Industrial Transformation

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A. Introduction

Industrial transformation is a long process, from agriculture, to manufacture, and then to service. The modern literature can be summarized as follows:

- Transition to modern economy and industrial transformation:
  - Hansen-Prescott (2002); Gollin-Parente-Rogerson (2003, 2007)
  - Ashraf-Galor (2011): Malthusian epoch
  - Young (2012): African miracle

- Activation of the industrial transformation process:

- Industrial transformation and economic growth:
  - Kongsamut-Rebelo-Xie (2002): transformation from agriculture to manufactur and to service
  - Matsuyama (2002): transformation to a mass-consumption economy
  - Buera-Kaboski (2009): shift from home to market and service society
  - Herrendorf-Rogerson-Valentinyi (2013): multi-sector structural transformation to meet various stylized facts
  - Buera-Kaboski-Rogerson (2015): the rise of skill-intensive sectors
  - Hu-Kunieda-Nishimura-Wang (2019): flying geese or middle income trap

- East Asian NICs: rapid growth + drastic industrial transformation
- Some African, South American and South Asian countries: low-growth trap in traditional industries
- Examples:
  - The emperor's new clothes were not made in Colombia (Morawetz 1981)
  - Morogoro Shoe Factory in Tanzania was shut down not too long after opening
  - Both Akosombo Dam in Ghana and $2 billion US Aid in Zambia were failed (Easterly 2001)
- Questions:
  - What could be the barrier to activation of a modern industry?
  - Are there any public policies that could overcome this barrier to entry?
- Shortcomings of the literature:
  - Conventional studies lack a formal model to explain the activation process
  - Big push theory largely ignores the underlying process of creating a modern industry
Key Features of the paper:
- Production in the modern industry requires high-skilled labor in addition to new technologies
- Modern sector needs industrial coordination (industry-wide networking) => the scale barrier
- Modern goods are not necessary for survival

1. A Simple Illustration

- Two sectors (i=1,2): traditional industry (sector 1) versus modern (sector 2)
- Homogeneous labor + perfectly mobile capital
- Production technologies: \( Y_i = A_i K_i^\alpha L_i^{1-\alpha} \) \( A_2 > A_1 > 0 \)
- Preferences: \( U = \ln(C_1) + \ln(C_2 + \theta) \), where \( \theta > 0 \) (\( C_2 \) is luxury good, though on-going increase in the standard of living implies \( \theta \) decreases over time)
- Factor allocation (FA) constraints: \( L_1 + L_2 \leq L \) and \( K_1 + K_2 \leq F \), where \( L \) is fixed and \( F \) depends on foreign aid/FDI
- Material balance conditions: \( C_1 = Y_1 \) and \( C_2 = Y_2 \)
- Real GNP: \( \text{GNP} = Y_1 + pY_2 \) (good 1 is numeraire)
Optimization: max $U$ s.t. (FA)

- factor allocation: $\frac{K_1}{L_1} = \frac{K_2}{L_2} = \frac{F}{L}$
- equalization of VMPK across sectors: $\frac{\alpha}{K_1} = \frac{\alpha A_2 K_2^{\alpha-1} L_2^{1-\alpha}}{A_2 K_2^{\alpha} L_2^{1-\alpha} + \theta}$
- results: $K_2 = \frac{1}{2} \frac{A_2 F^\alpha L_1^{1-\alpha} - \theta}{A_2 F^{\alpha-1} L_1^{1-\alpha}}$
  - industry 2 will emerge iff $A_2 F^\alpha L_1^{1-\alpha} - \theta > 0$
  - activation of a modern industry requires sufficient funding and labor in addition to the technological factor
  - modern industry always emerge if $\theta = 0$

2. A Complete Framework with External Effect and Heterogenous Labor

- Modern industry exhibits social IRS in the Romer (1986) convention and requires skilled labor
- Production technologies: $Y_1 = A_1 K_1^{\alpha_1} L_1^{1-\alpha_1}$ and $Y_2 = A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} K_2^{1-\alpha_2}$, where $\alpha_2 > \alpha_1$ and $A_2 > A_1$
- Heterogeneous labor (low-skilled with mass $N_1$ and high-skilled with mass $N_2$):
  - $L_2 \leq N_2$ (modern production requires skills)
  - $L_1 + L_2 \leq N_1 + N_2$ (the skilled can downgrade to work as unskilled)
• **Funds allocation:** \( K_1 + qK_2 \leq F \), where \( q > 1 \) as it is more costly to invest in modern industry) – \( q \) captures the effects of investment subsidies/tax rebates and tariff reductions

3. **Competitive equilibrium:**

• **Equalization of relative price and MRS:**

\[
p = \frac{A_1 K_1^\alpha_1 L_1^{1-\alpha_1}}{A_2 K_2 L_2^{1-\alpha_2} + \theta}
\]

• **Equalization of MVKs** \((MPK_1 = \frac{p}{q}MPK_2)\):

\[
\frac{\alpha_1}{K_1} = \frac{\alpha_2 A_2 L_2^{1-\alpha_2}}{A_2 (F - K_1) L_2^{1-\alpha_2} + q\theta}
\]

• **Solution for K1:**

\[
K_1 = \frac{\alpha_1}{(\alpha_1 + \alpha_2)} \left[ F + \frac{q\theta}{A_2 L_2^{1-\alpha_2}} \right], \text{ implying the } K_1/F \text{ ratio is}
\]

○ decreasing in \( F \) and \( L_2 \)

○ increasing in \( q \) and \( \theta \)

• **Results:** Industry 2 remains non-operative if \( F-K_1 \leq 0 \), or, \( \frac{\alpha_1}{F} \geq \frac{\alpha_2 A_2 N_2^{1-\alpha_2} q^\theta}{q\theta} \), i.e., if

○ less funding or skilled labor (low \( F \) or \( N_2 \))

○ imperfect copy of advanced technology (low \( A_2 \))

○ too expensive modern capital investment (high \( q \))

