained for central collisions (large $E_t$), we have explored the beam energy dependence of the parameter $c$ for $b < 0.25$. Schematic calculations with the statistical multifragmentation model indicate that the parameter $c$ provides additional constraints on the size of the emitting system. However, we could not provide a consistent description of the observed mean fragment energies and multiplicities and the parameter $c$. Future work will have to clarify whether an improved description can be obtained by allowing for emission from multiple sources or nonspherical decay configurations or whether explicit treatments of the time ordering of emissions or of the dynamics of emission mechanisms must be considered.

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