Chapter 11
A Real Intertemporal Model with Investment
Chapter 11 Topics

• Construct a real intertemporal model that will serve as a basis for studying money and business cycles in Chapters 12-14.
• Understand the investment decision of the firm.
• Show how macroeconomic shocks affect the economy.
• Focus on the implications of future expectations for current macroeconomic performance.
Real Intertemporal Model

- Current and future periods.
- Representative Consumer – consumption/savings decision
- Representative Firm – hires labor and invests in current period, hires labor in future
- Government – spends and taxes in present and future, and borrows on the credit market.
Representative Consumer’s Budget Constraints

Consumer’s current-period budget constraint:

\[ C + S^p = w(h - l) + \pi - T \]

Consumer’s future-period budget constraint:

\[ C' = w'(h - l') + \pi' - T' + (1 + r)S^p \]
Consumer’s Lifetime Budget Constraint

\[ C + \frac{C'}{1 + r} = w(h - l) + \pi - T + \frac{w'(h - l') + \pi' - T'}{1 + r} \]
Marginal Conditions for the Consumer

Current period: \( MRS_{t,c} = w \)

Future period: \( MRS_{t',c'} = w' \)

Intertemporal Choice: \( MRS_{c,c'} = 1 + r \)
Consumer’s Current Labor Supply Behavior

- Current labor supplied increases with the real wage (substitution effects are assumed to dominate income effects).
- Labor supply increases with an increase in the real interest rate, through an intertemporal substitution effect.
- An increase in lifetime wealth (e.g. taxes fall) reduces labor supply.
Figure 11.1
The Representative Consumer’s Current Labor Supply Curve
Figure 11.2
An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right
Figure 11.3
Effects of an Increase in Lifetime Wealth
The Current Demand for Consumption Goods

• MPC = marginal propensity to consume – the increase in demand for consumption goods induced by a one-unit increase in current real income.

• Intertemporal substitution: the demand for consumption goods decreases with an increase in the real interest rate (Chapter 9)

• An increase in lifetime wealth (e.g. taxes fall) increases the demand for consumption goods.
Figure 11.4
The Representative Consumer’s Current Demand for Consumption Goods Increases with Income
Figure 11.5
An Increase in the Real Interest Rate from $r_1$ to $r_2$ Shifts the Demand for Consumption Goods Down
Figure 11.6
An Increase in Lifetime Wealth Shifts the Demand for Consumption Goods Up
Firm’s Current and Future Production Technology

\[ Y = zF(K, N) \]
\[ Y' = z' F(K', N') \]
Evolution of the Firm’s Capital Stock

\[ K' = (1 - d)K + I \]
Firm’s Current and Future Profits

\[ \pi = Y - wN - I \]

\[ \pi' = Y' - w' N' + (1 - d) K' \]
The Firm Maximizes the Present Value of Profits

\[ V = \pi + \frac{\pi'}{1 + r} \]
The Firm’s Labor Demand

- As in Chapter 4, the firm’s labor demand schedule is the marginal product of labor for the firm, which is downward sloping.
Figure 11.7
The Demand Curve for Current Labor Is the Representative Firm’s Marginal Product of Labor Schedule

$w = \text{Current Real Wage}$

$N^d$ = Current Employment
Figure 11.8
The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity $z$ and in the Current Capital Stock $K$
The firm invests to the point where the marginal benefit from investment equals the marginal cost.
Marginal Cost of Investment

The marginal cost of investment is 1, as the firm gives up one unit of current profits for each unit it invests, so:

$$MC(I) = 1$$
Marginal Benefit of Investment

- The marginal benefit of investment is the marginal product of future capital plus the quantity of capital that will be left in the future after depreciation, all discounted back to the present:

\[ MB(I) = \frac{MP_K + 1 - d}{1 + r} \]
Optimal Investment Rule for the Firm

- The firm’s optimal investment rule, obtained by equating the marginal benefit and marginal cost of investment:

\[ MP_K' - d = r \]
Figure 11.9
Optimal Investment Schedule for the Representative Firm
Figure 11.10
The Optimal Investment Schedule Shifts to the Right if Current Capital Decreases or Future Total Factor Productivity Is Expected to Increase
Data for Christine’s Orchard

Table 11.1  Data for Christine’s Orchard

<table>
<thead>
<tr>
<th>$K' = \text{trees in the future}$</th>
<th>$I$</th>
<th>$Y'$</th>
<th>$V$</th>
<th>$MP'_{K} - d$</th>
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<td>7</td>
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</table>
• Suppose many firms in the economy – some good, some bad.
• Bad firms borrow in the credit market, consume the proceeds as executive compensation, then default.
• Lending rate of interest is $r$, loan rate is $r^l$
• Asymmetric information: Lenders cannot distinguish good from bad firms.
Asymmetric Information and Investment

- $r^l - r = x$, where $x$ is a default premium to compensate for borrowers who default.
- $x$ is an interest rate spread that reflects the severity of the asymmetric information problem.
- For firms that borrow to invest,

$$MP_K' - d - x = r$$
Asymmetric Information and Investment

- When the asymmetric information problem worsens, as in the financial crisis, $x$ increases.
- Given the interest rate, this reduces the borrowing firm’s optimal quantity of investment.
Figure 11.11
The Effect of an Increased Default Premium on a Firm’s Optimal Investment Schedule

\[ r = \text{Real Interest Rate} \]

\[ I^d = \text{Demand for Investment Goods} \]

\[ MP_K' - d - x_1 \]

\[ MP_K' - d - x_2 \]
Figure 11.12
Investment and the Interest Rate Spread

![Graph showing investment and interest rate spread over time. The x-axis represents years from 1940 to 2020, and the y-axis represents the percentage deviation from the trend for spread. The graph includes lines for investment and spread, with deviations ranging from -30 to 30%.]
Figure 11.13
Scatter Plot: Investment vs. Interest Rate Spread
The Government’s Present-Value Budget Constraint

\[ G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \]
Figure 11.14
Determination of Equilibrium in the Labor Market
Given the Real Interest Rate $r$
Figure 11.15
Construction of the Output Supply Curve
Figure 11.16
An Increase in Current or Future Government Spending Shifts the $Y^s$ Curve
Figure 11.17
An Increase in Current Total Factor Productivity Shifts the $Y^s$ Curve
Figure 11.18
The Demand for Current Goods