○ too much preference bias (high \( \theta \))
In order for the modern industry to be activated, to have sufficient funding \((F-K_1>0)\) is necessary but not sufficient, as it does not account for skilled labor reallocation, driven by the relative wage \((\Omega=W_2/W_1)\):

\[
\Omega(L_2) = \frac{\alpha_1(1-\alpha_2)}{(1-\alpha_1)\alpha_2} \left[ \frac{L - L_2}{L_2} \right] \left[ \frac{(\alpha_1 + \alpha_2) A_2 L_2^{1-\alpha_2} F}{\alpha_1(A_2 L_2^{1-\alpha_2} F + q_\theta)} - 1 \right]
\]

- increasing in \(F\) and \(A_2\)
- decreasing in \(q, \theta, L_2\)

Staged development:

- **I**: \(\Omega(N_2)<0\): industry 2 is unprofitable
- **II**: \(0<\Omega(N_2)<1\): the relative wage is too low for skilled workers to participate in industry 2
- **III**: when \(\Omega(N_2)\geq1\): activation of the modern industry 2, which requires:
  - sufficient capital funding
  - sufficient skilled labor
  - good access to new technologies
  - appropriate matches between capital and labor
4. Development Policy Recommendation

- Short-term policy prescriptions:
  - external assistance to raise $F$ (EIB loans/U.S. Aid)
  - technological transfer from developed countries to advance $A_2$
  - immigration of high skill to increase $N_2$

- Long-term policy prescriptions:
  - better education/training
  - greater saving incentives
  - more R&D investments


- Endogenous skill accumulation
- Endogenous saving & funds/capital accumulation
- Endogenous modern technology advancement (accompanied by exogenous traditional technology evolution)
- Model: one-period non-overlapping generations with intergenerational transfers and transmissions
- Effects of better initial modern technology
- Effects of higher initial skills
• Effects of reduction in capital allocation barriers

- **Stylized fact:**
  - declining agriculture employment share
  - inverted-U manufacture employment share
  - rising service employment share
1. The Model

- Three sectors: A = agriculture, M = manufacture (numeraire), S = service
- Production technologies and factor allocation constraints:

\[ A_t = B_A F(\phi_t^A K_t, N_t^A X_t), \]

\[ M_t + \dot{K}_t + \delta K_t = B_M F(\phi_t^M K_t, N_t^M X_t), \]

\[ S_t = B_S F(\phi_t^S K_t, N_t^S X_t), \]

\[ \phi_t^A + \phi_t^M + \phi_t^S = 1, \]

\[ N_t^A + N_t^M + N_t^S = 1, \]

- Preference: \[ U = \int_0^\infty e^{-\rho t} \frac{[(A_t - \bar{A})^\beta M_t^\gamma (S_t + \bar{S})^\theta]}{1 - \sigma} - 1 \] \[ dt \] , implying:
  - A is necessity with a minimum subsistence level of consumption
  - S is luxury
• Resource constraint: \[ M_t + \dot{K}_t + \delta K_t + P_A A_t + P_S S_t = B_M F(K_t, X_t) \]
• Production/consumption efficiency and competitive profit conditions:
  o labor allocation: \[ \frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S} = 1 \]
  o capital efficiency: \[ r = B_M F_1(k, 1) - \delta \]
  o optimal consumption allocation: \[ \frac{P_A(A_t - \bar{A})}{\beta} = \frac{M_t}{\gamma} \] and \[ \frac{P_S(S_t + \bar{S})}{\theta} = \frac{M_t}{\gamma} \]
  o competitive profit: \[ P_A = B_M / B_A \] and \[ P_S = B_M / B_S \]
• Results:
  o constant manufacture output growth: \[ \frac{\dot{M}}{M} = \frac{r - \rho}{\sigma} = g \] (Keynes-Ramsey)
  o declining agriculture and rising service output growth:
    \[ \frac{\dot{A}_t}{A_t} = g \frac{A_t - \bar{A}}{A_t}, \quad \frac{\dot{S}_t}{S_t} = g \frac{S_t + \bar{S}}{S_t} \]
  o negative agriculture, zero manufacture and positive service employment growth:
    \[ \dot{N}_t^A = -g \frac{\bar{A}}{B_A X_t F(k, 1)}, \quad \dot{N}_t^M = 0, \quad \dot{N}_t^S = g \frac{\bar{S}}{B_S X_t F(k, 1)} \]
• Problem: fail to explain the inverted-U manufacture employment share
D. Transformation to a Mass-Consumption Economy: Matsuyama (2002)

- Main idea: activation of industries driven by demands
- Key Features:
  - content of luxuries changes over time
  - increased productivity reduces prices of goods and enlarges customer-base (scale economies)
  - trickle-down: scale effect from the high-income demand makes luxuries more affordable to the low-income
  - trickle-up: scale effect from the low-income demand reduces the high-income cost and enables the latter to move up to consume better goods
  - such processes require suitable level of inequality

1. The Basic Setup

- Sectors:
  - 0 = food (homogeneous, divisible)
  - 1-J = manufacturing (heterogenous, indivisible)
Preferences: \[ U = \begin{cases} c & \text{if } c \leq 1 \\ 1 + \sum_{k=1}^{k} \left( \prod_{j=1}^{j} x_j \right) + \eta l & \text{if } c > 1 \end{cases} \]

- good 0 (captured by c) is necessary up to a subsistence level (= 1)
- good i < j is needed for consuming j
- leisure is the numeraire that takes care of the jumps due to indivisibility of manufacturing goods (\(\eta p i < 1\))

Budget Constraint (BC): \[ p_0c + \sum_{j=1}^{\eta} p_j x_j + l \leq I. \]

2. Consumer Optimization and Aggregate Demand:

- Optimization problem:
  \[
  \max_{k \in \{1, \ldots, J\}} U = \begin{cases} c & \text{if } c \leq 1 \\ 1 + k + \eta l & \text{if } c > 1 \end{cases} \\
  \text{s.t. } p_0c + \sum_{j=1}^{k} p_j x_j + l \leq I.
  \]
● Consumption Pattern
  - low-income: \( I < P_0 \Rightarrow c = \frac{I}{P_0}, l = 0, x_j = 0 \) (\( j = 1, \ldots, J \))
  - middle-income: \( P_k \leq I < P_{k+1} \Rightarrow c = 1, l = I - P_k, x_j = 1 \) (\( j = 1, \ldots, k \)), \( x_j = 0 \) (\( j = k + 1, \ldots, J \))
  - high-income: \( I \geq P_j \Rightarrow c = 1, l = I - P_j, x_j = 1 \) (\( j = 1, \ldots, J \))

where \( P_k = \sum_{j=0}^{k} p_j \) = minimum income required for consuming good \( k \)

● Income Distribution \( F(I) \):
  - only those with \( I > P_k = \sum_{j=0}^{k} p_j \) can purchase \( j \)
  - such mass gives a measure of the aggregate demand (AD) for good \( j \):
    \[ C_j = 1 - F(P_j) = 1 - F\left(\sum_{i=0}^{j} p_i\right) \]
    - aggregate demand depends on distribution
    - only mass (not income) matters
    - Hicks-Allen demand complementarity from \( i \leq j \) to \( j \)
    - mass \( C_j \) is decreasing in \( j \) (as the degree of necessity becomes lower)
3. Production:

- Food is CRS; manufacture is IRS
  - unit labor requirements \( a_0(t) = a_0 \) and \( a_j(t) = A_j(Q_j(t)) \), where \( A_j' < 0 \) and the value of production is:

\[
Q_j(t) = \delta_j \int_{-\infty}^{t} C_j(s) \exp[\delta_j(s - t)]ds \leq 1
\]

with \( \delta_j \) captures both the speed of learning and the rate of depreciation of learning experience

- this implies the following learning dynamics (LD):

\[
\dot{Q}_j(t) = \delta_j[C_j(t) - Q_j(t)]
\]

4. Equilibrium

- Competitive Profit (CP) Conditions: \( p_0 = a_0, \ p_j(t) = A_j(Q_j(t)) \)

- (CP) + (AD) => \( C_j(t) = 1 - F\left(a_o + \sum_{i=1}^{j} A_i(Q_i(t))\right) = D_j(Q(t)), \) which together with (LD) yields, (DY):

\[
\dot{Q}_j(t) = \delta_j[D_j(Q(t)) - Q_j(t)] = \Psi_j(Q(t)), \text{ with } \Psi_{ij} \geq 0
\]
(complementarity)
Main findings:

- trickle-down:
  
  \[ \text{demand } C_j \uparrow \]
  
  \[ \Rightarrow \text{ } p_j \downarrow \]
  
  \[ \Rightarrow \text{ those initially with } I < I_j \text{ can now afford to consume } j \]

- trickle-up:
  
  \[ \text{demand } C_i \uparrow \]
  
  \[ \Rightarrow \text{ } p_i \downarrow \]
  
  \[ \Rightarrow \text{ those initially with } I_j > I_i \text{ can now save expenses on } i \text{ and move up to purchase } h > j \]


- Multi-sector DGE model of structural transformation to meet various stylized facts concerning, most importantly,
  
  - economic development
  
  - regional income convergence
  
  - aggregate productivity trends
  
  - hours worked
  
  - wage inequality
● Key indicators:
  ○ performance measure: real GDP per capita (not per worker)
  ○ structural transformation measure:
    - employment shares
    - value-added or consumption shares
      ■ benchmark: nominal shares – local currency for production or consumption
      ■ alternative: real shares – international goods/services flows

1. Stylized Facts

● Sectoral shifts:
  ○ agriculture down
  ○ manufacture hump-shaped
  ○ service up
- Developed countries: 1800-2000

![Graphs showing employment and value added across different sectors for developed countries from 1800 to 2000. The graphs illustrate the relationship between log of GDP per capita and the share of employment and value added in agriculture, manufacturing, and services.]
• 5 Non-EU and aggregate of 15 EU: 1970-2007
• 15 EU: 1970-2007
- Real vs. Nominal: 5 Non-EU and aggregate of 15 EU (1970-2007)
- World Bank vs. UN-PWT 6.3

**World Bank**

**UN-PWT 6.3**

![Graphs showing the relationship between log of GDP per capita and share in total employment for Agriculture, Manufacturing, and Services.](image)
• Consumption measures:
  - US/UK
  - ICP-PWT 6.3
2. The Basic Model

- Extending the two-sector growth model of Uzawa (1963) and Greenwood-Hurcowitz-Krussell (1997)

- Production
  - Production of consumption and investment goods:

  \[ C_t = k_{ct}^\theta (A_{ct} n_{ct})^{1-\theta} \]
  \[ X_t = k_{xt}^\theta (A_{xt} n_{xt})^{1-\theta} \]

  - Capital evolution:
  \[ K_{t+1} = (1 - \delta)K_t + X_t \]
  \[ K_t = k_{ct} + k_{xt} \]

  - Factor allocation:
  \[ 1 = n_{ct} + n_{xt} \]

  - Relative factor demand:
  \[ \frac{k_{it}}{n_{it}} = \frac{\theta}{1 - \theta} \frac{W_t}{R_t} = K_i \quad \forall \ i = c, x \]

  - Relative price and factor prices:
  \[ P_t = \left( \frac{A_{xt}}{A_{ct}} \right)^{1-\theta} \]
  \[ R_t = \theta K_t^{\theta-1} A_{xt}^{1-\theta} \]
  \[ W_t = (1 - \theta)K_t^\theta A_{xt}^{1-\theta} \]
- Aggregate output: 
\[ Y_t = X_t + P_t C_t = K_t^\theta (A_x)^{1-\theta} (n_x + n_c) = K_t^\theta A_{xt}^{1-\theta} \]

- Households
  - maximization:
    \[ \max_{\{C_t, K_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \log C_t \quad \text{s.t.} \quad P_t C_t + K_{t+1} = (1 - \delta + R_t)K_t + W_t \]
  - Euler:
    \[ \frac{P_t C_t}{\beta P_{t-1} C_{t-1}} = 1 - \delta + R_t \]

- Technological progress:
  \[ \frac{A_{it+1}}{A_{it}} = 1 + \gamma_i \quad \forall i = c, x \]