Demand for Current Goods

Slope = MPC

$C^d(r) + I^d(r) + G$

$Y = \text{Current Income}$

$Y_1$

$45^\circ \text{ line}$
Figure 11.19
Construction of the Output Demand Curve

\[ Y = \text{Current Income} \]

(a) Demand for Current Goods

(b) \( Y^d \)

\[ r = \text{Real Interest Rate} \]
What Shifts the Output Demand Curve to the Right?

- A decrease in the present value of taxes.
- An increase in future income.
- An increase in future total factor productivity.
- A decrease in the current capital stock.
Figure 11.20
The Output Demand Curve Shifts to the Right if Current Government Spending Increases
Figure 11.21
The Complete Real Intertemporal Model
The Equilibrium Effects of a Temporary Increase in G

- Question: What is the rightward shift in the output demand curve due to an increase in G?
- Answer: The curve shifts to the right one-for-one with the increase in G.
- The demand multiplier is one.

\[
\frac{Y_2^d - Y_1^d}{G_2 - G_1} = 1
\]
Equilibrium Effects

- Output increases, real interest rate increases, real wage falls, consumption and investment decrease, employment rises.
- Government spending crowds out both consumption and investment.
Figure 11.22
A Temporary Increase in Government Purchases
Important Points

- The total multiplier – the ratio of the equilibrium increase in Y, to the increase in G – must be less than 1.
- In rudimentary Keynesian analysis, the multiplier is greater than 1. Why? That type of analysis does not take account of: (i) how the extra output is to be produced; (ii) the effect of taxes (present and future) on consumption.
Other Important Points

• Extra government spending comes at a cost – crowds out private consumption and investment spending, and we all have to work harder.

• In rudimentary Keynesian analysis, increases in G are costless – in fact Y and C increase as a result. The conclusion seems to be that G should be infinitely large – obviously silly.
A Decrease in the Current Capital Stock, K

- This could arise due to a war or natural disaster.
- Output may rise or fall, depending on how large the output demand effect is relative to the output supply effect.
- The real interest rate rises, the real wage falls, employment may rise or fall.
Figure 11.23
The Equilibrium Effects of a Decrease in the Current Capital Stock

(a) $w = \text{Current Real Wage}$

(b) $r = \text{Real Interest Rate}$

$N = \text{Current Employment}$

$Y = \text{Current Output}$

$N_2^d$ $N_2^s(r_2)$

$N_1^d$ $N_1^s(r_1)$

$Y_1^d$ $Y_1^s$

$Y_2^d$ $Y_2^s$
Current Total Factor Productivity Increases

- Real interest rate falls, consumption and investment rise, employment rises, real wage rises.
- Productivity shocks are a potential explanation for business cycles – see Chapter 13.
Figure 11.24
The Equilibrium Effects of an Increase in Current Total Factor Productivity
Total Factor Productivity Expected to Increase in Future – A News Shock

- Output demand curve shifts right.
- Real interest rate rises, investment increases, consumption may rise or fall, employment rises, real wage falls, output rises.
- Important in explaining investment boom in the 1990s in the United States.
Figure 11.25
The Effect on Average Labor Productivity of an Increase in $z$. 

![Diagram showing the effect of an increase in $z$ on average labor productivity.](image)
Figure 11.26
The Equilibrium Effects of an Increase in Future Total Factor Productivity
Figure 11.27
Percentage Deviations from Trend in Investment and a Relative Stock Price Index, 1957–2012
Credit Market Frictions and the Financial Crisis

- One feature of the financial crisis was more severe credit market frictions – asymmetric information and limited commitment (as studied in Chapter 10).
- More severe credit market frictions reduce the demand for goods by consumers, and increase labor supply.
- Output demand curve shifts to the left, output supply curve to the right.
- Real interest rate falls, output could rise or fall (but demand effect should be stronger).
Figure 11.28
The Effect of More Severe Credit Market Frictions
Figure 11.29
The Effect of More Severe Credit Market Frictions
Sectoral Shocks and Labor Market Mismatch

• “Jobless recoveries” a feature of the last 3 recessions.
• Could be due to sectoral reallocation – changes in labor markets and the structure of production giving rise to movement of factors of production across sectors.
• Adds friction to the labor market – labor demand and labor supply curves shift to the left. Output supply shifts to the right.
Figure 11.30
The Effects of a Sectoral Shock
Figure 11.31
The Effects of a Sectoral Shock on Average Labor Productivity
Figure 11.32
Real GDP in Two Recessions
Figure 11.33
Employment in Two Recessions

- 1981–1982 Recession
- 2008–2009 Recession
Figure 11.34
Average Labor Productivity in Two Recessions

- 2008–2009 Recession
- 1981–1982 Recession

Quarters from Start of Recession:
- 0
- 2
- 4
- 6
- 8
- 10
- 12

Average Labor Productivity, First Quarter of Recession = 100