- Economic growth:
  \[ \frac{Y_{t+1}}{Y_t} = (1 + \gamma_x)^\theta (1 + \gamma_c)^{1-\theta} = 1 + \gamma_x \] (no common growth)

- BGP rental rate:
  \[ \frac{1}{\beta} (1 + \gamma_c) = 1 - \delta + R \]

- Unique initial factor proportion:
  \[ K_0 = \left[ \frac{\beta \theta}{(1 + \gamma_x) - \beta (1 - \delta)} \right]^{\frac{1}{1-\theta}} A_{x0} \]

3. The General Model

- Nonhomothetic consumption aggregator:
  \[ C_t = \left[ \omega^\delta_a (c_a - \tilde{c}_a)^{\epsilon - \delta} + \omega^\delta_m (c_m)^{\epsilon - \delta} + \omega^\delta_s (c_s + \tilde{c}_s)^{\epsilon - \delta} \right]^{\frac{\epsilon - \delta}{\epsilon - 1}} \]
4 sectors:

\[ c_{it} = k_{it}^\theta (A_{it} n_{it})^{1-\theta}, \quad i \in \{a, m, s\} \]

\[ X_t = k_{xt}^\theta (A_{xt} n_{xt})^{1-\theta} \]

Factor allocation:

\[ K_t = k_{at} + k_{mt} + k_{st} + k_{xt} \]

Relative factor demand:

\[ \frac{k_{it}}{n_{it}} = K_t \]

Relative price:

\[ p_{it} = \left( \frac{A_{xt}}{A_{it}} \right)^{1-\theta}, \quad i = a, m, s \]

Aggregate output:

\[ Y_t = p_{at} c_{at} + p_{mt} c_{mt} + p_{st} c_{st} + X_t = K_t^\theta A_{xt}^{1-\theta} \]

Household maximization:

\[
\max \left\{ \sum_{t=0}^{\infty} \beta^t \log \left[ \omega_a^\frac{1}{\varepsilon} (c_{at} - \bar{c}_a)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_m^\frac{1}{\varepsilon} (c_{mt})^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^\frac{1}{\varepsilon} (c_{st} + \bar{c}_s)^{\frac{\varepsilon-1}{\varepsilon}} \right] \right\}
\]

s.t. \[ p_{at} c_{at} + p_{mt} c_{mt} + p_{st} c_{st} + K_{t+1} = (1 - \delta + R_t)K_t + W_t \]

aetemporal condition:

\[ \frac{1}{C_t} = \lambda_t \left[ \omega_a (p_{at})^{1-\varepsilon} + \omega_m (p_{mt})^{1-\varepsilon} + \omega_s (p_{st})^{1-\varepsilon} \right]^{1-\varepsilon} \]

price aggregator:

\[ P_t \equiv \left[ \omega_a (p_{at})^{1-\varepsilon} + \omega_m (p_{mt})^{1-\varepsilon} + \omega_s (p_{st})^{1-\varepsilon} \right]^{1-\varepsilon} \]

budget allocation:

\[ p_{at} c_{at} + p_{mt} c_{mt} + p_{st} c_{st} = P_t C_t + p_{at} \bar{c}_a - p_{st} \bar{c}_s \]
- **Trick:** dividing optimization into two separate steps
  - **intertemporal:**
    $$\max_{\{c_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \log C_t \quad \text{s.t.} \quad P_t C_t + K_{t+1} = (1 - \delta + r_t)K_t + w_t - p_{at} \bar{c}_a + p_{st} \bar{c}_s$$
  - **atemporal:**
    $$\max_{c_{at}, c_{mt}, c_{st}} \left[ \frac{1}{\omega_a} (c_{at} - \bar{c}_a)^{\frac{1}{\varepsilon}} + \frac{1}{\omega_m} (c_{mt})^{\frac{1}{\varepsilon}} + \frac{1}{\omega_s} (c_{st} + \bar{c}_s)^{\frac{1}{\varepsilon}} \right]^{\varepsilon}$$
    $$\text{s.t.} \quad p_{at} c_{at} + p_{mt} c_{mt} + p_{st} c_{st} = P_t C_t + p_{at} \bar{c}_a - p_{st} \bar{c}_s$$
  - **optimizing conditions:**
    - **atemporal tradeoff:**
      $$\frac{p_{at}^\varepsilon}{p_{mt}^\varepsilon} \frac{c_{at} - \bar{c}_a}{c_{mt}} = \frac{\omega_a}{\omega_m}$$
      $$\frac{p_{st}^\varepsilon}{p_{mt}^\varepsilon} \frac{c_{st} + \bar{c}_s}{c_{mt}} = \frac{\omega_s}{\omega_m}$$
    - **relative expenditure:**
      $$\frac{P_t C_t}{p_{mt} c_{mt}} = \left[ \frac{\omega_a}{\omega_m} \left( \frac{A_{mt}}{A_{at}} \right)^{(1-\theta)(1-\varepsilon)} \right] + 1 + \frac{\omega_s}{\omega_m} \left( \frac{A_{mt}}{A_{st}} \right)^{(1-\theta)(1-\varepsilon)}$$

- **Special cases:**
  - **Kongsamut-Rebelo-Xie (2001):** \( \gamma_i = \gamma_j \) for all \( i, j = a, m, s \) and \( \varepsilon = 1 \)
  - **Ngai-Prescott (2007):** \( \bar{c}_a = \bar{c}_s = 0 \)
4. Quantitative Analysis

- Cross country differences: TFPs, capital shares, elasticities of substitution, degrees of necessity/luxury, relative incomes, relative prices
- Sectoral TFP
Key relationship: declines in agricultural employment shares captured by,

\[
1 - n_{at} = \frac{1 - s_a(c_{at})}{1 + p_R(A_{at}, A_{nt})s_k(k_{at}, k_{nt})s_X(c_{nt}, X_t)}
\]

- \( 1 - s_a(c_{at}) = 1 - \frac{\bar{c}_a}{c_{at}} \) captures the income effect
- \( p_R(A_{at}, A_{nt}) = \frac{\omega_a}{\omega_n} \left( \frac{A_{nt}}{A_{at}} \right)^{1-\varepsilon} \) captures the relative price/productivity effect
- \( s_k(k_{at}, k_{nt}) = \left( \frac{1 - \theta_a}{1 - \theta_n} \right) \left( \frac{k_{nt}^{\theta_n}}{k_{at}^{\theta_a}} \right)^{1-\varepsilon} \) captures the capital deepening effect
- \( s_X(c_{nt}, X_t) = \frac{X_t}{c_{nt} + X_t} \) captures the saving/investment effect a la Laitner

Key parametrization: follow Dennis-Iscan (2009) and Buera-Kaboski (2009)

Results:
- overall good fitness with data especially with a uptrend in \( \bar{c}_a \)
- income effect matters most prior to 1950, but afterward relative price/productivity and capital deepening effects become important

Question: insufficient analysis on the transition to service society with accelerated shares in many advanced economies during the post-WWII period
5. Extensions

- International trade: Matsuyama (2009) 2-country iceberg model of structural transformation, but abstracting from capital
- Labor mobility:
  - labor composition:
    - skill vs. unskilled (Acemoglu, Laing-Palivos-Wang & others)
    - manual vs. nonmanual (Autor & others)
  - occupational mobility vs. sectoral mobility: changing sectors could incur a cost as large as 75% of annual wage income (Lee-Wolpin 2006), but within sector occupational switch is much less costly
- Labor intensity:
  - labor hours: Prescott-McGrattan-Rogerson (2004), Rogerson (2008) – EU 5% higher than US in 1956, but 30% lower in 2003 due to higher taxes
- Goods mobility:
  - transport costs: Collin-Rogerson (2010), Adamopoulos (2011)

- After take-off but before reaching a sustained growth path, countries may experience in flying geese pattern of development (Akamatsu 1962; Baumol 1991) or fall into middle income traps a certain periods of time.
- Empirical evidence for the existence of middle income trap has been provided by Eichengreen, Park and Shin (2013) among others:
  - Middle income trap is identified as a substantive fall (2% or more) in per capita real income growth of a previously fast growing (3.5% or higher) middle income country (with per capita real income exceeding US$10,000 in 2005 constant PPP prices) for a considerable duration (7 year before and after the structural break).
  - Upon checking a number of possible drivers, they find that more educated at secondary and higher education levels is a robust factor leading to lower likelihood for a country to fall into the middle income trap.
- But there is no unified theory under which not only a poverty trap but a middle income trap may also exist.
- This paper fills the gap by proposing human capital/knowledge capital to play a key role in the possibility of falling into a middle income trap.
Model

Economy

- Identical infinitely lived agents.
  - The population is normalized to one.
- Time is discrete from 0 to $\infty$.
- A representative agent produces general goods.
  - Technology: Cobb-Douglas production function
  - Inputs: physical capital, labor, and human capital (more generally including knowledge capital and know-how)
  - The general goods are consumed or used for investments by the representative agent.
Investment projects

- The representative agent is endowed with two types of investment projects
  - To produce physical capital. A one-for-one simple linear technology produces capital from general goods.
  - To produce human capital. The human capital is also produced from general goods, but its formation is subject to technology choice: the representative agent chooses the best technology among a number of technologies for the human capital formation.
Agents

A representative agent solves the following maximization problem for her lifetime utility:

$$\max \sum_{\tau=t}^{\infty} \delta^{\tau-t} \ln c_{\tau}$$

subject to

$$y_{\tau} = Ah_{\tau-1}^{\alpha} k_{\tau-1}^{\beta} l_{\tau}^{\gamma} = c_{\tau} + i_{\tau}^{h} + i_{\tau}^{k}$$ \hspace{1cm} (1)

$$k_{\tau} = g_{0}(i_{\tau}^{k})$$ \hspace{1cm} (2)

$$h_{\tau} = \max_{m=1,2,\ldots,M} \{g_{m}(i_{\tau}^{h}; \bar{h}_{\tau-1}, \bar{y}_{\tau})\},$$ \hspace{1cm} (3)

for $\tau \geq t$, where $\delta \in (0, 1)$ is the subjective discount factor and $c_{\tau}$ is consumption. In Eq. (1), $\alpha, \beta, \gamma \in (0, 1)$ and $\alpha + \beta + \gamma = 1$. 
\[ y_\tau = A h_{\tau-1}^\alpha k_{\tau-1}^\beta l_\tau^\gamma \] the production function of general goods.

- \( h_{\tau-1} \): human capital, \( k_{\tau-1} \): physical capital, \( l_\tau \): labor.
- The production takes one gestation period (time-to-build).

In Eqs. (1)-(3), \( i^h_\tau \) and \( i^k_\tau \) are investments for production of human capital and physical capital, respectively.

Both physical and human capital depreciates entirely in one period.
Agents (cont.)

Physical capital production

In Eq. (2), the production function for physical capital, \( g(i^k_\tau) \), is given by

\[
g_0(i^k_\tau) = i^k_\tau,
\]

which implies that physical capital is produced from the general goods with a one-for-one technology.
Human capital production

In Eq. (3), $g_m(i^h; \bar{h}_{\tau-1}, \bar{y}_\tau)$ is a production function for human capital when the representative agent applies the $m$th technology.

The agent chooses the best technology for human (or knowledge) capital formation and solves her maximization problem, exogenously given the externalities from the (average) past human capital, $\bar{h}_{\tau-1}$, and the (average) current-period output, $\bar{y}_\tau$, in the economy.
Assumption

\[(i) \frac{\partial g_m(i^h; \bar{h}_{t-1}, \bar{y}_t)}{\partial i^h_t} > 0 \quad (ii) \frac{\partial^2 g_m(i^h; \bar{h}_{t-1}, \bar{y}_t)}{\partial i^h_t \partial \bar{h}_{t-1}} > 0, \quad (iii) \frac{\partial^3 g_m(i^h; \bar{h}_{t-1}, \bar{y}_t)}{\partial i^h_t \partial \bar{h}_{t-1}^2} < 0, \quad \text{and} \quad (iv) \frac{\partial^2 g_m(i^h; \bar{h}_{t-1}, \bar{y}_t)}{\partial \bar{h}_{t-1} \partial \bar{y}_t} < 0.\]

- Assumption 1-(i) guarantees the positive marginal product of human capital investment, \(i^h_t\).
- Assumption 1-(ii) implies that the past knowledge accumulation, which is condensed into the past human capital formation \(\bar{h}_{t-1}\), has a positive external effect on the marginal product promoting the human capital formation.
- Assumption 1-(iii) implies that the effect of the positive externality diminishes as the past knowledge more accumulates.
- Assumption 1-(iv) mitigates the scale effect of human capital investment, \(i^h_t\). We assume that the past knowledge accumulation more significantly contributes to the human capital formation than the scale expansion of an economy.
To make the following analysis concrete, we specify $g_m(i^h_T; h_{T-1}, \bar{y}_T)$ as

$$g_m(i^h_T; h_{T-1}, \bar{y}_T) = \frac{B_m(h_{T-1})}{\bar{y}_T} i^h_T, \quad (5)$$

where $B_m(h_{T-1}) := \theta_m(h_{T-1} - \eta_m)^\sigma$ for $h_{T-1} \geq \eta_m$, with $\alpha < \sigma \in (0, 1)$, $\theta_m \in [1, \infty)$, and $\eta_m \in [0, \infty)$. 
Human or knowledge capital accumulation depends on the society’s existing stock in the spirit of the knowledge spillovers in Romer (1986).

However, the externality of $h_{t-1}$ is effective only when it exceeds a certain border, $\eta_m$.

Thus, one may view the presence of $\eta_m$ as a result of scale barrier in knowledge accumulation.

- The scale barrier in human capital considered here also captures the argument in Buera and Kaboski (2012) where higher skill is required for production of goods with greater complexity.
- Our scale barrier setup generates similar implication to the appropriate technology model in Caselli and Coleman (2006) where skilled labor abundant rich countries tend to choose technologies more efficient to skilled workers.
Assumption

(i) \(1 = \theta_1 < \theta_2 < \cdots < \theta_M\) and (ii) \(0 = \eta_1 < \eta_2 < \cdots < \eta_M\).

We assume that there is a trade-off between the productivity, \(\theta_m\), and the scale barrier, \(\eta_m\).
Equilibrium

Transformed system:

\[ q_t^k = \frac{1}{\delta(1 - \gamma)} q_{t-1}^k - \frac{\beta}{1 - \gamma}, \]  

(E1)

where \( q_t^k := p_t^k k_t = \lambda_t k_t \), which is the value of physical capital in \( t \).

\[ q_t^h = \frac{1}{\delta(1 - \gamma)} q_{t-1}^h - \frac{\alpha}{1 - \gamma}, \]  

(E2)

where \( q_t^h := p_t^h h_t \), which is the value of human capital in period \( t \).
Equilibrium

From FOCs & Tech Choice, we have

\[ h_t = \alpha \max_m \{ B_m(h_{t-1}) \} \left[ \frac{1}{1 - \gamma} - \frac{\beta \delta}{1 - \gamma} \left( \frac{1}{q_{t-1}^k} \right) \right], \quad (E3) \]

which can then be used to obtain

\[ k_t = \beta Ah_{t-1}^\alpha k_{t-1}^\beta \left[ \frac{1}{1 - \gamma} - \frac{\beta \delta}{1 - \gamma} \left( \frac{1}{q_{t-1}^k} \right) \right]. \quad (E4) \]
Technology choice

Define such cutoffs as $w_1$ and $w_2$ so that $B_1(w_1) = B_2(w_1)$ and $B_2(w_2) = B_3(w_2)$. It is straightforward to show that $w_1$ and $w_2$ satisfy

$$w_1^\sigma = \theta_2(w_1 - \eta_2)^\sigma$$

and

$$\theta_2(w_2 - \eta_2)^\sigma = \theta_3(w_2 - \eta_3)^\sigma,$$

respectively, from which $w_1$ and $w_2$ are uniquely determined as $w_1 = \frac{1}{\theta_2^\sigma} \eta_2 / (\theta_2^\sigma - 1)$ and $w_2 = (\frac{1}{\theta_3^\sigma} \eta_3 - \frac{1}{\theta_2^\sigma} \eta_2) / (\theta_3^\sigma - \theta_2^\sigma)$.

Proposition

Suppose that Assumption 2 holds. Then, in $\max_m \{B_m(h_{t-1})\}$, the representative agent optimally chooses the first technology ($m = 1$) if $0 \leq h_{t-1} < w_1$, the second technology ($m = 2$) if $w_1 < h_{t-1} < w_2$, and the third technology ($m = 3$) if $w_2 < h_{t-1}$.
Eqs. (E1)-(E4) form the dynamical system in equilibrium (D):

\[
\begin{align*}
q_t^h &= \frac{1}{\delta(1-\gamma)} q_{t-1}^h - \frac{\alpha}{1-\gamma} \\
q_t^k &= \frac{1}{\delta(1-\gamma)} q_{t-1}^k - \frac{\beta}{1-\gamma} \\
h_t &= \alpha \max_m \{ B_m(h_{t-1}) \} \left[ \frac{1}{1-\gamma} - \frac{\beta \delta}{1-\gamma} \left( \frac{1}{q_{t-1}^k} \right) \right] =: J_1(q_{t-1}^k, h_{t-1}, k_{t-1}) \\
k_t &= \beta Ah_t^\alpha k_{t-1}^\beta \left[ \frac{1}{1-\gamma} - \frac{\beta \delta}{1-\gamma} \left( \frac{1}{q_{t-1}^k} \right) \right] =: J_2(q_{t-1}^k, h_{t-1}, k_{t-1}).
\end{align*}
\]
The values of human and physical capital in the steady state, $q^h*$ and $q^k*$, are given by

$$q^h* = \frac{\alpha \delta}{1 - \delta(1 - \gamma)} \quad \text{and} \quad q^k* = \frac{\beta \delta}{1 - \delta(1 - \gamma)}.$$
Steady states

Since $B_j(h_{t-1})$ is concave, we can “potentially” obtain at most two steady states, say, $h_{j,1}^*$ and $h_{j,2}^*$, from each technology.

Summary

Propositions 2-4 imply that six steady states (including a trivial one) exist in the dynamical system if the following inequalities (IN) hold:

$$(\delta \alpha)^{\frac{1}{1-\sigma}} < \eta_2 < (1 - \sigma)\sigma^{\frac{\sigma}{1-\sigma}} (\delta \alpha \theta_2)^{\frac{1}{1-\sigma}}$$

$$(\delta \alpha \theta_2)^{\frac{1}{1-\sigma}} < \eta_3 < (1 - \sigma)\sigma^{\frac{\sigma}{1-\sigma}} (\delta \alpha \theta_3)^{\frac{1}{1-\sigma}},$$

in which each technology yields two steady states.
Steady states

It is straightforward to extend the analysis to the case in which the number of technologies is $M$ with $1 = \theta_1 < \cdots < \theta_M$ and $0 = \eta_1 < \cdots < \eta_M$. In this case, $2M$ steady states appear if the following inequalities hold:

$$
\left( \delta \alpha \right)^{\frac{1}{1-\sigma}} < \eta_2 < (1 - \sigma)\sigma^{\frac{1}{1-\sigma}} \left( \delta \alpha \theta_2 \right)^{\frac{1}{1-\sigma}} \\
\left( \delta \alpha \theta_2 \right)^{\frac{1}{1-\sigma}} < \eta_3 < (1 - \sigma)\sigma^{\frac{1}{1-\sigma}} \left( \delta \alpha \theta_3 \right)^{\frac{1}{1-\sigma}} \\
\vdots \\
\left( \delta \alpha \theta_{M-1} \right)^{\frac{1}{1-\sigma}} < \eta_M < (1 - \sigma)\sigma^{\frac{1}{1-\sigma}} \left( \delta \alpha \theta_M \right)^{\frac{1}{1-\sigma}}.
$$
Local dynamics

Linearization of the dynamical system ($\mathcal{D}$) with the $j$th technology around one of the steady states, \((q^{h*}, q^{k*}, h^*_{j,s}, k^*_{j,s})\) (s = 1 or 2), obtains the Jacobian as follows:

\[
\begin{pmatrix}
\frac{1}{\delta(1-\gamma)} & 0 & 0 & 0 \\
0 & \frac{1}{\delta(1-\gamma)} & 0 & 0 \\
0 & J_{1,\lambda}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \alpha\delta B'_j(h^*_{j,s}) & 0 \\
0 & J_{2,\lambda}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & J_{2,k}(q^{k*}, h^*_{j,s}, k^*_{j,s}) & \beta
\end{pmatrix},
\]

where $J_{n,q^k}(q^k, h, k) := \partial J_n(q^k, h, k)/\partial \lambda$ and $J_{n,k}(\lambda, h, k) := \partial J_n(\lambda, h, k)/\partial k$ for $n = 1, 2$.

The eigenvalues of this dynamical system are given by
\[
\rho_1 := 1/(\delta(1-\gamma)), \rho_2 := 1/(\delta(1-\gamma)), \rho_3 := \alpha\delta B'_j(h^*_{j,s}), \text{ and } \rho_4 = \beta.
\]
Local dynamics

Proposition

*Under Assumption 2, suppose that inequalities (IN) are satisfied. Consider the jth technology. Then, the following hold:*

- **There exists no equilibrium sequence,** \( \{q^h_t, q^k_t, h_t, k_t\} \), **around the lower steady state,** \((q^{h*}, q^{k*}, h^*_j, k^*_j, 1)\), **that converges to this steady state.**

- **There exists a unique equilibrium sequence,** \( \{q^h_t, q^k_t, h_t, k_t\} \), **around the higher steady state,** \((q^{h*}, q^{k*}, h^*_j, k^*_j, 2)\), **that converges to this steady state.**
Global analysis

In the local analysis, it is still unclear whether an equilibrium sequence exists around the higher steady state. Moreover, even if an equilibrium sequence exists around the higher steady state, the local analysis does not clarify where it goes.
Global analysis

It follows from the transversality condition, 
\[ \lim_{u \to \infty} \delta^u q^k_{t+u} = \lim_{u \to \infty} \delta^u q^h_{t+u} = 0, \]
that
\[ \lim_{u \to \infty} [\delta(1 - \gamma)]^u q^k_{t+u} = \lim_{u \to \infty} [\delta(1 - \gamma)]^u q^h_{t+u} = 0. \]
Therefore, we obtain 
\[ q^k_t = \beta \delta / (1 - \delta (1 - \gamma)) \]
and 
\[ q^h_t = \alpha \delta / (1 - \delta (1 - \gamma)), \]
respectively, which implies that the equilibrium sequences of \( \{ q^k_t, q^h_t \} \) are uniquely determined, being equal to 
\[ \{ q^k_t, q^h_t \} = \{ q^k*, q^h* \}. \]
Global analysis

Then, from Proposition 1,

\[ h_t = \begin{cases} 
\alpha \delta h_{t-1}^\sigma & \text{if } 0 \leq h_{t-1} < w_1 \\
\alpha \delta \theta_2 (h_{t-1} - \eta_2)^\sigma & \text{if } w_1 \leq h_{t-1} < w_2 \\
\alpha \delta \theta_3 (h_{t-1} - \eta_3)^\sigma & \text{if } w_2 \leq h_{t-1}
\end{cases} \quad (h) \]

and

\[ k_t = \beta \delta A h_{t-1}^\alpha k_{t-1}^\beta. \quad (k) \]
Global analysis

Figure 1. Phase diagram of $h_t$
Figure 2. Conditional phase diagram of $k_t$ given $h_{t-1}$
Proposition

Suppose that Assumption 2 holds. Then, there exist only two steady states in the dynamical system, which are a trivial one and the high steady state of the third technology if and only if the following parameter condition holds:

\[(\delta x) \frac{1}{1-\sigma} > \max\{\Phi_1, \Phi_2\},\]

where

\[\Phi_i := \frac{\left(\theta_i^{\frac{1}{\sigma}} \eta_i + 1 - \theta_i^{\frac{1}{\sigma}} \eta_i\right)^{\frac{1}{1-\sigma}}}{\theta_i^{\frac{1}{1-\sigma}} \theta_{i+1}^{\frac{1}{1-\sigma}} \left(\theta_i^{\frac{1}{\sigma}} - \theta_{i+1}^{\frac{1}{\sigma}}\right) \left(\eta_i + 1 - \eta_i\right)^{\frac{\sigma}{1-\sigma}}}.\]
Take-off and flying geese

Figure 3. Flying geese pattern
Define the set of all globally available technologies as $T := \{1, \cdots, M\}$ and the set of “interior” technologies as $I := \{2, \cdots, M - 1\}$.

**Proposition**

Suppose that Assumption 2 holds. Then, there exist nontrivial steady states in the dynamical system, featuring middle income trap at various technologies in $J \subseteq I$ if and only if the following parameter condition holds:

$$
\min_{j \in J} \{ \Phi_j \} > (\delta \alpha)^{\frac{1}{1-\sigma}} > \max_{m \in T \setminus J} \{ \Phi_m \}.
$$

(C)
Middle income trap

Figure 4. Trap in the $j$th technology
Middle income trap

Figure 5. Restricted middle income trap
The necessary condition for trap at the jth technology is:

\[
\frac{\left( \theta_{j+1}^{\frac{1}{\sigma}} \eta_{j+1} - \theta_j^{\frac{1}{\sigma}} \eta_j \right)^{\frac{1}{1-\sigma}}}{\theta_j^{\frac{1}{1-\sigma}} \theta_{j+1}^{\frac{1}{1-\sigma}} \left( \theta_{j+1}^{\frac{1}{\sigma}} - \theta_j^{\frac{1}{\sigma}} \right) (\eta_{j+1} - \eta_j)^{\frac{\sigma}{1-\sigma}}} > (\delta \alpha)^{\frac{1}{1-\sigma}}
\]
Middle income trap

Let $g_{\theta_j} = \frac{\theta_{j+1}}{\theta_j}$ and $g_{\eta_j} = \frac{\eta_{j+1}}{\eta_j}$ capture, respectively, the productivity gap and the barrier gap between the $j+1$th and the $j$th technologies. Then the above inequality reduces to

$$\frac{(\eta_j)^{1-\sigma} \left[ (g_{\theta_j})^{\frac{1}{\sigma}} g_{\eta_j} - 1 \right]}{\theta_j (g_{\theta_j})^{\frac{1}{\sigma}} (g_{\eta_j} - 1)^{\sigma}} > \delta \alpha$$

It is straightforward to show that the left-hand side of this inequality is strictly decreasing in $\theta_j$ and strictly increasing in $\eta_j$ and $g_{\theta_j}$.

Proposition

Suppose that Assumption 2 holds. Then, a middle income trap of the $j$th technology for $1 < j < M$ is likely to arise if the productivity of the $j$th technology is not too large, the scale barrier of the $j$th technology is sufficiently high and the productivity gap between the $j+1$th and the $j$th technologies is sufficiently big.
Calibration

Solving \((\theta_j, \eta_j)\) from:

\[
\left( \frac{y_{t+1}}{A_{t+1}} \right)^{\frac{1}{\alpha}} (\beta \delta y_t)^{-\frac{\beta}{\alpha}} = \alpha \delta \theta_j \left( \left( \frac{y_t}{A_t} \right)^{\frac{1}{\alpha}} (\beta \delta y_{t-1})^{-\frac{\beta}{\alpha}} - \eta_j \right)^\sigma
\]

and

\[
\eta_j = \frac{(\beta \delta y_{t-1})^{-\frac{\beta}{\alpha}} (y_{t-1}/A_{t-1})^{\frac{1}{\alpha}} \left( \left( \frac{y_{t+2}/A_{t+2}}{y_{t+1}/A_{t+1}} \right)^{\frac{1}{\alpha}} \left( \frac{y_{t+1}}{y_t} \right)^{-\frac{\beta}{\alpha}} \right)^{\frac{1}{\sigma}} - (y_{t+1}/A_{t+1})^{\frac{1}{\alpha}} \left( \frac{y_t}{y_{t-1}} \right)^{-\frac{\beta}{\alpha}}}{\left( \left( \frac{y_{t+2}/A_{t+2}}{y_{t+1}/A_{t+1}} \right)^{\frac{1}{\alpha}} \left( \frac{y_{t+1}}{y_t} \right)^{-\frac{\beta}{\alpha}} \right)^{\frac{1}{\sigma}}} - 1
\]

Use data from PWT9.0, with 3 year MA and examine in 5 year intervals to identify flying geese periods and middle income trap
Calibration Result

- **Advanced countries**
  - US: trapped mid 60s/early 70s/early 80s/early 2000s; flying otherwise
  - UK: trapped late 70s-late 80s/2000s; flying otherwise
  - Germany: trapped late 70s-early 80s/mid-late 2000s; flying otherwise
  - Japan: trapped early 70s/early 80s/late 90s; flying otherwise

- **Fast growing economies:**
  - Hong Kong: trapped late 1990s; flying otherwise
  - Singapore: trapped late 70s-early 80s/late 90s; flying otherwise
  - South Korea: trapped early 70s/late 2000s; flying otherwise
  - Taiwan: trapped late 70s-early 80s; flying otherwise

- **Emerging growing economies:**
  - China: trapped late 50s-early 80s/mid 90s; flying mostly after mid 80s
  - Malaysia: trapped early 70s/late 90s; flying otherwise
  - Mexico: trapped late 60s/late 70s-80s/early 2000s; flying before mid 60s

- **Development laggards:**
  - Argentina: trapped early 60s/most of 70s and 80s; not much flying
  - Greece: trapped late 70s-early 90s/most of 2000s; flying before early 70s
  - Philippines: trapped mid 60s/mid 70s/mid 80s/mid 90s/late 2000s; not much flying